Chapter 1

Concept Application Exercises

Exercise 1.1

1. (i) The mass distribution is symmetrical about O. Hence the centre of mass remains at O as the excluded masses are symmetrically placed.

(ii) The mass distribution is symmetrical about the diagonal line passing through corner 1 and 3. The centre of mass shift towards greater mass between O to 3 diagonally.

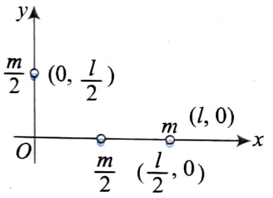
(iii) The mass distribution is symmetrical about Y-axis. The centre of mass shift towards greater mass along OY'.

(iv) The mass distribution is symmetrical about O. The centre of mass does not shift. It will remain at O.

(v) The mass distribution is symmetrical about the diagonal line passing through corner 2 and 4. The centre of mass shift towards greater mass between O to 4 diagonally. CM shifts diagonally from O to 4.

(vi) The mass distribution is symmetrical about O. The centre of mass does not shift. It will remain at O.

2. Convert the rod as the combination of two point masses $\frac{m}{2}, \frac{m}{2}$ placed at $x = \frac{l}{2}$ and $y = \frac{l}{2}$. Then this system consists three particles. Hence,

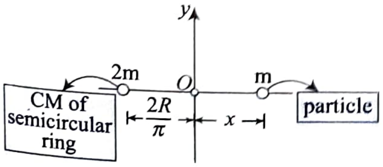


$$x_{cm} = \frac{\left(\frac{m}{2}\times 0 + \frac{m}{2}\times\frac{l}{2} + m\times l\right)}{\left(\frac{m}{2}+\frac{m}{2}+m\right)} = \frac{5l}{8}$$

$$\text{and } y_{cm} = \frac{\left(\frac{m}{2}\times\frac{l}{2} + \frac{m}{2}\times 0 + m\times 0\right)}{\left(\frac{m}{2}+\frac{m}{2}+m\right)} = \frac{l}{8}$$

the position of CM is $\left(\frac{5l}{8}, \frac{l}{8}\right)$.

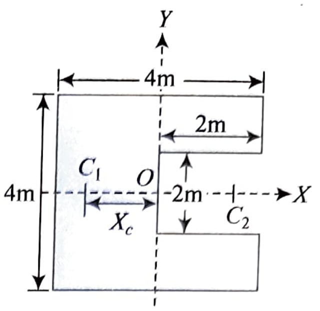
3. The ring is substituted by a particle of mass m at a distance $\frac{2R}{\pi}$ from the origin.



Now, the CM of the two-particle system is at origin, hence

$$2m\left(-\frac{2R}{\pi}\right) + mx = 0 \text{ or } x = \frac{4R}{\pi}$$

4.



The plate is symmetric about x-axis, thus the CM must be on the x-axis.

The CM of the complete plate (uncut) will be at origin and the CM of the cut-out will be at position (1m, 0).

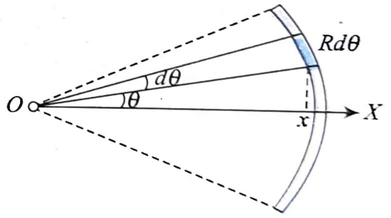
As the plate has uniform thickness, its mass will be proportional to its area.

Hence the position of centre of mass of the given structure.

$$x_{cm} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2} = \frac{4^2\times 0 - 2^2\times 1}{4^2 - 2^2} = -\frac{1}{3}\text{m s}$$

Hence the coordinate of CM is $\left(-\frac{1}{3}\text{m}, 0\right)$.

5. Considering element of angular width $d\theta$ as shown in figure, its x coordinate is $R\cos\theta$ and its mass is $dm = \frac{m}{\phi}\cdot d\theta$



Thus centre of mass of arc is given as

$$x_c = \frac{1}{m}\int x\,dm = \frac{1}{m}\int_{-\phi/2}^{+\phi/2}\frac{m}{\phi}\,d\theta(R\cos\theta)$$

$$= \frac{R}{\phi}\left(2\sin\frac{\phi}{2}\right) = \frac{2R\sin\frac{\phi}{2}}{\phi}$$

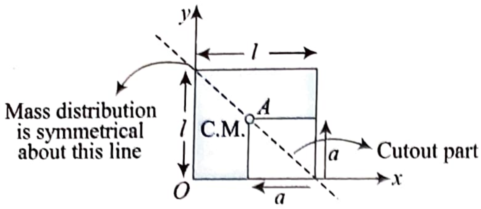
6. The mass distribution is symmetrical about x-axis, hence y-coordinate of centre of mass will be zero.

$$x_{cm} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

$$= \frac{(2r^2)\frac{r}{2} - \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)}{(2r^2) - \left(\frac{\pi r^2}{2}\right)} = \frac{2r}{3(4-\pi)}$$

Hence, the coordinate of centre of mass is $\left(\frac{2r}{3(4-\pi)}, 0\right)$.

7.



$$y_{\text{cm}} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{l^2 \times \dfrac{l}{2} - a^2 \times \dfrac{a}{2}}{l^2 - a^2} = a$$

$$\Rightarrow \quad l^3 - a^3 = 2a(l^2 - a^2)$$

$$\Rightarrow \quad (l-a)(l^2 + a^2 + la) = 2a(l+a)(l-a)$$

$$\Rightarrow \quad l^2 - al - a^2 = 0 \qquad \text{...(i)}$$

On solving Eq. (i) we get, $a = \dfrac{(\sqrt{5}-1)l}{2}$

Exercise 1.2

1. Block A slides with acceleration $g/2$ and block B slides with acceleration $\sqrt{3}g/2$. Now the acceleration of centre of mass of the system of blocks A and B can be given both in x and in y directions as

$$a_x = \frac{3 \times \dfrac{\sqrt{3}\,g}{2} \cos 60^\circ - 2 \times \dfrac{g}{2} \cos 30^\circ}{5} = \frac{\sqrt{3}g}{20}$$

and $$a_y = \frac{3 \times \dfrac{\sqrt{3}g}{2} \sin 60^\circ + 2 \times \dfrac{g}{2} \sin 30^\circ}{5} = \frac{11g}{20}$$

Thus acceleration of the centre of mass of the system is given as

$$a_{\text{CM}} = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(\frac{\sqrt{3}g}{20}\right)^2 + \left(\frac{11g}{20}\right)^2} = \frac{\sqrt{31}}{10}\,g$$

2. If m_1 and m_2 are placed at a distance l apart, their centre of mass will be located at a distance x from m_1, where

$$x = \frac{m_2 l}{m_1 + m_2}$$

If m_1 is displaced by l_1 towards C and m_2 is displaced by l_2 away from C, the new centre of mass C' now will be located at a distance x' from m_1, where

$$x' = \frac{m_2(l - l_1 + l_2)}{m_1 + m_2}$$

Displacement of the centre of mass is

$$\Delta x = x' + l_1 - x = \frac{m_1 l_1 + m_2 l_2}{m_1 + m_2}$$

3. First we write the three velocities in vectorial form, taking right direction as positive x-axis and upwards as positive y-axis.

$$\vec{v}_1 = -\frac{1}{2} v_1 \hat{i} - \frac{\sqrt{3}}{2} v_1 \hat{j}$$

$$\vec{v}_2 = v_2\, \hat{i}, \quad \vec{v}_3 = -\frac{1}{2} v_3 \hat{i} + \frac{\sqrt{3}}{2} v_3 \hat{j}$$

Thus the velocity of centre of mass of the system is

$$\vec{v}_{\text{CM}} = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3}{3}$$

$$= \left(v_2 - \frac{1}{2} v_1 - \frac{1}{2} v_3\right)\hat{i} + \frac{\sqrt{3}}{2}(v_3 - v_1)\hat{j}$$

which can be written as $\vec{v}_{\text{CM}} = v_x \hat{i} + v_y \hat{j}$

Thus displacement of the centre of mass in time t is

$$\Delta\vec{r} = v_x t\, \hat{i} + v_y t\, \hat{j}$$

If $v_1 = v_2 = v_3 = v$, we have

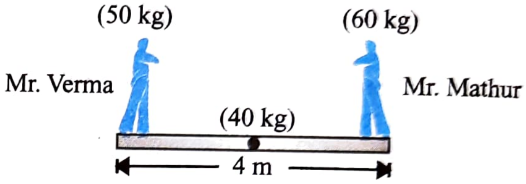
$$\vec{v}_{\text{CM}} = 0$$

Therefore, there is no displacement of centre of mass of the system.

4. When the ball falls into the hole, with respect to the flat car, the ball travels a horizontal distance l. During this motion, to conserve momentum and to maintain the position of centre of mass, the car moves towards left, say by a distance x. Thus, the total distance travelled by the ball towards right is $(l - x)$. As centre of mass remains at rest, the change in mass moments of the two (ball and car) about any point must be equal to zero. Hence

$$m(l - x) = Mx \Rightarrow x = \frac{ml}{M + m}$$

5.



Let displacement of the boat is $x_b = x$ towards right

The displacement of Mr. Verma: $x_v = (x + 2)$ m

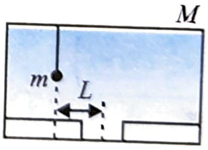
The displacement of Mr. Mathur: $x_m = (x - 2)$ m

As no external forces are acting on the system, the displacement of the centre of mass of the system should be zero.

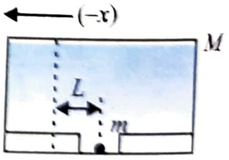
$$\Delta x_{\text{CM}} = \frac{M_b x_b + M_v x_v + M_m x_m}{(M_b + M_v + M_m)}$$

$$0 = \frac{40x + 50\,(x+2) + 60(x-2)}{150} \Rightarrow x = \frac{2}{15}\text{ m}$$

6. Let displacement of the cart be $x_c = -x$



Displacement of the bob is



$$x_b = (L - x)$$

$$(x_{\text{CM}})_{\text{system}} = 0$$

$$M_{\text{cart}}\, x_{\text{cart}} + M_{\text{bob}}\, x_{\text{bob}} = 0$$

$$M(-x) + m(L - x) = 0$$

$$x = \frac{mL}{m + M}$$

7. Let wedge move towards right by distance x; then

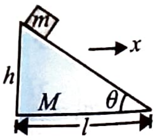
$$\Delta x_{\text{CM}} = \frac{m(l\cos\theta + x) + Mx + m(x - l)}{m + M + m}$$

$$0 = ml\cos\theta - ml + (m + M + m)x$$

$$x = \frac{ml - ml\cos\theta}{2m + M}$$

8. $l = h\cos\theta$

Let displacement of wedge be x towards right:



$$\Delta x_{\text{CM}} = \frac{m(l + x) + Mx}{M + m}$$

$$0 = ml + (M+m)x$$

$$x = \frac{-ml}{M+m} = -\frac{mh\cos\theta}{M+m}$$

9. $$\Delta x_{CM} = \frac{Mx + m(R - r + x)}{M+m}$$

$$0 = Mx + mx + m(R-r)$$

$$x = \frac{-m(R-r)}{M+m}$$

10. $$m_1\,\Delta\vec{x}_1 + m_2\,\Delta\vec{x}_2 = 0$$

$$m(R\sin\theta - x) - Mx = 0$$

$$x = \frac{mR\sin\theta}{m+M}$$

11. As no external force is acting on the system, the displacement of the centre of mass should be zero.

Let the displacement of the platform toward right be s,

$$\Delta\vec{x}_1 = \Delta\vec{x}_{1,p} + \Delta\vec{x}_p = (-x_1 + s)$$

$$\Delta\vec{x}_2 = \Delta\vec{x}_{2,p} + \Delta\vec{x}_p = (l - x_1 + s)$$

Hence,

$$0 = \frac{m_1(-x_1+s) + m_2(l - x_1 + s) + m_0 s}{m_1 + m_2 + m_0}$$

$$s(m_1 + m_2 + m_0) - m_1x_1 + m_2(l - x_1) = 0$$

$$s = \frac{m_1x_1 - m_2(l - x_1)}{m_1 + m_2 + m_0}$$

12. Initial coordinates of CM = (0, 0, 0)

x coordinates of CM after 3 s,

$$x_{CM} = 120 \times 3 = 360 \text{ m}$$

y coordinates of CM after 3s,

$$y_{CM} = -\frac{1}{2}gt^2 = -45 \text{ m}$$

After 3 s,

$$(m_1 + m_2)x_{CM} = m_1x_1 + m_2x_2$$

$$x_2 = \frac{30 \times 360 - 12 \times 300}{18} = 400 \text{ m}$$

Similarly, $(m_1 + m_2)\,y_{CM} = m_1y_1 + m_2y_2$

$$-30 \times 45 = 12 \times 24 + 18 \times y_2$$

$$y_2 = -91 \text{ m}$$

and in z coordinate, $m_1z_1 + m_2z_2 = 0 \Rightarrow z_2 = 32$ m

13. As no external force acts in z direction, hence z-coordinate of the centre of mass of the ball should be zero. To make z-coordinate zero other ball should fall symmetrically with respect to z-axis.

Hence, z-coordinate of other ball = –5 m.

The balls do not have any external force in x-direction. Hence in x-direction the centre of mass should move with constant velocity.

x-coordinate of centre of mass at $t = 2.0$ s = 200 × 2 = 400 m

Hence, $$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$400 = \frac{20 \times 250 + 20\,x_2}{20 + 20}$$

$$x_2 = 800 - 250 = 550 \text{ m}$$

Height fallen by centre of mass at $t = 2$ s,

$$h = \frac{1}{2} \times 10 \times (2)^2 = 20 \text{ m}$$

Hence, y-coordinate of centre of mass = 30 – 20 = 10 m

Hence, $$y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

$$10 = \frac{20 \times 0 + 20 \times y_2}{20 + 20}$$

$$\Rightarrow \quad y_2 = 20 \text{ m}$$

14. Initially, the whole system is at rest, so $v_{CM} = 0$. As there is no external force acting on the system, $\vec{v}_{CM}$ = constant = 0. So position of centre of mass of system remains fixed.

$$x_{CM} = \frac{mx_1 + Mx_2}{m + M} \qquad \text{...(i)}$$

where m is mass of the cannon balls and M that of the (car + cannon) system.

As $\Delta x_{CM} = 0$, therefore

$$m\,\Delta x_1 + M\Delta x_2 = 0 \qquad \text{...(ii)}$$

As cannon balls cannot leave the car, so maximum displacement of the balls relative to the car is L and in doing so the car will shift a distance $\Delta x_2 = D$ (say) relative to the ground, opposite to the displacement of the balls; then the displacement of balls relative to ground will be

$$\Delta x_1 = L - D \qquad \text{...(iii)}$$

Substituting the value of Δx_1 from Eq. (iii) in Eq. (ii), we get

$$m(L - D) - MD = 0$$

$$\Rightarrow \quad D = \frac{mL}{M+m} = \frac{L}{1 + \frac{M}{m}}$$

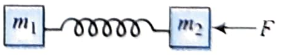
$$\Rightarrow \quad D < L$$

i.e., rail road car cannot travel more than L.

Exercise 1.3

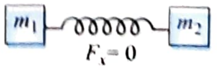
1. The net external force acting on the ball/earth system is zero. The gravitational forces that the ball and earth exert on each other are internal forces, or forces that the objects within the system exert on each other. The space probe is also an isolated system, since there are no external forces acting on it. Hence, only (ii) and (v) are isolated systems.

2. (a) When the spring is compressed, the vertical wall pushes the block with a force F (say) to counter-act the spring force on m_2. Hence, the net external force F acts on the system (m_1 + spring + m_2). Hence, we cannot conserve the linear momentum of the system.



(a)

(b) In this case (II), there is no external force acting on the system. Hence, we can conserve momentum of the system in horizontal direction, which is equal to $m_1v_0\hat{i}$.



(b)

3. Here x is the displacement, v is the speed, p is the momentum, a is the acceleration, KE is the kinetic energy, F is the force.

$$(F_{ext})_{system} = 0 \Rightarrow \vec{p}_1 + \vec{p}_2 = 0$$

$$m_1v_1 = m_2v_2 \Rightarrow \frac{v_1}{v_2} = \frac{m_2}{m_1}$$

$$m_1\int_0^t v_1 dt = m_2\int_0^t v_2 dt$$

where t is the time interval.

$$m_1x_1 = m_2x_2 \Rightarrow \frac{x_1}{x_2} = \frac{m_2}{m_1}$$

$$\frac{KE_1}{KE_2} = \frac{p_1^2/2m_1}{p_2^2/2m_2} = \frac{m_2}{m_1} \qquad (\because p_1 = p_2)$$

$$\frac{F_1}{F_2} = \frac{kx}{kx}$$

where x is the deformation of the spring.

$$\therefore \quad \frac{F_1}{F_2} = 1 \Rightarrow \frac{m_1a_1}{m_2a_2} = 1 \Rightarrow \frac{a_1}{a_2} = \frac{m_2}{m_1}$$

4. (a) Linear momentum is conserved since there are no external forces acting on the system. The fragments go off in different directions and their vector momenta add to zero.

(b) Kinetic energy is not conserved because the chemical potential energy initially in the explosive is converted into kinetic energy of the pieces of the bomb.

5. Momentum conservation is not violated if we choose as our system the planet along with you. When you receive an impulse forward, the Earth receives the same size impulse backwards. The resulting acceleration of the Earth due to this impulse is much smaller than your acceleration forward, but the planet's backward momentum is equal in magnitude to your forward momentum.

6. In empty space, the centre of mass of a rocket plus fuel system does not accelerate during a burn, because no outside force acts on this system. The rocket body does accelerate as it blows exhaust containing momentum out of its back.

According to the text's basic expression for rocket propulsion, the change in speed of the rocket body will be larger than the speed of the exhaust relative to the rocket, if the final mass is less than 37% of the original mass.

7. Here net force on the block will be zero, hence its momentum or speed will remain conserved.

8. The velocity of shell at the highest point is

$$v = u \sin\theta = 100 \times \sin 30° = 50 \text{ m/s}$$

Let m be the mass of the shell. Then the mass of the lighter fragment is $m/3$ and that of heavier fragment is $2m/3$.

Initial momentum of the shell before explosion is

$$mv = 50m$$

As no external force is acting on the shell, we can conserve momentum of the shell before and after its explosion.

Note: An external force of gravity is present here during explosion but as explosion is an instantaneous phenomenon, so due to very short duration impulse of gravity is negligible and cannot cause any change in momentum. Thus in all very short duration explosions or collisions, we always ignore the presence of gravity just before and after the occurrence.

In x-direction,

$$mv = \frac{2m}{3}v_2\cos\theta \Rightarrow v_2\cos\theta = \frac{3}{2}v \qquad \text{...(i)}$$

in y-direction,

$$0 = \frac{m}{3}v_1 - \frac{2m}{3}v_2\sin\theta \Rightarrow v_2\sin\theta = \frac{v_1}{2} \qquad \text{...(ii)}$$

Squaring and adding Eqs. (i) and (ii), we get

$$v_2^2 = \frac{9}{4}v^2 + \frac{1}{4}v_1^2$$

$$\Rightarrow \quad v_2 = \frac{1}{2}\sqrt{9v^2 + v_1^2} = \frac{1}{2}\sqrt{9 \times 2500 + 40000} = 125 \text{ m/s}$$

9. Apply conservation of momentum:

$$\Rightarrow \quad m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = 0$$

$$m(2\hat{i} + 5\hat{j} - 6\hat{k}) + 2m\vec{v}_2 + m(-4\hat{i} + 3\hat{j} + 2\hat{k}) = 0$$

$$\vec{v}_2 = \hat{i} - 4\hat{j} + 2\hat{k}$$

10. (a) Let v_1 be the new velocity of the car. Then according to the principle of conservation of linear momentum,

$$Mu = (M + m)v_1$$

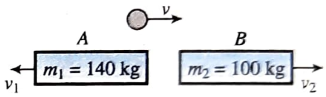
$$\Rightarrow \quad v_1 = \frac{m}{M+m}u \qquad \text{...(i)}$$

(b) Let v_2 be the new velocity of the car relative to the earth; then velocity of the man relative to the earth $= (v_2 - v_{rel})$

$$\therefore \quad (M + m)v_1 = Mv_2 + m(v_2 - v_{rel})$$

$$\therefore \quad v_2 = v_1 + \frac{mv_{rel}}{M+m}$$

11. Applying conservation principle of momentum for trolley A and bag



$$mv - m_1v_1 = 0$$

or $\quad mv = m_1v_1$

or $\quad 10v = 140\,v_1$

or $\quad v = 14v_1 \qquad \text{...(i)}$

also, $\quad v_1 + v = 4$

or $\quad v_1 + 14v_1 = 4$

$$\therefore \quad v_1 = \frac{4}{15}\text{ms}^{-1},\ v = 14v_1 = \frac{56}{15}\text{ms}^{-1}$$

Applying conservation principle of momentum for bag and trolley B,

$$mv = (m + m_2)v_2$$

$$\therefore \quad v_2 = \frac{mv}{m+m_2} = \frac{10 \times \frac{56}{15}}{110} = \frac{56}{15 \times 11}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{4}{15 \times 56} \times 15 \times 11 = \frac{11}{14}$$

12. In the problems involving mass ejection and mass addition, the ejected and added mass both are part of a system. When the men jump they push on the floor of the buggy (action); the same force acts on them as well (reaction).

This action–reaction pair is internal force for the system of man and buggy. Similarly, when men jump in, there is an action–reaction force between them and the buggy.

Let buggy '1' moves to the right and buggy '2' to the left, when the men jump. Man in '1' brings momentum, mv_1 directed towards right in buggy '2'.

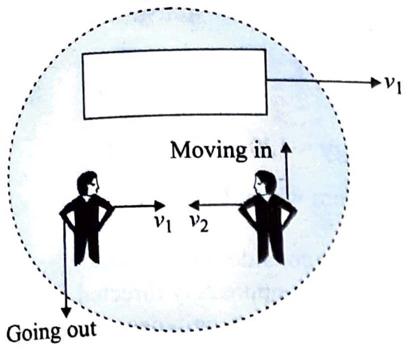
Similarly, the man in '2' brings momentum mv_2 in '2' directed towards left.

For system 1:

$$p_i = (M + m)v_1 - mv_1 - mv_2 = Mv_1 - mv_2$$

$$p_f = 0$$

From law of conservation of momentum,



System 1

$$p_i = p_f$$

$$Mv_1 - mv_2 = 0 \qquad \text{...(i)}$$

For system 2:

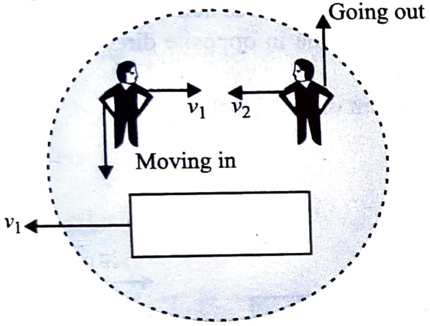
$$p_i = (M+m)\,v_2 - mv_1 - mv_2$$

$$p_f = (M+m)\,v$$

From law of conservation of momentum,

$$p_i = p_f$$

$$(M+m)v_2 - mv_1 - mv_2 = (M+m)v \qquad \text{...(ii)}$$



System 2

On solving Eqs. (i) and (ii) simultaneously, we obtain

$$v_1 = \frac{mv}{(M-m)};\ v_2 = \frac{Mv}{(M-m)}$$

13. (a) Let v be the velocity of the car relative to the earth, when the man runs to the left, then conservation of linear momentum gives

$$(M+m)v_0 = Mv + m(v - v_{\text{rel}})$$

$$\therefore \quad v = v_0 + \frac{m}{M+m}v_{\text{rel}} \qquad \text{...(i)}$$

Change in velocity, $\Delta v = v - v_0 = \dfrac{mv_{\text{rel}}}{M+m}$

(b) When there are n men on the car and they jump off together, then velocity of the car after jumping off, from Eq. (i), will be

$$v_1 = v_0 + \frac{mn}{M+mn}v_{\text{rel}} \qquad \text{...(ii)}$$

If the men jump one by one, the mass of the system will go on changing till the last man jumps.
Let v' be the velocity when one man jumps off the car, then from Eq. (ii)

$$v' = v_0 + \frac{m}{M+(n)\,m}v_{\text{rel}}$$

When the second man also jumps off, then

$$v'' = v' + \frac{m}{M+(n-1)\,m}v_{\text{rel}}$$

$$= v_0 + \frac{mv_{\text{rel}}}{M+(n)\,m} + \frac{mv_{rel}}{M+(n-1)\,m}$$

When the last man jumps, then final velocity

$$v_2 = v_0 + \frac{mv_{\text{rel}}}{M+(n)\,m} + \frac{mv_{\text{rel}}}{M+(n-1)\,m} + \dots + \frac{mv_{\text{rel}}}{M+m} \qquad \text{....(iii)}$$

Comparing Eqs. (ii) and (iii), we note that

$$v_2 > v_1$$

i.e., the men would impart greater velocity to the car by running and jumping off one by one.

Exercise 1.4

1. (a) Velocity of centre of mass

$$\vec{v}_{\text{cm}} = \frac{(2m)(v\hat{i}) + m(-v\hat{j})}{2m+m} \Rightarrow \vec{v}_{\text{cm}} = \frac{v}{3}(2\hat{i} - \hat{j})$$

$$|\vec{v}_{\text{cm}}| = \frac{v}{3}\sqrt{4+1} = \frac{\sqrt{5}}{3}v$$

$$K_{\text{cm}} = \frac{1}{2}(m+2m)|\vec{v}_{\text{cm}}|^2 = \frac{1}{2}3m\left(\frac{5v^2}{9}\right) = \frac{5}{6}mv^2$$

(b) Kinetic energy w.r.t. centre of mass $K_{\text{int}} = \dfrac{1}{2}\dfrac{m_1m_2}{m_1+m_2}|\vec{v}_1 - \vec{v}_2|^2$

$$= \frac{1}{2}\frac{m\times 2m}{m+2m}|v\hat{i} - (-v\hat{j})|^2 = \frac{2}{3}mv^2$$

2. The result obtained in the last problem will help us produce an easy solution to this problem.
In the COM frame (a reference frame travelling with the velocity of the centre of mass of the system) the initial mechanical energy of the system is equal to the spring potential energy as velocity of both blocks is zero.
This energy gets converted into KE when the spring regains its natural length.

$$\frac{1}{2}kx^2 = \frac{1}{2}\mu v_r^2 \qquad \text{...(i)}$$

Here, μ = reduced mass of the blocks $= \dfrac{(m)(2m)}{m+2m} = \dfrac{2}{3}m$

and v_r = relative velocity of the two.
Substituting in Eq. (i), we get

$$kx^2 = \mu v_r^2$$

$$kx^2 = \frac{2m}{3}v_r^2 \Rightarrow v_r = \sqrt{\frac{3k}{2m}}x$$

3. At maximum expansion in spring both the rings moves with same velocity.
Applying work energy theorem from CM from

$$W_{\text{total}} = (\Delta K)_{\text{cm}} = \left(\frac{1}{2}\mu v_{\text{rel}}^2\right)_{\text{final}} - \left(\frac{1}{2}\mu v_{\text{rel}}^2\right)_{\text{initial}}$$

$$-\frac{1}{2}kx_{\text{max}}^2 = 0 - \frac{1}{2}\mu v_0^2 \qquad [\mu = \text{Reduced mass}]$$

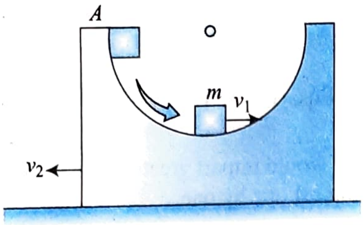
$$\Rightarrow \quad x_{\text{max}} = \sqrt{\frac{\mu}{x}}\cdot v_0 = \sqrt{\frac{2m}{3k}}\,v_0$$

4. At the time when slipping stops, both the block and plank move with common velocity.
Now applying work energy theorem from centre of mass frame.

$$W_{\text{ext}} = (\Delta K)_{\text{cm}} = \left(\frac{1}{2}\mu v_{\text{rel}}^2\right)_{\text{final}} - \left(\frac{1}{2}\mu v_{\text{rel}}^2\right)_{\text{initial}}$$

$$W_{\text{friction}} = -\frac{1}{2}\frac{m\times 3m}{(m+3m)}\cdot v_0^2 = -\frac{3}{8}mv_0^2$$

5. Conserving linear momentum in horizontal direction.



$$mv_1 = 9mv_2 \text{ or } v_1 = \frac{v_0}{3} \qquad \ldots\text{(i)}$$

Now applying work energy theorem from centre of mass frame

$$W_{\text{total}} = (\Delta K)_{\text{cm}} = \left(\frac{1}{2}\mu v_{\text{rel}}^2\right)_{\text{final}} - \left(\frac{1}{2}\mu v_{\text{rel}}^2\right)_{\text{initial}}$$

$$mgR = \frac{1}{2}\frac{m\cdot 9m}{(m+9m)}(v_1+v_2)^2 = \frac{9m}{20}\left(\frac{4}{3}v_2\right)^2$$

$$v_2 = \sqrt{\frac{5}{4}gR}$$

6. As the time of maximum stretch in spring both the blocks moves with common velocity. Hence the relative velocity of the blocks becomes zero. Now applying work energy theorem in centre of mass frame.

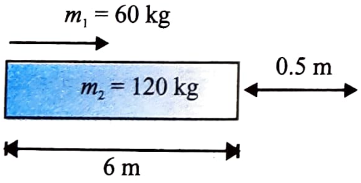
$$W_{\text{total}} = (\Delta K)_{\text{cm}} = \left(\frac{1}{2}\mu v_{\text{rel}}^2\right)_{\text{final}} - \left(\frac{1}{2}\mu v_{\text{rel}}^2\right)_{\text{initial}}$$

$$-\frac{1}{2}kx_m^2 = 0 - \frac{1}{2}\left(\frac{m\cdot 3m}{m+3m}\right)(v_0+3v_0)^2$$

$$kx_m^2 = \frac{3m}{4}\times 16v_0^2$$

$$x_m = 2\sqrt{\frac{3m}{k}}v_0$$

7.



(a) Displacement of man w.r.t. boat, $x_{m,b} = 6$ m

$$x_b = x_{m,b}\left(\frac{m_1}{m_1+m_2}\right) = -6\times\frac{60}{60+20} = -2 \text{ m}$$

Therefore, boat displaces by 2 m away from the pier.

∴ Separation = 2.5 m

(b) $\vec{v}_{mb} = \vec{v}_m - \vec{v}_b$

$$m_1\vec{v}_m + m_2\vec{v}_b = 0$$

$$m_1(\vec{v}_{mb} + \vec{v}_b) + m_2\vec{v}_b = 0$$

$$m_1v_{mb} = (m_1+m_2)v_b$$

$$v_b = \left(\frac{m_1}{m_1+m_2}\right)v_{mb}$$

$$v_b = \frac{60}{180}\times 3 = 1 \text{ m/s}$$

KE of the man $= \frac{1}{2}m_mv_m^2 = \frac{1}{2}\times 60\times(3-1)^2 = 120$ J

KE of the boat $= \frac{1}{2}\times 120\times(1)^2 = 60$ J

Total KE energy = 120 + 60 = 180 J

KE of the system when the boat is tied $= \frac{1}{2}\times 60\times 3^2 = 270$ J

8. (a) Whomsoever we consider the aggressor, brother and sister exert equal in magnitude oppositely directed forces on each other, to give each other equal magnitudes of momentum. We take the eastward component of the equation

Total original momentum = total final momentum for the two-sibling system

$$0 = 80 \text{ kg } (-3 \text{ m/s}) + (40 \text{ kg})v \Rightarrow v = 6.0 \text{ m/s}$$

i.e., she moves at 6.0 m/s east.

(b) Original chemical energy in the girl's body = total final kinetic energy

$$U_{\text{chemical}} = (1/2)(80 \text{ kg})(3 \text{ m/s})^2 + (1/2)(40 \text{ kg})(6.0 \text{ m/s})^2 = 1080 \text{ J}$$

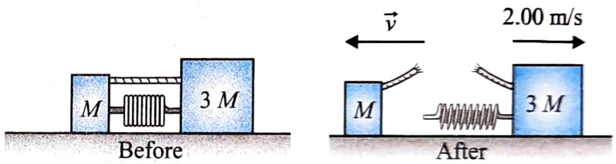
(c) System's momentum is conserved with the value zero. The net forces on the two siblings are of equal magnitude and opposite directions. Their impulses add to zero. Their final momenta are of equal magnitude in opposite directions, to add as vectors to zero.

9. (a) For the system of two blocks $\Delta p = 0$

or $p_i = p_f$

Therefore,

$$0 = Mv_M + (3M)(2.00 \text{ m/s})$$



(a) (b)

Solving, $v_M = -6.00$ m/s (motion toward the left)

(b) $\frac{1}{2}kx^2 = \frac{1}{2}Mv_M^2 + \frac{1}{2}(3M)v_{3M}^2 = 8.40$ J

(c) The original energy is in the spring. A force had to be exerted over a distance to compress the spring, transferring energy into it by work.

(d) Momentum of the system is conserved with the value zero. The forces on the two blocks are of equal magnitude and opposite directions. Their impulses add to zero. The final momenta of the two blocks are of equal magnitude and opposite directions. Plenty of motion afterward is due to the fact that energy stored in the spring converts into the kinetic energy of blocks.

10. (a) Let m_1 and m_2 be the masses of bullet and the rifleman; v_1 and v_2 their respective velocities after the first shot.

Initially, the rifleman and bullet are at rest, therefore initial momentum of the system = 0.

As external force is zero, momentum of the system is constant i.e., initial momentum = final momentum

$$0 = m_1v_1 + m_2v_2$$

$$\text{or } v_2 = -\frac{m_1v_1}{m_2} = -\frac{(10\times 10^{-3} \text{ kg})(800 \text{ m/s})}{100 \text{ kg}} = -0.08 \text{ m/s}$$

As mass of the bullet is negligible as compared to (rifle + man),

∴ Velocity acquired after 10 shots = $10v_2 = 0.8$ m/s

i.e., the velocity of rifleman is 0.8 m/s in a direction opposite to that of the bullet.

(b) The momentum acquired by the rifleman is

$p = m_2 v_2 = 100 \times 0.8$ kg m/s = 80 kg m/s

This momentum is acquired in 10 s, therefore the average force exerted $= \frac{\Delta p}{\Delta t} = \frac{80}{10} = 8$ N

(c) Kinetic energy of the rifleman,

$$E_1 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \times 100 \times (0.8)^2 = 32 \text{ J}$$

Kinetic energy of 10 bullets:

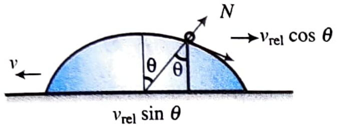
$$E_2 = 10\left[\frac{1}{2} \times 10 \times 10^{-3}\right](800)^2 = 32000 \text{ J}$$

Kinetic energy of bullets is 1000 times the kinetic energy of rifleman.

11. Actual velocity of ball is v_1 is horizontal direction.

$\therefore \quad v_1 - (-v) = v_{\text{rel}} \cos\theta$

$\therefore \quad v_1 + v = v_{\text{rel}} \cos\theta$ ...(i)



Applying conservation principle of momentum in horizontal direction.

$p_{ix} = p_{fx}$

$0 = mv_1 - 4mv$

$\therefore \quad v_1 = 4v$ ...(ii)

Applying mechanic energy conservation principle

Loss in PE = gain in KE

$$mg(r - r\cos\theta) = \frac{1}{2}m\left[\sqrt{v_1^2 + (v_{\text{rel}}\sin\theta)^2}\right]^2 + \frac{1}{2}4mv^2 \quad \text{...(iii)}$$

After solving Eqs. (i), (ii) and (iii)

$$v_{\text{rel}} = \frac{5v}{\cos\theta}$$

$$\therefore \quad \omega_{\text{rel}} = \frac{v_{\text{rel}}}{R} = \frac{5v}{R\cos\theta}$$

Exercises

Single Correct Answer Type

1. (3) When man will walk, friction force will act forward, man shifts right and the plank does not move (since lower surface is rough). Therefore, centre of mass shifts towards right.

2. (3) There is no external force in horizontal direction. Hence centre of mass will not shift horizontally.

3. (4) Mass distribution is symmetrical about x-axis, hence y-coordinate of C.M. will be zero.

x-coordinate of C.M.

$$x_{\text{cm}} = \frac{M_1x_1 + M_2x_2}{M_1 + M_2} = \frac{\frac{M}{2}\left(-\frac{a}{2}\right) + \frac{M}{2}\left(\frac{a}{3}\right)}{M} = -\frac{a}{12}$$

Hence coordinate of centre of mass of Fig. (b) will be $\left(-\frac{a}{12}, 0\right)$

4. (2) Mass distribution is symmetrical about x-axis, hence y-coordinate of C.M. will be zero.

x-coordinate of C.M.

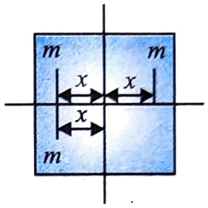
$$x_{\text{cm}} = \frac{(M \times 0) + \left(-\frac{M}{4} \times \frac{-L}{3}\right) + \left(\frac{M}{4} \times \frac{4L}{6}\right)}{M} = \frac{L}{4}$$

Hence coordinate of centre of mass of Fig. (b) will be $\left(\frac{L}{4}, 0\right)$

5. (2) If we take both the planks and packets as a system, the system has no external force in horizontal direction. The linear momentum of the system will be conserved. As plank A is moving with greater speed than plank B, the packet transfer from plank A will carry greater linear momentum than the packet transferred from plank B. It means the linear momentum of plank B will increases and the linear momentum of plank A will decrease.

6. (2) The mass distribution is symmetrical about the line $y = x$, hence the possible x and y coordinates of the centre of mass should be equal.

7. (2)



$$x_1 = \left(\frac{mx}{M - m}\right), x_2 = \left(\frac{2mx}{M - 2m}\right), x_3 = \left(\frac{mx}{M - 3m}\right)$$

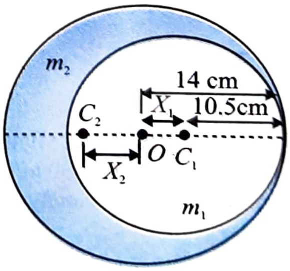
$$x_1 < x_3 < x_2$$

8. (1) The force has to be applied at the centre of mass of the system for pure translation motion.

$$x = \frac{M(0) + M\left(\frac{L}{2}\right)}{M + M} = \frac{L}{4}$$

9. (3) C_1 is the centre of mass of cut portion and C_2 that of remaining portion. We have to find x_2.

$x_1 = 14 - 10.5 = 3.5$ cm



Mass will be proportional to area. So mass of the whole disc is

$M = k\pi\,(14)^2$

Mass of cut portion $m_1 = k\pi\,(10.5)^2$

Mass of the remaining portion $m_2 = M - m_1 = k\pi\,(14^2 - 10.5^2)$

$= k\pi\,(24.5) \times (3.5)$

Now, $m_1x_1 = m_2x_2$

$$\Rightarrow x_2 = \frac{m_1x_1}{m_2} = \frac{k\pi\,(10.5)^2 \times 3.5}{k\pi\,(24.5) \times 3.5} = 4.5 \text{ cm}$$

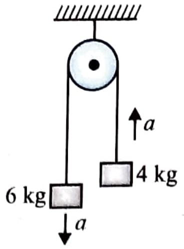
10. (2) The rod will balance about centre of mass; so distance of centre of mass from the left end,

$$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{100 \times 25 + 50 \times 55}{100 + 50} = 35 \text{ cm}$$

Therefore, distance from centre of rod = 35 – 25

= 10 cm to the right

11. (3) Acceleration of blocks,



$$a = \frac{6-4}{6+4} g = \frac{g}{5}$$

$$a_{CM} = \frac{-6 \times a + 4a}{6+4}$$

$$= -\frac{a}{5} = -\frac{g}{25}$$

12. (1) Using $\vec{v}_{CM} = \hat{i}\, v_x + \hat{j}\, v_y$

where $v_x = \dfrac{mv_{x_1} + mv_{x_2} + mv_{x_3}}{3m} = 0$

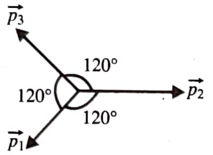
$$v_y = \frac{mv_{y_1} + mv_{y_2} + mv_{y_3}}{3m} = 0$$

Net velocity is zero (using vector theory) i.e., CM is at rest. So, displacement of C.M. is zero.

13. (4) Net force on centre of mass is zero. Therefore, centre of mass always remains at rest.

14. (4) As no external force acts, so centre of mass remains at rest.

15. (4) $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3}{m_1 + m_2 + m_3}$

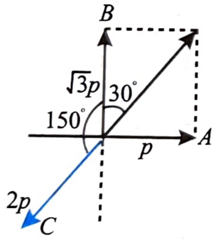


$$\Rightarrow \vec{v}_{cm} = \frac{\text{Total momentum}}{\text{Total mass}}$$

Here total momentum of system is zero, because momentum of each particle is same in magnitude and they are symmetrically oriented as shown.

So $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

16. (4) The momentum of C will be $2p$ in a direction making angle 150° with that of B.



17. (1) Suppose m is the mass of each piece flying off perpendicular to one another with same speed u (= 30 m/s). Then $3m$ is the mass of the third piece. Let v be the velocity of the third piece.

According to figure,

$3\,mv\cos\theta = mu$,

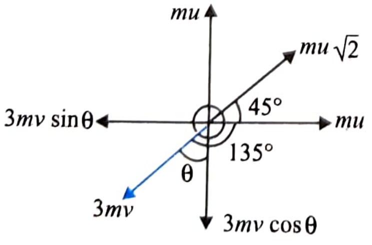
$3\,mv\sin\theta = mu$

$\tan\theta = 1$ or $\theta = 45°$

Hence, $3\,mv\cos 45° = mu$

or $\dfrac{3v}{\sqrt{2}} = u = 30$ m/s

or $v = 10\sqrt{2}$ m/s



(inclined at 135° w.r.t. direction of each one)

18. (1) Let the three mutually perpendicular directions be along xy and z-axis, respectively.

$$\vec{p}_1 = mv_0\hat{i},\quad \vec{p}_2 = mv_0\hat{j}$$

where $\frac{1}{2}mv_0^2 = E_0$, $\vec{p}_3 = mv_0\hat{k}$ and $\vec{p}_4 = m\vec{v}$

By linear momentum conservation,

$$0 = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4$$

or $\vec{v} = -v_0(\hat{i} + \hat{j} + \hat{k})$

or $v = v_0\sqrt{1^2 + 1^2 + 1^2} = v_0\sqrt{3}$

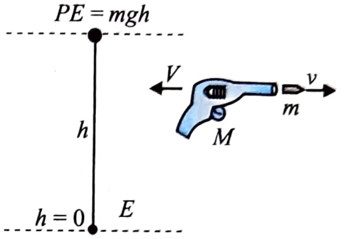
Total energy $= 3\left(\frac{1}{2}mv_0^2\right) + \frac{1}{2}mv^2 = 3E_0 + 3E_0 = 6E_0$

19. (3) $u_1 = \dfrac{p}{m_1}$, $u_2 = \dfrac{p}{m_2}$

$$E = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

$$= \frac{1}{2}\left[m_1\frac{p^2}{m_1^2} + m_2\frac{p^2}{m_2^2}\right] = \frac{p^2(m_1 + m_2)}{2m_1m_2}$$

20. (1) If E is the energy of explosion, then since it is just sufficient to carry the bullet to a height h, so



We have $E = mgh$. Now, if v and V are the respective velocities of the bullet and the gun, respectively, then from the law of conservation of linear momentum,

$$mv + M(-V) = 0 \Rightarrow v = \frac{MV}{m} \qquad \text{...(i)}$$

Also, $E = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$

$\Rightarrow mgh = \frac{1}{2}m\left(\frac{MV}{m}\right)^2 + \frac{1}{2}MV^2$ [from Eq. (i)]

$$\Rightarrow 2mgh = \frac{M^2V^2}{m} + MV^2$$

$$\therefore\ V = \left[\frac{2m^2gh}{M(M+m)}\right]^{1/2}$$

21. (4) Conservation of momentum during explosion yields

$$\Rightarrow \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0 \Rightarrow \vec{p}_3 = |\vec{p}_1 + \vec{p}_2|$$

$$\Rightarrow m_3\vec{v}_3 = |m_1\vec{v}_1 + m_2\vec{v}_2|$$

since $\vec{v}_1 \perp \vec{v}_2 \Rightarrow \vec{v}_3 = \sqrt{\dfrac{m_1^2v_1^2 + m_2^2v_2^2}{m_3}}$

$$\Rightarrow \vec{v}_3 = \sqrt{\left(\frac{m_1}{m_3}\right)^2 v_1^2 + \left(\frac{m_2}{m_3}\right)^2 v_2^2}$$

$$= \sqrt{\left(\frac{1}{3}\right)^2 (15)^2 + \left(\frac{1}{3}\right)^2 (15)^2} \quad (\because m_1 : m_2 : m_3 = 1 : 1 : 3)$$

$$\Rightarrow v_3 = 5\sqrt{2}\ \text{ms}^{-1}$$

22. (4) $u_x = 20\sqrt{2}\cos 45° = 20$ m/s

$u_y = 20\sqrt{2}\sin 45° = 20$ m/s

After 1 s, horizontal component remains unchanged while vertical component becomes

$$v_y = u_y - gt = 20 - 10 = 10\ \text{m/s}$$

Due to explosion, one part comes to rest. Hence, from conservation of linear momentum, vertical component of second part will become v_{y1} = 20 m/s. Therefore, maximum height attained by the second part will be

$H = h_1 + h_2$

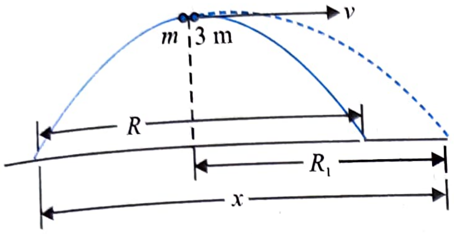
Here, h_1 = height attained in 1 s $= (20)(1) - \frac{1}{2} \times 10 \times 1^2 = 15$ m

and h_2 = height attained after 1 s $= \frac{v_{y1}^2}{2g} = \frac{(20)^2}{2 \times 10} = 20$ m

$\therefore\ H = 15 + 20 = 35$ m

23. (2) Apply conservation of momentum at highest point:

$$4\,mu \cos\theta = 3\,mv \Rightarrow v = \frac{4}{3} u \cos\theta \qquad \text{...(i)}$$



$R = (u \cos\theta)T$

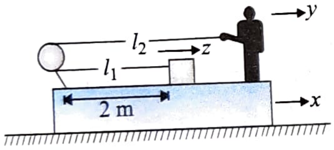
$R_1 = v\frac{T}{2} = \frac{4}{3}\frac{u \cos\theta\, T}{2} = \frac{2R}{3}$

$\Rightarrow x = \frac{R}{2} + R_1 = \frac{R}{2} + \frac{2R}{3} = \frac{7R}{6}$

24. (2) Let x be displacement of wedge w.r.t. ground, y be displacement of man w.r.t. ground and z be displacement of block w.r.t. ground. As the length of string will remain same, so

$$(l_1 + z - x) + (l_2 + y - x) = l_1 + l_2$$

$$\Rightarrow z + y = 2x \qquad \text{...(i)}$$



Also, if the pulley and the block meet, then we can write

$$x = z + 2 \qquad \text{...(ii)}$$

$$\Delta x_{cm} = 0 \Rightarrow \frac{Mx + 2My + Mz}{4M} = 0$$

$$\Rightarrow x + 2y + z = 0 \qquad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii),

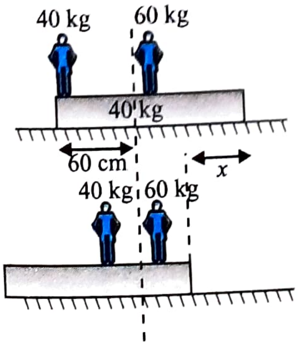
$$x = \frac{-1}{2}\text{ m},\ y = \frac{3}{2}\text{ m},\ z = \frac{-5}{2}\text{m}$$

25. (2) Till time t_0, friction on car will be in forward direction and on boat in backward direction. So the boat will not move till t_0. Let v be the velocity of car + boat system just after applying the brakes. From conservation of momentum,

$$(m + M)v = mv_0 \Rightarrow v = \frac{mv_0}{m + M}$$

Time taken is $t_0 + L/v = t_0 + \frac{L(m + M)}{mv_0}$

26. (3) Taking the origin at the centre of the plank,



$$m_1\Delta x_1 + m_2\Delta x_2 + m_3\Delta x_3 = 0 \qquad (\because \Delta x_{CM} = 0)$$

(Assuming the centres of the two men are exactly at the axis as shown in figure.)

$$60 \times 0 + 40 \times 60 + 40(-x) = 0 \Rightarrow x = 60 \text{ cm}$$

Hence, A and B meet at the right end of the plank.

27. (2) In the absence of external force, v_{CM} = constant. Here initially $v_{CM} = 0$; so $v_{CM} = 0$ throughout, i.e., position of centre of mass, remains unshifted.

28. (1) Motion of centre of mass is exactly similar to the motion of a body had it not exploded.

$$u_x = u\cos\theta = \frac{10}{\sqrt{2}}\text{ m/s},\ u_y = u\sin\theta = \frac{10}{\sqrt{2}}\text{ m/s}$$

$$v_x = u_x = \frac{10}{\sqrt{2}}\text{ m/s}$$

(since there is no change in the horizontal velocity)

$$v_y^2 - u_y^2 = 2(-g)(h)$$

$$\Rightarrow v_y^2 = \frac{100}{2} - 2 \times 10 \times 1 = 30$$

Therefore, net velocity of CM $= \sqrt{v_x^2 + v_y^2}$

$$= \sqrt{\frac{100}{2} + 30} = \sqrt{80} = 4\sqrt{5}\text{ m/s}$$

29. (4) As long as no external force acts on the system of 'man + boat', its centre of mass will remain stationary.

30. (3) Method 1: Acceleration of 3 kg:

$$a_2 = \frac{6}{3} = 2\text{ m/s}^2$$

Velocity of 3 kg at $t = 5$ s: $v_2 = u_2 + a_2t = 0 + 2 \times 5 = 10$ m/s

$$v_{CM(t=5s)} = \frac{2 \times 0 + 3 \times 10}{2 + 3} = 6\text{ m/s}$$

Method 2:

$$a_{CM} = \frac{F}{m_1 + m_2} = \frac{6}{2 + 3} = 1.2\text{ m/s}^2$$

$$v_{CM} = u_{CM} + a_{CM}t = 0 + 1.2 \times 5 = 6\text{ m/s}$$

31. (3) $v_{CM} = \frac{5 \times 14}{5 + 2} = 10$ m/s

Velocity of the 5 kg block w.r.t. CM $= 14 - 10 = 4$ m/s
Velocity of the 2 kg block w.r.t. CM $= 0 - 10 = -10$ m/s

32. (3) $\vec{a}_{CM} = \frac{\vec{F}}{m_1 + m_2} = \frac{16\hat{i} + 8\hat{j}}{8 + 8} = \hat{i} + \frac{1}{2}\hat{j}$

$$|\vec{a}_{CM}| = \sqrt{1^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{5}}{2}\text{ m/s}^2$$

$$\tan\theta = \frac{a_y}{a_x} = \frac{\frac{1}{2}}{1} = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

33. (2) As no external force acts on the ball + box system, hence velocity of the system remains constant.

34. (2) At maximum compression when m sticks to M,

$$mv = (M + m)V$$

$$V = \frac{mv}{M + m}$$

Fraction of KE stored in spring, $f = \frac{K_i - K_f}{K_i}$

$$\therefore\ f = 1 - \frac{\frac{1}{2}(M + m)V^2}{\frac{1}{2}mv^2} \Rightarrow f = 1 - \left(\frac{m}{M + m}\right) = \frac{M}{M + m}$$

35. (3) As the car stops, so velocity of bullet after firing is same w.r.t. ground and w.r.t. car. Conserving momentum in horizontal direction.

$$(M+m)V = mv\cos\alpha \quad\Rightarrow\quad V = \frac{mv\cos\alpha}{M+m}$$

36. (4) Using conservation of linear momentum, we have

$$m_2v_0 = (m_1+m_2)v \quad\Rightarrow\quad v = \frac{m_2}{m_1+m_2}v_0$$

From work–energy conservation,

$$\frac{1}{2}m_2v_0^2 - \frac{1}{2}(m_1+m_2)\frac{m_2^2v_0^2}{(m_1+m_2)^2} = \frac{1}{2}kx^2$$

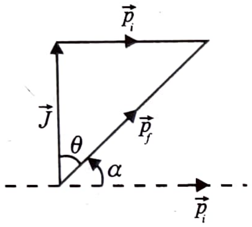
$$x = v_0\sqrt{\frac{m_1m_2}{k(m_1+m_2)}}$$

37. (3) From impulse–momentum theorem,

$$\vec{J} = \vec{p}_f - \vec{p}_i \Rightarrow \vec{p}_f = \vec{J} + \vec{p}_i$$

Here $p_i = 3000 \times 300 = 9\times 10^5$ kg m/s

$J = 4000 \times 225 = 9 \times 10^5$ kg m/s



$$\tan\theta = \frac{p_i}{J} = 1 \Rightarrow \theta = 45°$$

So, the angle by which the space probe rotates is

$$\alpha = \frac{\pi}{2} - \theta = 45°$$

38. (4) As only the gravity force is acting on the system, the centre of mass of the system follows a parabolic path.

At $t = 2$ s,

$$x_{CM} = 30\cos 37° \times 2 = 48\text{ m}$$

$$y_{CM} = 30\sin 37° \times 2 - \frac{1}{2}\times 10 \times 2^2 = 16\text{ m}$$

Let coordinates of the second piece, i.e., 6 kg piece be (x, y). Then

$$x_{CM} = 48 = \frac{6x + 4\times 105}{10} \Rightarrow x = 10\text{ m}$$

$$y_{CM} = 16 = \frac{6y + 4\times 43}{10} \Rightarrow y = -2\text{ m}$$

Negative value of y shows that the second piece collides the ground before $t = 2$ s.

39. (4) Initially, both the blocks are under the gravity effect, so

$$a_{CM} = \frac{4g+2g}{4+2} = g$$

But once the jerk in the string occurs and string becomes taut, the 4 kg block moves down with acceleration $g/3$ and 2 kg block moves up with acceleration $g/3$. So,

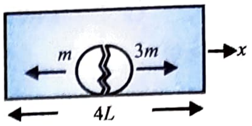
$$a_{CM} = \frac{4\times\frac{g}{3} - \frac{2g}{3}}{6} = \frac{g}{9}$$

40. (4) $mv_0 = (m + m')\,v$, where m' is mass of snow that is added.

$$v' = \frac{mv_0}{m+m'}$$

Since Ram throws sideways, no impulse acts in the direction of motion.

41. (4) Let box moves towards right by a distance x, then displacement of $2m$ is $x + 2L$ and displacement of m is $x - 2L$ towards right.



$$\Delta x_{cm} = \frac{m_1\Delta x_1 + m_2\Delta x_2 + m_3\Delta x_3}{m_1+m_2+m_3}$$

$$\Rightarrow\quad 0 = 3mx + 2m(x+2L) + m(x-2L)$$

$$\Rightarrow\quad x = -\frac{L}{3}$$

42. (2) For the given situation, block should be able to reach just at highest point of M, and their relative velocity is to become zero. Let their common velocity is v_C.

$$\text{COLM: } (M+m)\,v_C = mv \Rightarrow v_C = \frac{mv}{M+m}$$

$$\text{COME: } \frac{1}{2}(M+m)v_C^2 + mgh = \frac{1}{2}mv^2$$

Solve to get $v = \sqrt{2\left(1+\frac{m}{M}\right)gh}$

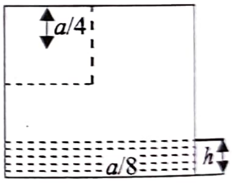
43. (2) P_x (before collision) = P_x (after collision)

$\Rightarrow\ -4 + b = 3 + 4 \Rightarrow b = 11$

P_y (before collision) = P_y (after collision)

$5 - 2 = a + 2 \Rightarrow a = 1$

44. (3) Let h be the height of water surface. Then,



$$a^2h = a\times\frac{a}{2}\times\frac{a}{2}$$

$$h = \frac{a}{4}$$

CM gets lowered by

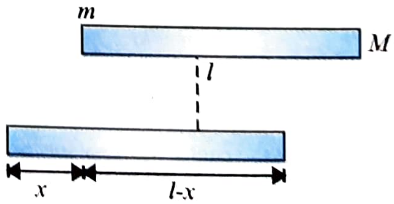
$$a - \left(\frac{a}{4} + \frac{a}{8}\right) = a - \frac{3a}{8} = \frac{5a}{8}$$

Work done by gravity $= mg\,\dfrac{5a}{8} = \dfrac{5}{8}mga$

45. (2) Statement (i) is correct because in this case, spring would be compressing. So PE in the spring will be increasing. It means KE of the system will be decreasing as the total mechanical energy of the system is constant.

Further, momentum of the system will always be conserved. It will neither be increasing nor be decreasing.

46. (1)



$$m(l-x) = Mx \Rightarrow x = \frac{ml}{m+M}$$

Distance moved by insect w.r.t. ground is

$$l - x = l - \frac{ml}{m+M} = \frac{Ml}{m+M}$$

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{Ml}{t\,(M+m)}$$

47. (2) In both CM and ground frame, $K_{\max}$ is there, when x is zero in spring, which occurs simultaneously.

$$v_{CM} = \frac{m(v_0) + 0}{5m} = \frac{v_0}{5}$$

$$K_{\max\,CM} = \frac{1}{2} m \left(\frac{4v_0}{5}\right)^2 + \frac{1}{2}(4m)\left(\frac{v_0}{5}\right)^2 = \frac{2}{5} m v_0^2$$

$$K_{\max\,\text{ground}} = \frac{1}{2} m v_0^2$$

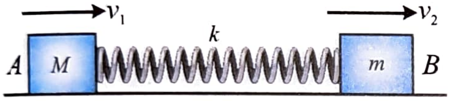
$$K_{\min\,CM} = 0$$

$$K_{\min\,\text{ground}} = \frac{1}{2}(m + 4m)\, v_{CM}^2 = \frac{m v_0^2}{10}$$

$$K_{\max\,m} = \frac{1}{2} m v_0^2 \text{ (ground frame)}$$

$K_{\min\,m} = 0$ (ground frame when energy is shared by spring and $4m$ only and m will reverse direction of motion)

48. (1) The velocity of B will be maximum when spring comes to its natural length. Let at that time their velocities are v_1 and v_2 as shown.



Conservation of momentum:

$$2v_1 + 1v_2 = (2+1)\times 1 \quad \Rightarrow \quad 2v_1 + v_2 = 3 \qquad \text{...(i)}$$

Conservation of energy:

$$\frac{1}{2}2v_1^2 + \frac{1}{2}1v_2^2 = \frac{1}{2}600\left(\frac{10}{100}\right)^2 + \frac{1}{2}3(1)^2 \Rightarrow 2v_1^2 + v_2^2 = 9 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$v_1 = 2 \text{ m/s},\ v_2 = -1 \text{ m/s} \quad \text{or} \quad v_1 = 0,\ v_2 = 3 \text{ m/s}$$

The second solution is possible.

49. (2) Till then m remains on B, B will continue to accelerate due to contact force between m and B. Hence, velocity of B will be maximum when finally m leaves B.

50. (3) Momentum of the system in x-direction is conserved because all the forces on the system in x-direction are balanced. But the force in y-direction is not balanced. So, trajectory of motion of centre of the system is x-constant.

51. (3) Since velocity of man w.r.t. trolly is greater than velocity of trolly w.r.t. earth, after the man turns back displacement of the man will decrease, so maximum displacement will be at the moment when man turns back.

$$\therefore\ t = \frac{L}{1.5\,V}$$

$$\text{Displacement} = (v + 1.5\,v)t = 2.5v \times \frac{L}{1.5v} = \frac{5}{3}L$$

52. (3) At the time of maximum compression, the speeds of blocks will be the same. Let that speed be v and maximum compression be x.

Applying conservation of momentum,

$$(m_1 + m_2)\,v = m_1 v_1 + m_2 v_2$$

$$\Rightarrow v = 4 \text{ m/s}$$

Applying conservation of mechanical energy,

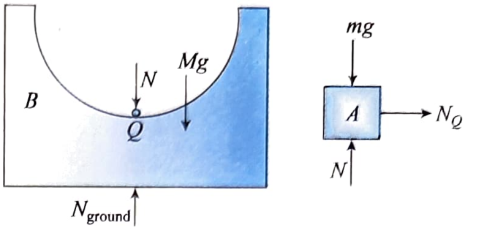
$$\frac{1}{2}kx^2 + \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

Solving, we get $x = 0.02$ m.

53. (3) (1) There is external force acting on the system of 'A' and 'B' in vertical direction, also 'A' has acceleration it means the centre of mass of system will also accelerating.

(2) When A reaches Q the acceleration of A is $\frac{v_Q^2}{R}$ in frame of 'B' so acceleration of centre of mass will not be zero.

(3) From F.B.D. of the block at Q

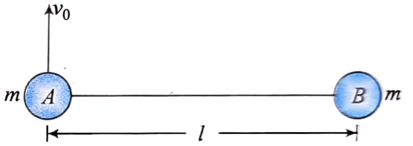


$$N - mg = \frac{mv_Q^2}{R} \Rightarrow N = mg + \frac{mv_Q^2}{R}$$

So net normal force will be greater than combined weight.

(4) When 'A' is at Q acceleration of B is zero as no external force acts on the system in horizontal direction.

54. (2) The acceleration of C.M. will be g(downward)



$$\text{Initial velocity of C.M., } v_{cm} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

$$= \frac{mv_0 + m \times 0}{(m + m)} = \frac{v_0}{2}$$

At maximum height the velocity of C.M. should be zero, using

$$v_{cm}^2 = u_{cm}^2 + 2a_{cm}h$$

$$0 = \left(\frac{v_0}{2}\right)^2 - 2gh_c \Rightarrow h_c = \frac{v_0^2}{8g}$$

55. (2) The linear momentum of the system will remain conserved in horizontal direction. The initial linear momentum of the system is zero. Let velocity of the particle relative to wedge be v,

$$0 = M(-V) + m(v\cos 60^\circ - V)$$

$$\Rightarrow \quad v = \frac{(M+m)V}{m\cos 60^\circ} = \frac{5 \times 0.1}{0.2 \times (1/2)} = 5 \text{ m/s}$$

56. (3) If take person 'A', 'B' and boat as system. There is no external force acting on the system. Hence the C.M. of system will not displaced. Let the displacement of the boat is Δx (rightward), then

$$x_{cm} = 0 = \frac{m_A \Delta x_A + m_B \Delta x_B + m_{\text{boat}} \Delta x_{\text{boat}}}{m_A + m_B + m_{\text{boat}}}$$

$$0 = \frac{80(\Delta x + 10) + 50(\Delta x - 10) + 70\,\Delta x}{80 + 50 + 10}$$

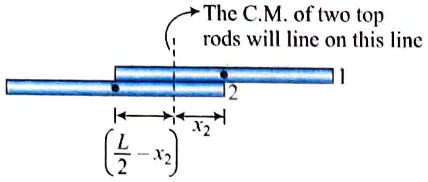
$$\Rightarrow \quad (80 + 50 + 70)\Delta x + 800 - 500 = 0$$

$$\Rightarrow \quad 200\,\Delta x + 300 - 0$$

$$\text{or} \quad \Delta x = -\frac{3}{2} \text{ m}$$

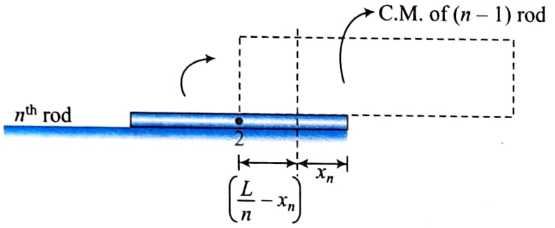
or 1.5 m towards left

57. (4) The C.M. of the top rod should lie on the edge of second top most rod, i.e., the overhang of top most rod should be $\frac{L}{2}$. The C.M. of two top rod should lie on the edge of third top most rod. Let the overhang of second top most rod be x_2.


The C.M. of two top rods will line on this line
1
2
x_2
$\left(\frac{L}{2}-x_2\right)$

$$Mx_2 = M\left(\frac{L}{2}-x_2\right) \Rightarrow x_2 = \frac{L}{4}$$

If n rods are placed on the table then the C.M. of top $(n-1)$ rods should lie on the edge of n^{th} rod.



$$M\left(\frac{L}{n}-x_n\right) = Mx_n \Rightarrow x_n = \frac{L}{2n}$$

Hence $n = 3$

Hence maximum offset $= \frac{L}{2}+\frac{L}{4}+\frac{L}{6} = \frac{11}{12}L$

Multiple Correct Answers Type

1. (1), (3)

(1) As $\Sigma\vec{p} = 0$ and $\vec{p}_1 + \vec{p}_2 \neq 0$, $\vec{p}_3$ cannot be zero.

(2) For any system of two or more than two particles, KE can change due to work done by internal forces while the centre of mass remains at rest

(3) $\vec{a}_{CM} = 0 \Rightarrow \Sigma\vec{F}_{\text{ext}} = 0$

(4) It is possible to have changing speed of CM without any work done by external forces on the system. When a person accelerates himself on a rough horizontal surface without any flipping between his shoes and ground, work done by friction, normal reaction and weight all are zero but speed of centre of mass changes.

2. (2), (3)

The general form of Newton's second law is

$$\vec{F}_{\text{ext}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

The form $\vec{F}_{\text{ext}} = m\vec{a}$ is valid only if $\frac{dm}{dt} = 0$, i.e., mass does not change with time. Hence, option (1) is incorrect.

Option (2) is correct because a body at rest may have potential energy yet no momentum. Option (3) is also correct. A body has momentum if it has mass and velocity and a body having mass and velocity must have kinetic energy. Option (4) is incorrect because the relative velocity remains unchanged in magnitude and gets reversed in direction:

$$v_2 - v_1 = -(u_2 - u_1)$$

Hence, the correct options are (2) and (3).

3. (1), (2), (3), (4)

(1) $v_1 = 20 - gt$

$v_2 = -20 - g(t-2)$

$v_1 - v_2 = [20 - gt] + 20 + g(t-2) = 20$ m/s

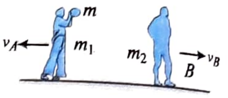
Clearly, it is time independent, i.e., relative velocity between the two stones remains constant.

(2) In both the cases, the total energy remains the same.

(3) and (4). These are simple concepts.

4. (2), (4)

Let after the process of catching is over, final velocities are as shown.



$m \to$ mass of ball

$m_1 \to$ mass of A

$m_2 \to$ mass of B

Apply conservation of momentum for entire system.

$$0 = -(m_1 + m)v_A + m_2 v_B$$

$$\Rightarrow (m_1 + m)v_A = m_2 v_B \qquad \text{...(i)}$$

Hence $m_1 > m_2$

$$\Rightarrow \frac{v_B}{v_A} = \frac{m_1 + m}{m_1} > 1 \Rightarrow v_B > v_A$$

$$p_A = m_1 v_A, \quad p_B = m_2 v_B \qquad \text{...(ii)}$$

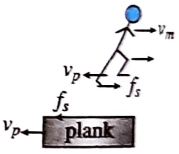
Clearly from (i) and (ii), $p_B > p_A$

$$K_A = \frac{1}{2}m_1 v_A^2 = \frac{1}{2}p_A v_A$$

$$K_B = \frac{1}{2}m_2 v_B^2 = \frac{1}{2}p_B v_B$$

Clearly $K_B > K_A$

5. (1), (3), (4)



At contact point of man and plank, the relative motion is zero. As plank moves backward, man's foot also moves backwards, but friction on man acts forward. Hence work done by friction on man is negative. But with respect to plank, foot will be at rest, hence no work done by friction on man w.r.t. plank.

6. (1), (4)

$$v_{\text{cm}-x} = \frac{mv_1\cos\theta_1 + m(-v_2\cos\theta_2)}{m+m} = 0$$

So horizontal velocity of centre of mass is zero. Hence centre of mass will move in vertical direction.

Now $v_2\cos\theta_2 = v_1\cos\theta_1$

$$\Rightarrow \frac{v_2}{v_1} = \frac{\cos\theta_1}{\cos\theta_2} > 1 \qquad (\because \theta_2 > \theta_1)$$

$$\Rightarrow v_2 > v_1$$

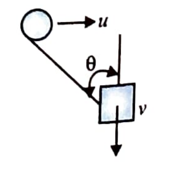
$$\Rightarrow v_2 \sin\theta_2 >> v_1 \sin\theta_1$$

At any time, particle '2' will be at greater height than particle '1'. So particle '2' will be above centre of man level.

7. (2), (3)

The initial velocity of CM is upward. The acceleration of the CM is 'g' downward.

8. (1), (4)



$v\cos\theta = u\sin\theta$

From conservation of energy,

$$mgl\cos\theta = 2\left(\frac{1}{2}mu^2\right) + \frac{1}{2}mv^2$$

$$u=\sqrt{\frac{2gl\cos\theta}{2+\tan^2\theta}}=\sqrt{\frac{gl}{5}}\text{, using }\theta=60°$$

$$v=\sqrt{\frac{3gl}{5}}$$

9. (1), (2), (3), (4)

Since the system of two blocks and a spring is placed on a smooth horizontal floor, therefore no external horizontal force acts on the system.

Since the system is initially at rest, therefore centre of mass of the system will remain stationary and the block will oscillate with the same frequency. It means option (4) is correct.

Since the system is initially at rest and centre of mass stationary, therefore centre of mass cannot move or the momentum of the system remains equal to zero. It means, momenta of two blocks are numerically equal but they are in opposite directions. Therefore, ratio of magnitude of velocity of the two blocks will be in inverse ratio of their masses. Hence, option (1) is correct.

Since ratio of mass is constant, therefore the ratio of their velocity will remain constant. It means that option (2) is also correct.

Since the system is on smooth horizontal floor, therefore no energy loss takes place. Hence, linear momentum and energy of the system remain conserved. Therefore, option (3) is also correct.

10. (3), (4)

Using conservation of linear momentum,

$$mv_0=(M+m)v$$

$$v=\frac{mv_0}{M+m}$$

KE in centre of mass frame:

$$KE_c=\frac{1}{2}m(v_0-v)^2+\frac{1}{2}Mv^2=\frac{1}{2}\left(\frac{Mm}{M+m}\right)v_0^2$$

From energy conservation,

$$\frac{1}{2}mv_0^2-\frac{1}{2}(m+M)v^2=\frac{1}{2}kx^2$$

$$\Rightarrow x=v_0\sqrt{\frac{mM}{k(M+m)}}$$

11. (1), (3), (4)

Impulse = change in momentum

$$\Rightarrow 6\times4=10(v-5)\Rightarrow v=7.4\text{ ms}^{-1}$$

w.r.t. observer B, speed will be $v-2=7.4-2=5.4\text{ ms}^{-1}$

12. (1), (2), (3)

It can be shown that $K_0=K_{cm}+\frac{1}{2}MV_{cm}^2$ where M is the total mass of the system and V_{cm} is velocity of centre of mass with respect to ground.

Due to internal changes K_{cm} can change but V_{cm} will remain same. Hence only K_{cm} portion of kinetic energy can be transformed to some other form of energy. Thus statements (1), (2) and (3) are correct.

13. (1), (2)

$$\vec{a}_{cm}=\frac{m_1\vec{a}_1+m_2\vec{a}_2}{(m_1+m_2)}=\frac{m(2a\hat{i})+2m(-a\hat{i})}{m+2m}=0$$

Hence option (1) is correct.

Net force on the system $\vec{F}_{net}=m\vec{a}_{cm}=0$

Hence option (2) is correct.

As nothing is given about the velocities of the particles, hence we can not say anything about velocity of C.M.

We do not know about the nature of internal forces in the system. So we can not say that bodies must interact with each other.

14. (1), (2), (3), (4)

Velocity of centre of mass

$$v_c=\frac{mv_0+M\times0}{(m\times M)}=\frac{mv_0}{(m+M)}$$

Hence option (1) is correct.

Kinetic energy w.r.t C.M.

$$K=\frac{1}{2}\mu v_{rel}^2=\frac{1}{2}\frac{Mm}{(M+m)}(v_0-0)^2=\frac{1}{2}\frac{Mmv_0^2}{M+m}$$

Hence option (2) is correct.

If we apply W.E. theorem from C.M frame. In case of maximum work done by spring force, the deformation of the spring should be maximum. At this time both the blocks moves with same velocity.

$$W_{total}=\Delta K$$

$$W_{spring}=\left(\frac{1}{2}\mu v_{rel}^2\right)_{final}-\left(\frac{1}{2}\mu v_{rel}^2\right)_{initial}$$

$$=0-\frac{1}{2}\frac{Mm}{M+m}(v_0^2)=-\frac{1}{2}\frac{Mm}{M+m}v_0^2$$

Hence option (3) is correct.

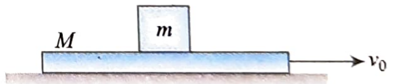
$$\text{Also }\frac{1}{2}kx_{max}^2=\frac{1}{2}\frac{Mm}{(M+m)}v_0^2$$

$$x_{max}=v_0\sqrt{\frac{Mm}{k(M+m)}}$$

Hence option (4) is also correct.

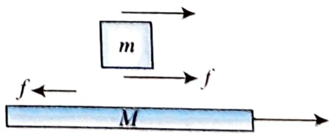
15. (1), (2), (3), (4)

If we take the 'block and plank' as system no external force acts on the system. The velocity of C.M. will be constant always.



$$v_c=\frac{m\times0+M\times v_0}{(M+m)}=\frac{Mv_0}{(M+m)}$$

The direction of the friction on m is in forward direction and on M will be in backward direction. Hence work done by friction of the block will be negative.



If we apply W.E. theorem from C.M. frame

$$W_{friction}=\Delta K=\left(\frac{1}{2}\mu v_{rel}^2\right)_{final}-\left(\frac{1}{2}\mu v_{rel}^2\right)_{initial}$$

$$=0-\frac{1}{2}\frac{Mm}{(M+m)}(v_0-0)^2$$

$$\Rightarrow W_{friction}=\frac{-1}{2}\frac{Mm}{(M+m)}v_0^2$$

16. (1), (3)

Let hollow sphere moves a distance 'x' leftwards.

As no external force is applied in horizontal direction.

$$\Delta x_{CM} = 0$$

$$\Rightarrow \frac{Mx + m(x - (R - r))}{M + m} = 0$$

$$\Rightarrow Mx + mx = m(R - r)$$

$$x = \frac{m}{M + m}(R - r) = \text{displacement of bigger sphere}$$

Displacement of smaller sphere $= x - (R - r) = \dfrac{M(R - r)}{M + m}$

17. (1), (2), (4)

Let common velocity is v. Using conservation of linear momentum,

$$\frac{m}{2}u = \left(m + \frac{m}{2}\right)v \Rightarrow v = \frac{u}{3}$$

Hence option (1) is correct.

Initial kinetic energy of the block B,

$$K_B = \frac{1}{2}\left(\frac{m}{2}\right)u^2 = \frac{1}{4}mu^2$$

Using work energy theorem, $W_{f.} = \Delta K$

$$W_f = \frac{1}{2}\left(\frac{3m}{2}\right)\frac{u^2}{9} - \frac{mu^2}{4} = \frac{mu^2}{6} = \frac{2}{3} \times \frac{1}{4}mu^2$$

Hence option (2) is correct.

Force of friction between blocks, $f = \mu\left(\dfrac{m}{2}\right)g$

Acceleration of A to right, $a_A = \dfrac{\mu mg}{2(m)} = \dfrac{\mu g}{2}$

Acceleration of B to left, $a_B = \dfrac{\mu mg}{2\left(\dfrac{m}{2}\right)} = \mu g$

Acceleration of A relative to B, $a_{AB} = a_A - (-a_B)$

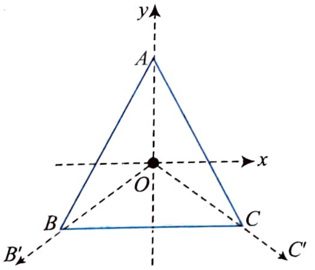
$$a_{AB} = \frac{\mu g}{2} + \mu g = \frac{3\mu g}{2}$$

Hence option (4) is correct.

18. (1), (3), (4)

When BC is removed, frame is symmetric about y-axis

$\Rightarrow$ C.M. shifts on y-axis only.



When AB is removed, frame is symmetric about OC' line

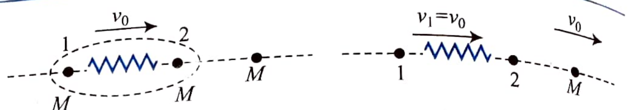
$\Rightarrow$ C.M. shifts on fourth quadrant.

When AC is removed, frame is symmetric about OB' line

$\Rightarrow$ C.M. shifts on third quadrant.

19. (1), (2), (3), (4)

$$v_{\text{cm}} = \frac{2Mv_0}{M + M + M} = \frac{2v_0}{3}$$



Just before collision

Just after collision

The collision of the molecule with atom,

Velocity of C.M. of molecule after collision

$$(v_{\text{cm}})_{\text{molecule}} = \frac{Mv_0 + M \times 0}{2M} = \frac{v_0}{2}$$

At the time of maximum compression of the spring both the atoms of the molecules moves with same velocity equal to centre of mass.

Now using mechanical energy theorem.

$$\Delta K + \Delta U = 0$$

$$\left[\frac{1}{2}(2M)\left(\frac{v_0}{2}\right)^2 - \frac{1}{2}Mv_0^2\right] + \Delta U = 0$$

$$\Rightarrow \Delta U = \frac{Mv_0^2}{4}$$

After collision linear momentum of the molecule

$$p_{\text{molecule}} = 2M \times 2m = 2M \times \frac{v_0}{2} = Mv_0$$

20. (1), (2), (3)

(1) No external force is acting so momentum conserved.

(2) In frame of centre of mass momentum is always zero.

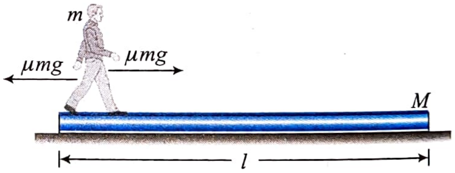
(3) v_c is constant.

(4) KE in frame of centre of mass will keep on converting in spring potential energy.

21. (1), (2), (4)

Maximum acceleration of man

$$a_m = \frac{\mu mg}{m} = \mu g \text{ (towards right)}$$



Maximum acceleration of the plank

$$a_m = \frac{\mu mg}{M} = \mu\frac{m}{M}g \text{ (toward left)}$$

When the acceleration of the man is maximum the acceleration of the plank will also be maximum. Hence option (1) is correct.

Relative acceleration of the man w.r.t. plank

$$a_{m,p} = \mu g - \left(-\frac{\mu m}{M}g\right) = \mu g\left[\frac{m + M}{M}\right]$$

Let time taken by man in crossing the plank be t, then

$$l = \frac{1}{2}\left[\mu g\left(\frac{m + M}{M}\right)t^2\right] \Rightarrow t = \sqrt{\frac{2Ml}{(M + m)\mu g}}$$

Hence option (2) is correct.

Displacement of C.M. of the plank, $\Delta x_{\text{plank}} = \dfrac{ml}{(M + m)}$

As no external force is acting on the system and C.M. of the system was initially at rest hence velocity of C.M. of the system should be zero.

22. (2), (3)

Since there is no external force on the system ('A' + 'B' + plank) is horizontal direction therefore centre of mass of the system remain at rest.

Let displacement of plank is x (towards right), then

$$50(x+5)+70(x-5)+80x=0$$

or $x = 0.5\text{ m} = 50\text{ cm}$

23. (1), (2), (3)

Let the velocity of the cannon be v (towards left). Linear momentum of the system will be conserved.

$$0=-nmv+m(v_0-v) \Rightarrow v=\frac{v_0}{n+1}$$

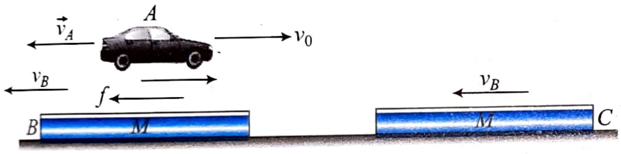
Velocity of the shell $v_s = v_0 - v = v_0 - \dfrac{v_0}{n+1} = \dfrac{nv_0}{n+1}$

Total energy released = change in K.E of the system.

$$\Delta KE=\frac{1}{2}\times nm\left(\frac{v_0}{n+1}\right)^2+\frac{1}{2}m\left(\frac{nv_0}{n+1}\right)^2=\frac{1}{2}mv_0^2\frac{n}{n+1}$$

24. (1), (3)

The friction force between the car 'A' and plank 'B' will be respectively for the motion of the plank 'B', towards left. As both the plank 'B' and 'C' are connected by string, both the plank will move with equal velocity towards left.



Let the velocity of the car 'A' and plank 'B' when the car moves with velocity v_0 be v_A (right) and v_B (left) respectively then

$$\vec{v}_A=\vec{v}_{A.B}+\vec{v}_B \Rightarrow v_A=v_0-v_B$$

or $v_A+v_B=v_0$ ...(i)

The linear momentum of the system car 'A', plank 'B' and plank 'C' should be zero.

$$0=mv_A+M(-v_B)+M(-v_B)$$

$mv_A = 2Mv_B$ ...(ii)

From (i) and (ii), we get

$$v_A=\frac{2Mv_0}{(m+2M)} \text{ and } v_B=\frac{mv_0}{(m+2M)}$$

After the car brakes to a stop, the tension in the string connecting A, B becomes zero. Applying conservation of momentum to A and C

$$mv_A - Mv_B = (m+M)v'_A$$

We find the velocity of A (to right)

$$\Rightarrow v'_A=\frac{mMv_0}{(m+M)(m+2M)}$$

Linked Comprehension Type

For Problems 1–3

1. (1) Physically possible explosions are those in which both particles move in opposite directions. i.e., signs of velocities are opposite.

$\Rightarrow$ II, IV and V

2. (2) If $m_A = m_B$, magnitude of velocities are same

$\Rightarrow$ II

If $m_A > m_B \Rightarrow v_A < v_B \Rightarrow$ V

If $m_A < m_B \Rightarrow v_A > v_B \Rightarrow$ IV

3. (4) As in VI momentum is not conserved.

$\Rightarrow$ Force must be acting on it.

For Problems 4–5

4. (3) **5. (1)**

Since rear buggy stops and velocity of man relative to this buggy is v, therefore, absolute velocity of man at the time of jump is v. Applying law of conservation of momentum on the system of rear buggy and man,

$mv = (150 + m)\,4$ ...(i)

Kinetic energy of man and front buggy (just before collision) is

$$E_1=\frac{1}{2}mv^2+\frac{1}{2}\times 150\times 4^2$$

Let velocity of the front buggy, after collision, be v_c.

Applying law of conservation of momentum on the system of front buggy and man, we get

$mv + 150 \times 4 = (m + 150)v_c$ ...(ii)

Kinetic energy of the system after collision is

$E_2=\frac{1}{2}(m+150)\,v_c^2$ ...(iii)

But $E_1 - E_2 = 2700$ J

From above three equations, $v_c = 7$ m/s

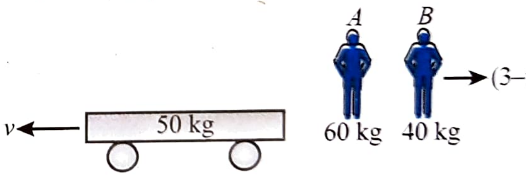
$m = 50$ kg

$v = 16$ m/s

For Problems 6–8

6. (4) Centre of mass will remain at rest because there is no external force on the system of A, B and platform.

7. (1) When both jump simultaneously.

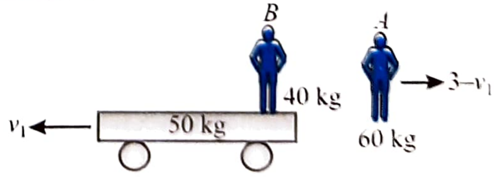


Let speed of the platform be v, then speed of A and B will be $3-v$ as shown because relative velocity is 3 m/s. Conserving momentum is

$$50v = (60+40)(3-v)$$

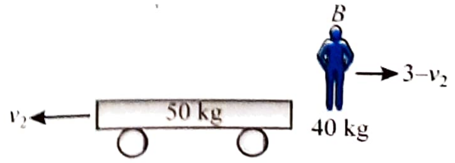
$\Rightarrow v = 2$ m/s

8. (4) After A jumps:



$(50 + 40)\,v_1 = 60\,(3 - v_1) \Rightarrow v_1 = 1.2$ m/s

After B jumps:



$(50 + 40)\,v_1 = 50v_2 - 40\,(3 - v_2)$

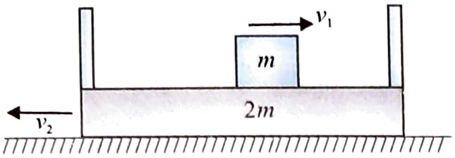
$\Rightarrow 90 \times 1.2 = 50v_2 - 120 + 40v_2$

$\Rightarrow v_2 = \dfrac{228}{90} = 2.53$ m/s

For Problems 9–11

9. (3) $mv_1 = 2mv_2$

$\Rightarrow v_1 = 2v_2$



$$\frac{1}{2}mv_1^2 + \frac{1}{2} \times 2mv_2^2 = \frac{1}{2}kx^2$$

Solving, we get

$$v_1 = 2\left(\sqrt{\frac{k}{6m}}\right)x,\ v_2 = \left(\sqrt{\frac{k}{6m}}\right)x$$

Relative velocity:

$$v_1 + v_2 = 3\left(\sqrt{\frac{k}{6m}}\right)x = x\sqrt{\frac{3}{2}\frac{k}{m}}$$

10. (1) Momentum of both will be same in magnitude and opposite in direction at any time.

11. (1) $\mathrm{KE}_i = \frac{1}{2}mv_1^2 + \frac{1}{2} \times 2mv_2^2 = \frac{1}{2}kx^2$

$$\mathrm{KE}_f = \frac{1}{2}m\left(\frac{v_1}{2}\right)^2 + \frac{1}{2} \times 2m\left(\frac{v_2}{2}\right)^2 = \frac{1}{4}\left[\frac{1}{2}kx^2\right]$$

$$\text{Loss} = \mathrm{KE}_i - \mathrm{KE}_f = \frac{3}{8}kx^2$$

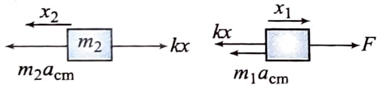
For Problems 12–14

12. (4) $F = (m_1 + m_2)a_{cm} \Rightarrow a_{cm} = \frac{F}{m_1 + m_2}$

13. (1) Centre of mass has acceleration, so it cannot be inertial frame of reference.

Kinetic energy becomes minimum in centre of mass frame.

14. (3)



$$x_1 + x_2 = x \qquad \text{...(i)}$$

$$(F - m_1a_{cm})x_1 + m_2a_{cm} \cdot x_2 - \frac{1}{2}kx^2 = 0 \qquad \text{...(ii)}$$

$$\text{where } a_{cm} = \frac{F}{m_1 + m_2} \qquad \text{...(iii)}$$

Solve to get answer.

For Problems 15–17

15. (2) Since the centre of mass will remain at rest.

$$m(5l - x) = Mx$$

$$x = \frac{5ml}{(M + m)}.$$

16. (3) $Mv_M - mv_m = 0$ ...(i)

$$\frac{1}{2}kl^2 = \frac{1}{2}mv_m^2 + \frac{1}{2}Mv_M^2 \qquad \text{...(ii)}$$

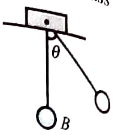
Solving Eqs. (i) and (ii), $v_M = \sqrt{\frac{km}{M(M+m)}}\,l$

17. (3) $v_m = \frac{M}{m}\sqrt{\frac{km}{M(M+m)}}\,l = \sqrt{\frac{kM}{m(M+m)}}\,l$

$$\mathrm{KE} = \frac{1}{2}mv_m^2 = \frac{kMl^2}{2(M+m)}$$

For Problems 18–21

18. (1) If ball and blocks are taken as system, then in horizontal direction no external force is present. Hence, centre of mass remains in rest in horizontal direction.



$\therefore\ m(L\sin\theta - x) - mx = 0$

or $mL\sin\theta - mx - mx = 0$

$$\therefore\ x = \frac{L}{2}\sin\theta$$

19. (4) Loss in PE = gain in KE

$$mg(L - L\cos\theta) = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

Applying momentum conservation principle in horizontal direction.

$$p_{ix} = p_{fx}$$

$$0 = mv_1 - mv_2$$

$$\therefore\ v_1 = v_2 = v$$

$$\therefore\ mg(L - L\cos\theta) = mv^2$$

$$\therefore\ v = \sqrt{gl(1 - \cos\theta)}$$

$$T - mg = \frac{m(2v)^2}{l}$$

$$\therefore\ T = mg + \frac{4mg\,l(1 - \cos\theta)}{l}$$

$$\therefore\ T = mg + 4mg(1 - \cos\theta) = mg(5 - \cos\theta)$$

20. (1) $v_{max} = \sqrt{gl(1 - \cos\theta)}$

(From solution of previous problem.)

21. (3) $y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$

$$\therefore\ \Delta y_{cm} = \frac{m_1\Delta y_1 + m_2\Delta y_2}{m_1 + m_2}$$

$$\Delta y_{cm} = \frac{0 - mL(1 - \cos\theta)}{m + m}$$

$$\therefore\ \Delta y_{cm} = \frac{L}{2}(1 - \cos\theta) \text{ in downward direction.}$$

For Problems 22–24

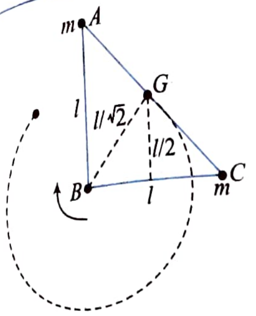
22. (1) **23. (1)** **24. (1)**

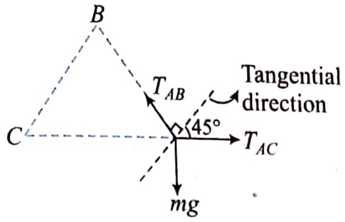
From the figure, the centre of mass of two body system (G) moves along a partial circular path with radius $\frac{l}{\sqrt{2}}$. Hence the maximum shift of centre of mass from initial position is $2 \times \frac{l}{\sqrt{2}} = \sqrt{2}\,l$.

The centre of mass is at lowest position when AC is horizontal. Since gravitational potential energy of system is least at this position, its kinetic energy shall be maximum.

Thus the speed of both objects shall be maximum at this position and as a result the tangential acceleration of both object is zero. Further as the system moves from initial position to position of maximum kinetic energy, the vertical fall in centre of mass of system is

$$\frac{l}{\sqrt{2}}(1 + \cos 45°) = \frac{l}{\sqrt{2}} + \frac{l}{2}$$





From conservation of energy, Gain in K.E. = loss in P.E.
Therefore total acceleration of mass A

$$\frac{1}{2}mv^2+\frac{1}{2}mv^2=(2m)gl\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)$$

or $v=\sqrt{gl(1+\sqrt{2})}$

Therefore total acceleration of mass A = normal acceleration of

mass A $=\frac{v^2}{l}=g(1+\sqrt{2})$

The free body diagrams of mass A when rod is horizontal is because tangential acceleration is zero.

$$T_{AC}\cos 45°=mg\sin 45°$$

or $T_{AC}=mg$

Matrix Match Type

1. i. → b., c.; ii. → a., d.; iii. → c.; iv. → a.

i. Internal forces may affect the motion of individual particles of a system, but not that of the centre of mass.

ii. Centre of momentum of a body can be inside or outside the body.

iii. Kinetic energy of a system of two blocks may increase, decrease or remain same with time, but it will have no effect on motion of the centre of mass.

iv. Centre of mass of a solid cylinder is inside the cylinder.

2. i. → d.; ii. → d.; iii. → c.; iv. → a., b., c.

i. in this option there is net momentum towards left after explosion which is not possible.

ii. in this option there is net momentum in vertical direction after explosion which is not possible.

iii. in vertical direction: $m_1v_1=m_2v_2\sin\theta$
Since $v_1>v_2$, so we should have $m_1<m_2$

iv. all three a., b. and c. are possible.

3. i. → a., c., d.; ii. → a., b., c., d.; iii. → a.; iv. → a., b.

$$v_{CM}=\frac{3\times 2+6\times 0}{3+6}=\frac{2}{3}\text{ m/s}$$

So velocity of centre of mass is always $\frac{2}{3}$ m/s.

i. If velocity of 3 kg block is $\frac{2}{3}$ m/s, then velocity of 6 kg block is also $\frac{2}{3}$ m/s. In this situation, spring will be maximum elongated. Their relative velocity will be zero.

ii. When the speed is $\frac{2}{3}$ m/s, velocity can be $\frac{2}{3}$ m/s, if

$$v_{3\text{ kg}}=-\frac{2}{3}\text{ m/s.}$$

$$v_{CM}=\frac{2}{3}=\frac{3\times\left(-\frac{2}{3}\right)+6v_2}{3+6}\Rightarrow v_2=\frac{4}{3}\text{ m/s}$$

Let deformation in spring be x. Then

$$\frac{1}{2}\times 3(2)^2=\frac{1}{2}\times 3\left(\frac{2}{3}\right)^2+\frac{1}{2}\times 6\left(\frac{4}{3}\right)^2+\frac{1}{2}kx^2$$

$$\Rightarrow x=0$$

iii. Minimum speed of 3 kg block is zero. At this speed, spring will have some deformation, but not maximum.

iv. Speed of 6 kg block will be maximum when spring returns to its normal length after passing through elongation.

4. i. → b.; ii. → a.; iii. → c.; iv. → a., b., d.

For 'A': Pure translational motion

For 'B': Pure rotational motion about centre of mass.

For 'C': No motion as $\sum\vec{F}=0$ and $\sum\vec{\tau}=0$

For 'D': Combined rotation and translational motion.

5. i. → a.; ii. → b.; iii. → a., c.; iv. → b., d.

i. If velocity of block A is zero, from conservation of momentum, speed of block B is $2u$. Then KE of block $B=\frac{1}{2}m(2u)^2=2mu^2$ is greater than net mechanical energy of the system. Since this is not possible, velocity of A can never be zero.

ii. Since initial velocity of B is zero, it shall be zero for many other instants of time.

iii. Since momentum of system is non-zero, KE of system cannot be zero. Also KE of system is minimum at maximum extension of the spring.

iv. The potential energy of spring shall be zero whenever it comes to natural length. Also PE of spring is maximum at maximum extension of the spring.

6. i. → b.; ii. → a., b.; iii. → c.; iv. → d.

i. Initial velocity of centre of mass of given system is zero and net external force is in vertical direction. Since there is shift of mass downwards, the centre of mass has only downward shift.

ii. Obviously, there is shift of centre of mass of given system downwards. Also the pulley exerts a force on string which has a horizontal component towards right. Hence, centre of mass of system has rightward shift.

iii. Both block and monkey move up, hence centre of mass of the given system shifts vertically upwards.

iv. Net external force on given system is zero. Hence, centre of mass of the given system remains at rest.

7. (1) Velocity of C.M. of system

$$v_{\text{cm}}=\frac{m_1v_1+m_2v_2}{m_1+m_2}=\frac{(1)(10)+(2)(-5)}{3}=0$$

It means the linear momentum of the C.M. will be zero.

$$p_{\text{cm}}=0$$

Net force on the system is zero hence, v_{cm} and p_{cm} will remain constant.

Velocity of the block and plank keep on changing initially and finally both of them stop as v_{cm} was zero.

It means the linear momentum and kinetic energy of the block first decreases than becomes zero.

8. (2) For (i) when man 1 alone jump to left

$$P_i=P_f\Rightarrow 0=60(1-v)+(80+20)v$$

$$\Rightarrow\quad v=-\frac{3}{8}$$

is opposite to velocity of Man 1 i.e., $\frac{3}{8}$ m/s towards right.

For (ii) $0=80(1+v)+(60+20)v\Rightarrow v=-\frac{1}{2}$ m/s

For (iii) $80(1+v)+60(-1+v)+20v=0$

$80-60+800+60v+20v=0$

$-20=16v;\ \Rightarrow v=-\dfrac{1}{8}\,\text{m/s}$

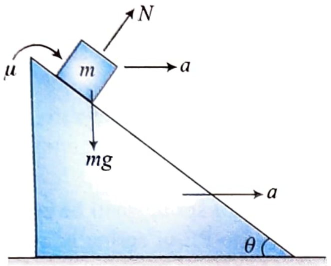
For (iv) After jump of Man 1

Velocity of plank + Man 2 is $\dfrac{3}{8}$ m/s towards right

Now $(80+20)\dfrac{3}{8}=80(1+v)+20v$

$v=-\dfrac{17}{40}\,\text{m/s}$

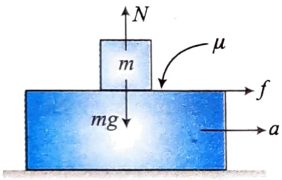
9. (4) For (i, ii):



Block will not slide on incline, so $F_{\text{net}}=ma$

Net reaction force $\left(\sqrt{N^2+f^2}\right)>ma$

For (iii):



$a<\mu g$

$\Rightarrow$ there is no sliding and $a_{\text{block}}=a$

so, $f=ma$ and $N=mg$

Net reaction force $=\sqrt{N^2+f^2}>ma$

For (iv):

$a<\mu g$

$\Rightarrow$ there is sliding and $a_{\text{block}}<a$

so, $f<ma$ and $N=mg$

So, net reaction force $\sqrt{N^2+f^2}$ may be less than *ma* or equal to *ma* or more than *ma*

10. (3) For (i): $a_{CM}=0$

$F_{\text{net external}}=0\Rightarrow$ momentum is conserved

$\Rightarrow$ work done by external = 0

$\Rightarrow$ mechanical energy is conserved

Potential energy gets converted to K.E.

$\Rightarrow$ KE ≠ constant

For (ii): $a_{CM}=0$

$F_{\text{net external}}=0\Rightarrow$ momentum is conserved

$\Rightarrow$ work done by external = 0

$\Rightarrow$ mechanical energy is conserved

KE gets stored in the masses during collision as PE

$\Rightarrow$ KE ≠ constant

For (iii): $a_{CM}=0$

$F_{\text{net external}}=0\Rightarrow$ momentum is conserved

$\Rightarrow$ work done by external = 0

$\Rightarrow$ mechanical energy is conserved

KE gets stored in the masses during collision as PE

$\Rightarrow$ KE ≠ constant

For (iv): $a_{CM}\neq 0$

$F_{\text{net external}}\neq 0$

$\Rightarrow$ mechanical energy is conserved

K.E. gets conserved in PE of spring $\Rightarrow$ KE ≠ constant.

11. (2) As no external forces are acting on the system in horizontal direction and the system is initially at rest. The velocity of centre of mass of system of block and wedge in horizontal direction is always zero.

(i) At maximum compression, the velocities of block and wedge must be same and equal to velocity of centre of mass Hence same and equal to velocity of centre of mass Hence velocity of both is zero and also minimum

(ii) At natural length of spring the velocity of system is zero as given.

(iii) At the instant acceleration of wedge is zero its velocity must be maximum. Hence velocity of block must be maximum.

(iv) When elastic energy stored in spring is least it must be in state of natural length.

Numerical Value Type

1. (8) As $\vec{F}_{\text{ext}}=0;\ \vec{a}_{\text{cm}}=0$

$$\vec{v}_{\text{cm}}=\frac{2\times 8\hat{i}-3\times 2\hat{i}}{2+3}=2\hat{i}$$

$$x_{\text{cm}}=\frac{2\times 3+3\times 8}{2+3}=6\,\text{m}$$

As $\vec{v}_{\text{cm}}$ is constant.

Therefore, CM will move 2 m in 1 s.

$\therefore\ 6+2=8$ m.

2. (5) Initial momentum

$$\vec{p}_i=-mv\hat{j}+mv\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)+mv\left(-\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)$$

Final momentum

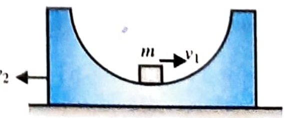
$$\vec{p}_f=-mv\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)+m\vec{v}_c$$

Now $\vec{p}_f=\vec{p}_i$

$$\Rightarrow\ \vec{v}_c=-v\hat{j}+v(\sqrt{3}\hat{i}+\hat{j})+v\left(-\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)$$

$$\Rightarrow\ \vec{v}_c=v\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)\Rightarrow v_C=5\,\text{m s}^{-1}$$

3. (2) Let smaller block is moving with speed v_1 relative to bigger block when it reaches the bottommost position and at this instant bigger block is moving at v_2 (say) then using conservation of momentum in horizontal direction we have



$$6\,mv_2 = m(v_1 - v_2) \qquad \text{...(i)}$$

Now using energy conservation

$$mga = \frac{1}{2}m(v_1 - v_2)^2 + \frac{1}{2}(6m)v_2^2 \qquad \text{...(ii)}$$

Solving (i) and (ii), we get

$$2\,ga = 36\,v_2^2 + 6\,v_2^2$$

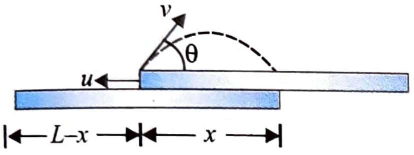
$$v_2 = \sqrt{\frac{ga}{21}} = \sqrt{\frac{10 \times 8.4}{21}} = 2\ \text{ms}^{-1}$$

4. (2) Displacement of wedge

$$\Delta x_{\text{wedge}} = \frac{m(2h\cot\theta + d)}{(M+m)} \text{ towards left}$$

$$\left(\frac{m}{2m+m}\right)(4 + 2\times1\times1) = \frac{1}{3}(6) = 2\ \text{m}$$

5. (5) Apply conservation of momentum in horizontal direction:



$$mv\cos\theta - mu = 0 \quad \Rightarrow \quad u = v\cos\theta$$

$$L - x = ut,\ x = v\cos\theta t$$

Solve to get, $x = \dfrac{L}{2}$

$$x = \frac{v^2\sin 2\theta}{g} \Rightarrow \frac{L}{2} = \frac{v^2 \sin 2\theta}{g}$$

$$\Rightarrow\ v = \sqrt{\frac{gL}{2\sin 2\theta}} \text{ for minimum } v,\ \sin 2\theta = 1$$

$$v_{\min} = \sqrt{\frac{gL}{2}} = \sqrt{\frac{10\times5}{2}} = 5\ \text{m/s}$$

6. (5) Let v_1 and v_2 are the velocities along the spring of A and B when spring regains it natural length then

$$2v_1 = 8v_2 \qquad \text{...(i)}$$

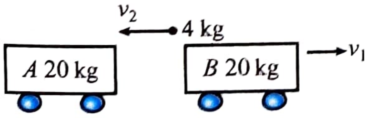
$$\frac{1}{2}2v_1^2 + \frac{1}{2}8v_2^2 = \frac{1}{2}1000\left(\frac{20}{100}\right)^2 \qquad \text{...(ii)}$$

From (i) and (ii), $v_1 = 4\ \text{ms}^{-1}$, $v_2 = 1\ \text{ms}^{-1}$

Ground frame velocity of $A = \sqrt{v^2 + v_1^2} = \sqrt{3^2 + 4^2} = 5\text{ms}^{-1}$

7. (1) All the velocities shown in diagrams are w.r.t. ground.

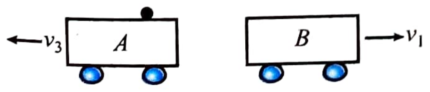
After first jump:



$20v_1 = 4v_2$ and $v_1 + v_2 = 6$ (given)

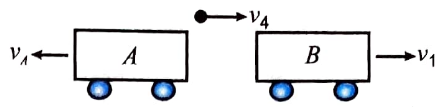
Solve to get $v_1 = 1$ m/s, $v_2 = 5$ m/s

When child arrives on A:



$$(20 + 4)\,v_3 = 4v_2 \Rightarrow v_3 = \frac{5}{6}\ \text{m/s}$$

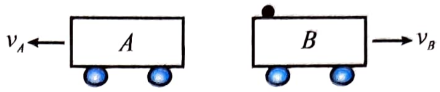
After the second jump:



$$v_4 + v_A = 6,\ 24v_3 = 20v_A - 4v_4$$

Solve to get $v_A = \dfrac{11}{6}$ m/s, $v_4 = \dfrac{25}{6}$ m/s

when child arrives on B:



$$24v_B = 4v_4 + 20v_1$$

$$\Rightarrow\ 24v_B = 4\left(\frac{25}{6}\right) + 20\times1 \ \Rightarrow\ v_B = \frac{55}{36}\ \text{m/s}$$

$$\text{Now } \frac{6v_B}{5v_A} = \frac{6\times55\times6}{36\times5\times11} = 1$$

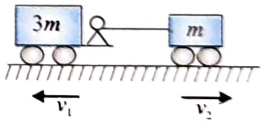
8. (3) To save himself, the man throws his jacket in opposite direction to the lake. According to momentum conservation, he himself gets a velocity in the direction of the lake. During the motion as gravity is the only external force on the system (man plus jacket), centre of mass will not be displaced horizontally. Thus, centre of mass of the system falls vertically and when the man falls in the lake, jacket falls at a point such that the centre of mass of the man and the jacket will be directly below the point from where the man jumps).

As it is given that man falls at a distance d from this point, it implies that jacket will fall at a distance x in the opposite direction such that

$$mx = Md \Rightarrow x = \frac{M}{m}d = 29\ \text{m},$$

man has to travel a distance $x + d = 29 + 1 = 30$ m to pick his jacket.

9. (9) Trolleys gain momentum due to force applied by man which will be internal force for the system of trolleys and man and there is no other external force. Here we assume that man applies force for a very short time, during which effect of friction can be neglected.



Momentum just before pushing = momentum just after pushing.

$$0 = 3\,mv_1 - mv_2 \Rightarrow v_1 = \frac{v_2}{3}$$

From work–energy theorem for individual trolleys,

$$f_1S_1 = \frac{1}{2}3mv_1^2,\quad f_2S_2 = \frac{1}{2}mv_2^2$$

Here $f_1 = \mu 3mg$, $f_2 = \mu mg$

$$\text{Solve to get } \frac{S_2}{S_1} = \left(\frac{v_2}{v_1}\right)^2 = 9$$

10. (2) Using conservation of linear momentum

$$(2m + m)V = 2m\times0 + mv\cos\alpha$$

$$V = \frac{mv\cos\alpha}{3m} = \frac{1}{3}v\cos\alpha = \frac{1}{3}\times12\times\cos60^\circ = 2\ \text{ms}^{-1}$$

11. (1670) External forces acting on the system are weights of the men, weight of the ladder and tension supporting the ladder.

$$F_{\text{ext}} = \Sigma m\vec{a}$$

$$(T - m_Ag - m_Bg - m_Cg - m_Lg) = m_Aa_A + m_Ba_A + m_Ca_A + m_La_A$$

Substituting given values of masses

$$m_A = 40\text{ kg}, m_B = 60\text{ kg}, m_C = 50\text{ kg}, m_L = 20\text{ kg}$$

given values of accelerations $g = 10\text{ m/s}^2$

$$a_A = -2\text{ m/s}^2, a_B = 0\text{ m/s}^2, a_C = 1\text{ m/s}^2 \text{ and } a_L = 0\text{ m/s}^2$$

we obtain $T - 400 - 600 - 500 - 200 = -80 + 0 + 50 + 0$

$$T = 1670\text{ N}$$

12. (5) Velocity of C.M. of 'A' and 'B', $v_{cm} = \frac{m(0) + 2m(v)}{3m} = \frac{2v}{3}$

Distance of block B from C.M., $r = \frac{ml}{m + 2m} = \frac{l}{3}$

As there is no external force in horizontal direction, it means C.M. will move with constant velocity and the masses will rotate by C.M.

The mass 'B' will rotate about C.M. in radius r with speed v_B.

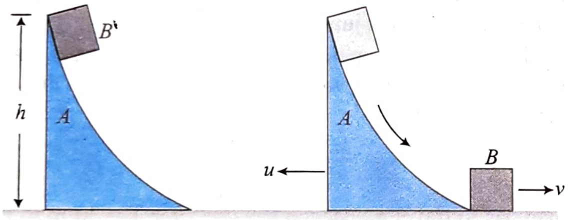
The speed of the mass 'B' w.r.t centre of mass $v_B = v - \frac{2v}{3} = \frac{v}{3}$

The tension will provide required centripetal force for rotation of mass 'B'

$$T = \frac{(2m)v_B^2}{r} = \frac{(2m)(v/3)^2}{l/3} = \frac{2mv^2}{3l}$$

Substituting the values we get, $T = 5$ N

13. (25.00)



Initial state Final state

(a)

Let u and v be the speed of wedge A and block B at just after the block B gets off the wedge A. Applying conservation of momentum in horizontal direction, we get

$$mu = mv \quad \text{...(i)}$$

Applying conservation of energy between initial and final state as shown in Fig. (a), we get

$$mgh = \frac{1}{2}mu^2 + \frac{1}{2}mv^2 \quad \text{...(ii)}$$

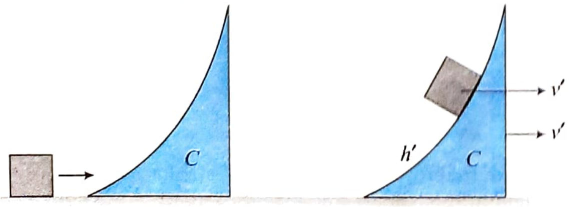
Solving (i) and (ii) we get

$$v = \sqrt{gh} \quad \text{...(iii)}$$

At the instant block B reaches maximum height h' on the wedge C (Fig. (b)), the speed of block B and wedge C are v'.

Applying conservation of momentum in horizontal direction, we get

$$mu = (m + m)v' \quad \text{...(iv)}$$



Initial state Final state

(b)

Applying conservation of energy between initial and final state

$$\frac{1}{2}mv^2 = \frac{1}{2}(m + m)v'^2 + mgh' \quad \text{...(v)}$$

Solving Eqs. (iii), (iv) and (v), we get $h' = \frac{h}{4}$

Hence $h' = \frac{100}{4} = 25\text{cm}$

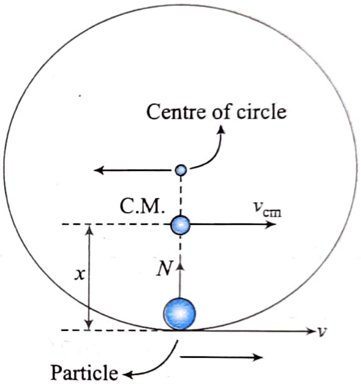
14. (6) $W_{\text{spring}} + W_{100\text{ N}} = \Delta K(\text{on } A \text{ and } B)$

$$W_{\text{spring}} + (100)\left(\frac{10}{100}\right) = \frac{1}{2}(2)(2)^2$$

$$W_{\text{spring}} = 4 - 10 = -6\text{ J}$$

15. (6) There is no external force acting on the system 'ring + particle', it means the C.M. will move with constant velocity and centre of ring and particle will rotate about C.M.

The radius of rotation of the particle, $x = \frac{MR}{(M + m)}$.



The normal reaction between the particle and ring provides required centripetal force for circular motion of the particle about C.M.

Hence $N = \frac{m(v - v_{cm})^2}{x}$

$$= \frac{m\left[v - \frac{mv}{M + m}\right]^2}{\frac{MR}{(M + m)}} = \frac{mMv^2}{(m + M)}$$

Substituting the value of m, M and v we get,

$$N = 6 \times 10^{-3}\text{ N}$$

16. (38.75)

$$a_{cm} = \frac{F_{ext}}{\text{mass}} = \frac{m_Pg + m_Qg}{m_P + m_Q} = g(\downarrow)$$

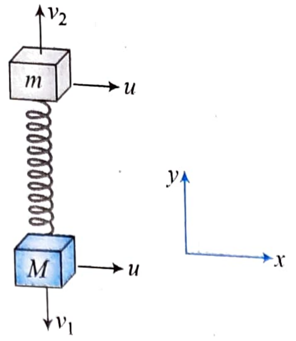
$$u_{cm} = \frac{m_Pu_P + m_Qu_Q}{m_P + m_Q} = \frac{4 \times 20 - 1 \times 5}{4 + 1} = 15\text{m/s}(\uparrow)$$

Initial height of COM above P is $h_0 = \frac{4 \times 0 + 1 \times 100}{5} = 20\text{m}$

Further height attained $h = \frac{u_{cm}^2}{2g} = \frac{(15)^2}{2 \times 10} = 18.75\text{m}$

Total height $H = 20 + 18.75 = 38.75$ m.

17. (5.00) When the string snaps, the component of velocity in x direction remains unchanged.



Linear momentum will remain conserved in y direction

$$mv_2 = Mv_1$$

$$2v_2 = 8v_1 \Rightarrow v_2 = 4v_1 \quad \dots\text{(i)}$$

Using conservation of mechanical energy,

$$\frac{1}{2}m(u^2+v_2^2)+\frac{1}{2}M(u^2+v_1^2)=\frac{1}{2}kx^2+\frac{1}{2}mu^2+\frac{1}{2}Mu^2$$

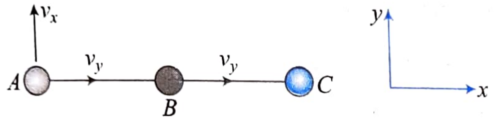
$$mv_2^2+Mv_1^2=kx^2 \qquad \text{(Using Eq. (i))}$$

$$2(4v_1)^2+8(v_1^2)=1000\times(0.2)^2$$

$$\therefore\ 40v_1^2=40 \text{ or } v_1=1 \text{ m/s}$$

The speed of block m, $v_m=\sqrt{u^2+v_2^2}=\sqrt{3^2+4^2}=5\text{ m/s}$

18. (6.00)



Particle A and C will collide when the strings BA and BC become parallel to y axis. At this time B will be moving along y with velocity v_y (say).

Component of velocity of A & C along y axis will also be v_y.

Particle A has a component of velocity along x axis as well (say v_x).

Momentum conservation along y direction gives

$$mv_0 = 3mv_y$$

$$\Rightarrow v_y=\frac{v_0}{3}=3\text{ m/s}$$

Energy conservation,

$$\frac{1}{2}mv_0^2=\frac{1}{2}m\left(\frac{v_0}{3}\right)^2+\frac{1}{2}(2m)\left(\left(\frac{v_0}{3}\right)^2+v_x^2\right)$$

$$\left[\therefore\ \text{Speed of } A \text{ and } C=\sqrt{v_y^2+v_y^2}=\sqrt{\left(\frac{v_0}{3}\right)^2+v_x^2}\right]$$

Solving, $v_x=3\sqrt{3}$ m/s

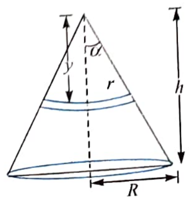
$\therefore$ Speed of A, $v_A=\sqrt{(3\sqrt{3})^2+3^2}=6$ m/s

Archives

JEE Main

Single Correct Answer Type

1. (2) $dm=\pi r^2\,.\,dy\,.\,r$

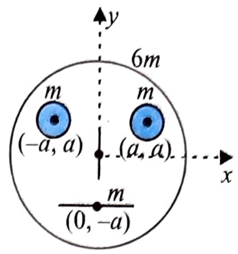


$$y_{CM}=\frac{\int y\,dm}{\int dm}=\frac{\int_0^h \pi r^2 dy\times\rho\times y}{\frac{1}{3}\pi R^2 h\rho}=\frac{3h}{4}$$

JEE Advanced

Single Correct Answer Type

1. (1) $Y_{CM}=\dfrac{6m\times0+m\times a+m\times a+m\times0+m\times(-a)}{10m}$

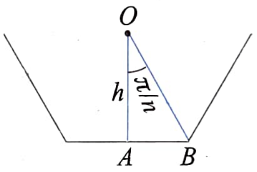


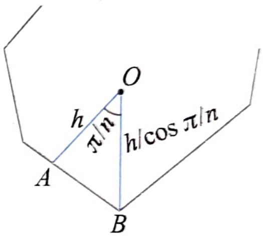
$$Y_{CM}=\frac{a}{10}$$

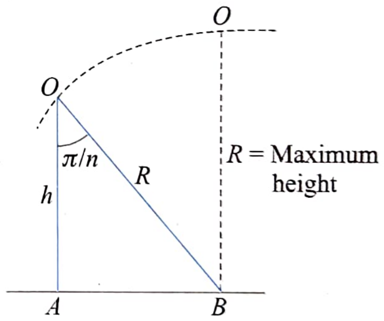
2. (3) Area under F–t curve = 4.5 kg m/s

$$\text{KE}=\frac{1}{2}(2)\left(\frac{4.5}{2}\right)^2=5.06\text{ J}$$

3. (3) $OA=h$







$$OB=\frac{h}{\cos\frac{\pi}{n}}$$

Initial height of $COM=h$

Final height of $COM=\dfrac{h}{\cos\left(\frac{\pi}{n}\right)}$

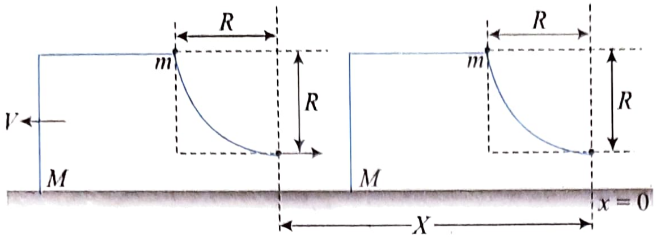
$$\therefore\ \Delta=\frac{h}{\cos\frac{\pi}{n}}-h$$

$$\Delta=h\left[\frac{1}{\cos\frac{\pi}{n}}-1\right]$$

Multiple Correct Answers Type

1. (1), (4)

There is no net external force acting on the system along the x-axis. Hence linear momentum of the system (block + point mass) should be conserved along x-direction.



$\therefore$ Along the x-axis

$mv + MV = 0$ ...(i)

From conservation of mechanical energy

Loss in GPE of particle of mass m = Gain in kinetic energy of both the block and point mass.

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2$$

$$\Rightarrow 2gh = \left(1 + \frac{m}{M}\right)v^2 \Rightarrow v = \sqrt{\frac{2gh}{1+\frac{m}{M}}} \quad \text{...(ii)}$$

Hence option (1) is correct.

From (i) and (ii)

$$V = -\frac{m}{M}v = -\frac{m\sqrt{2gh}}{\sqrt{M(m+M)}}$$

Block 'M' will move along –ve x-axis.

Hence option (2) is incorrect.

Displacement of block $\Delta x_M = -X$

Displacement of point mass $\Delta x_m = -(X - R)$

Since the location of centre of mass does not change along the x-axis

$$m.\Delta x_m + M.\Delta x_M = 0$$

$$-m(X - R) - MX = 0$$

$$-(M + m)X + mR = 0 \Rightarrow X = \frac{mR}{(M + m)}$$

Hence displacement of block $M = -\frac{mR}{(M + m)}$

Displacement of the mass 'm' $= -(X - R)$

$$\Rightarrow \Delta x_m = -\left(\frac{mR}{M + m} - R\right) = \frac{MR}{(M + m)}$$

Hence final position of 'm'

$$x_m = -x + \Delta x_m = -R + \frac{MR}{(M + m)} = -\frac{mR}{(M + m)}$$

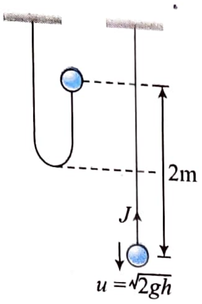
Hence option (3) is incorrect.

Chapter 2

Concept Application Exercises

Exercise 2.1

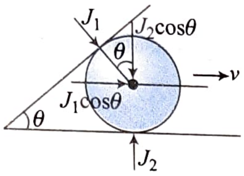
1. The string will become taut when the particle will fall through a distance 2 m in downward direction. So applying impulse momentum equation: $mu - J = 0$



$$J = mu = m\sqrt{2gh}$$

$$= 1\sqrt{2 \times 10 \times 2} = 2\sqrt{10}\ \text{kg m/s}$$

2. As the collision is elastic, so the sphere will rebound with the same speed after collision.



$$J_1 \cos\theta = J_2 \quad \text{...(i)}$$

$$-mv + J_1 \sin\theta = mv \quad \text{...(ii)}$$

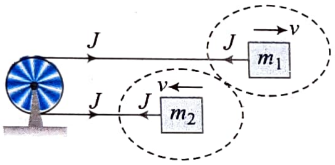
Solving Eqs. (i) and (ii), we get

$$J_2 = 2mv\cot\theta$$

$$J_1 = 2mv\ \text{cosec}\,\theta$$

3. During the impact, a large tension develops. The impulse of tension T_1 is equal to the change in momentum of m_1 which is given as

$$m_1 v_0 - J = m_1 v \quad \text{...(i)}$$



Similarly, the impulse of tension T_2 is equal to the change in momentum of m_2 which is given as

$$J = m_2 v \quad \text{...(ii)}$$

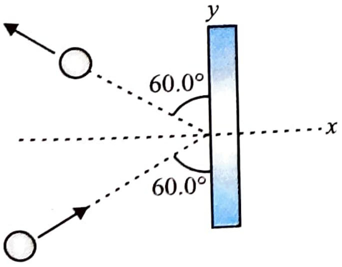
From Eqs. (i) and (ii), we have $v = \dfrac{m_1 v_0}{m_1 + m_2}$.

4. $\Delta\vec{p} = \vec{F}\,\Delta t$

$$\Delta p_y = m(v_{fy} - v_{iy}) = m(v \cos 60°) - mv \cos 60° = 0$$

$$\Delta p_x = m(-v \sin 60° - v \sin 60°) = -2mv \sin 60°$$

$$= -2(3.00\ \text{kg})(10.0\ \text{m/s})(0.866) = -52.0\ \text{kg m/s}$$



$$F_{\text{avg}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0\ \text{kg m/s}}{0.200\ \text{s}} = -260\ \text{N}$$

5. We have, Force $F = mnu$

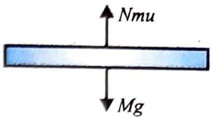
where m = mass of bullet, u = speed of bullet, n = number of bullets per unit time.

$$\Rightarrow \quad 200 = \frac{50}{1000} \times 2000 \times n$$

Hence, number of bullets fired per minute = 60 × 2 = 120.

6. The change in momentum of a bullet due to hitting the plate is $\Delta\vec{p} = 0 - m\vec{u}$. If N bullets be hitting the plate per unit time, the rate of change of momentum of the bullets is $-Nm\vec{u}$. This is also the force exerted by the plate during collision. Using Newton's third law of motion, an opposite force, $Nm\vec{u}$ will act on the plate due to the colliding bullets.

Considering the equilibrium of forces acting on the plate,



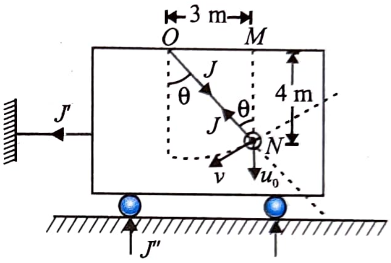
$$Nmu + Mg = 0$$

$$\Rightarrow \quad Nmu = Mg$$

$$N = \frac{Mg}{mu}$$

This is the number of bullets striking the plate per unit time.

7. From M to N the ball will fall a distance 4 m, then impulse will be developed in the string ON.



Velocity of the ball just before the string becomes tight

$$u_0 = \sqrt{2 \times 10 \times 4} = \sqrt{80}\ \text{m/s}$$

Velocity of the ball in the direction perpendicular to the impulse will remain same (just after and just before the impulse developed).

$$v = u_0 \sin\theta = \sqrt{80} \times \frac{3}{5} = \frac{12}{\sqrt{5}}\ \text{m/s}$$

$$J = mu_0 \cos\theta = 1 \times \sqrt{80} \times \frac{4}{5} = \frac{16}{\sqrt{5}}\ \text{kg m/s}$$

$$J' = J \sin\theta = \frac{48}{5\sqrt{5}}\ \text{kg m/s}$$

and impulse given by earth,

$$J'' = J \cos\theta = \frac{64}{5\sqrt{5}}\ \text{kg m/s}$$

Exercise 2.2

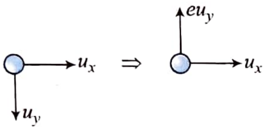
1. (d) According to law of conservation of energy, total energy always remain conserved. But due to dissipation of energy, kinetic energy will not be conserved in elastic collision. As no external force is acting, there momentum will remain conserved.
2. (c) As the collision is inelastic, kinetic energy will not be conserved but for bullet and target as a system total momentum will remain conserved.
3. (a) During elastic collision between two bodies at the point of maximum deformation, potential energy stored in body's deformation is maximum so kinetic energy of system is minimum. Thus during collision kinetic energy of system is not conserved. The kinetic energy is conserved just before and just after collision.
4. (c) Since external forces are absent, so the total momentum of the system remains constant. However, internal forces, may alter momentum of each particle. But since internal forces are always equal and opposite, so the change in momentum (during any time

interval) corresponding to these forces are also equal and opposite. In words, momentum is exchanged.

5. (b) Any type of finite forces are much smaller than impulsive force. Also if we take both the balls as system the impulse acting on the balls will be equal and opposite hence will not change the linear momentum of the system.
6. (d) The linear momentum of the system is always conserved in any type of collision. Because the loss of kinetic energy, the velocity of separation is less than velocity of approach. In case of perfectly inelastic collision the coefficient of restitution is zero.
7. The ball moves to right with a speed v; it comes back with a velocity $\vec{v}\,' = -ev\hat{i}$. Then the total time of journey is $\Delta t = \frac{l}{v} + \frac{l}{ev}$.

Hence, the average speed is $v_{av} = \frac{l+l}{\Delta t} = \frac{2l}{l/v + l/ev} = \frac{2ev}{e+1}$.

8. Just after collision x-component of velocity remain unchanged but y-component of velocity becomes e times.



Just before collision Just after collision

Now, $T = \frac{2u_y}{g}, H = \frac{u_y^2}{2g}$ and $R = \frac{2u_x u_y}{g}$

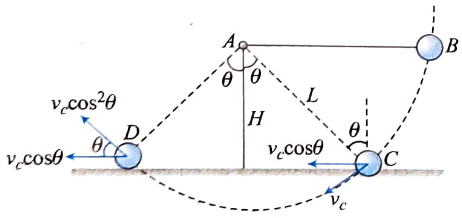
or $T \propto u_y$, $H \propto u_y^2$ and $R \propto u_y$

$$\therefore \quad \frac{T_1}{T_2} = a = \frac{u_y}{eu_y} = \frac{1}{e}$$

$$\frac{R_1}{R_2} = b = \frac{u_y}{eu_y} = \frac{1}{e}$$

$$\text{and} \quad \frac{H_1}{H_2} = c = \frac{(u_y)^2}{(eu_y)^2} = \frac{1}{e^2}$$

9. In perfectly inelastic collision with the horizontal surface the component parallel to the surface will remain unchanged. Similarly when the string becomes taut again, the component perpendicular to its length will remain unchanged.



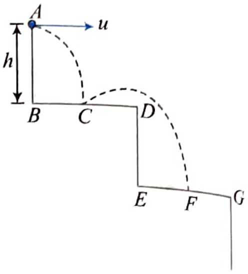
$$\cos\theta = \frac{H}{L} \Rightarrow v_c = \sqrt{2gH}$$

$$v_c\cos^2\theta = \left(\sqrt{2gH}\right)\frac{H^2}{L^2} = v(\text{say})$$

$$\therefore \quad h = \frac{v^2}{2g} = \frac{(2gH)\frac{H^4}{L^4}}{2g} = \frac{H^5}{L^4}$$

10. Let height of each step be h. The horizontal velocity of the ball during the motion remains constant.

Thus, the journey from C to F takes twice the time as taken from A to C.



Time of flight from A to C, $t = \sqrt{\frac{2h}{g}}$

And vertical component of velocity with which ball strikes at C is given by

$$v^2 = 0^2 + 2gh \Rightarrow v = \sqrt{2gh}$$

The velocity with which ball rebounds,

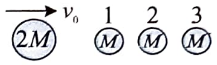
$$v_1 = ev$$

$$-h = ev(2t) - \frac{1}{2}g(2t)^2 \text{ or } -h = 2e(2h) - 4h$$

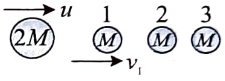
$$\text{or} \quad 3h = 4eh \Rightarrow e = \frac{3}{4}$$

Exercise 2.3

1. Assume some spacing between the balls

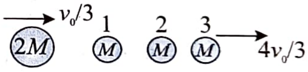


After collision of $2M$ with ball '1',

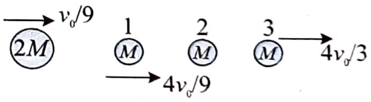


$$u = \left(\frac{2M - M}{2M + M}\right)v_0 = \frac{v_0}{3},\ v_1 = \frac{2\,(2M)\,v_0}{2M + M} = \frac{4}{3}v_0$$

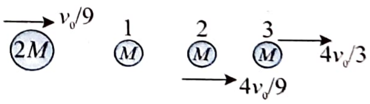
Now ball '1' collides with "2' and itself comes to rest, then '2' will collide with '3' and itself comes to rest, so we have



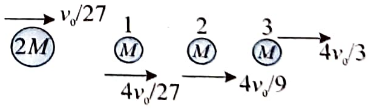
Now again $2M$ will collide with ball '1', after this collision we have



Now '1' will collide with '2' and itself comes to rest, so we have



Now $2M$ will again collide with '1', after this collision we have



So final velocities are as shown in above figure.

2. Let velocities of sphere 1 and 2 after collision be v_1 and v_2 respectively, then

$$\frac{v_2 - v_1}{u} = e \text{ and } mv_2 + mv_1 = mu$$

From above equations: $v_2 = \frac{(1+e)u}{2}$

Now sphere 2 will collide with sphere 3 and after collision velocity of 3 can be formed as

$$v_3 = \left(\frac{1+e}{2}\right)^2 u$$

Hence, similarly velocity of nth sphere can be formed as

$$v_n = \left(\frac{1+e}{2}\right)^{n-1} u$$

3. Since m_2 is at rest ($u_2 = 0$), velocity of m_1 after collision:

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1$$

Given, $v_1 < 0 \Rightarrow m_1 - em_2 < 0$

$\Rightarrow \quad m_1 < em_2 \Rightarrow m_1 < m_2$

4. $u_1 = 12$ m/s $\qquad m_1 = 4$ kg

$u_2 = 4$ m/s $\qquad m_2 = 8$ kg

Let v_1 and v_2 be the velocities after impact.

Conservation of momentum:

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

$\Rightarrow \quad 4v_1 + 8v_2 = 80 \qquad$ ...(i)

Newton's experimental law: $v_1 - v_2 = -e(u_1 - u_2)$

$v_1 - v_2 = -0.5\ (12 - 4) = -4 \qquad$ ...(ii)

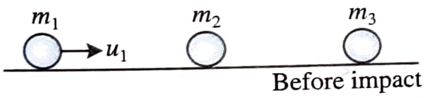
Solving Eqs. (i) and (ii), we get

$v_1 = 4$ m/s, $v_2 = 8$ m/s

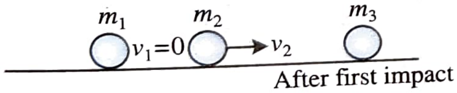
$$\text{Loss in KE} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

$$= \frac{1}{2}[4(144) + 8(16)] - \frac{1}{2}[4(16) + 8(64)] = 64\text{ J}$$

5.



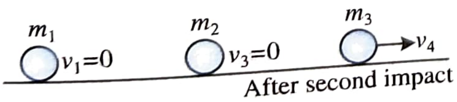
Before impact



After first impact

After first impact: $v_1 = 0$

$$\Rightarrow \quad \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 = 0 \Rightarrow m_1 = em_2 \qquad \text{...(i)}$$



After second impact

After second impact: $v_3 = 0$

$$\Rightarrow \quad \left(\frac{m_1 - em_3}{m_1 + m_3}\right)v_2 = 0 \qquad \text{...(ii)}$$

$\Rightarrow \quad m_2 = em_3$

From (i) and (ii), $m_2^2 = m_1m_3$

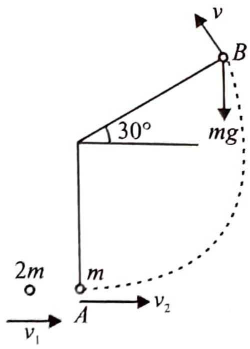
6. Velocity of B after collision:

$$v_2 = \frac{(1+e)m_1u_1}{m_1 + m_2} = \frac{(1+e)m \times 10}{m+m} \Rightarrow v_2 = 5\,(1+e)$$

Height raised by $B = \dfrac{v_2^2}{2g}$

Now, $\dfrac{v_2^2}{2g} \geq 3.2 \Rightarrow \dfrac{(5(1+e))^2}{2 \times 10} \geq 3.2 \Rightarrow e \geq \dfrac{3}{5}$

7. v_1 and v_2 are after collision.



$$v_2 = \frac{(1+e)2mu}{2m+m} = \frac{2u(1+e)}{3}$$

Since tension becomes zero at B, so

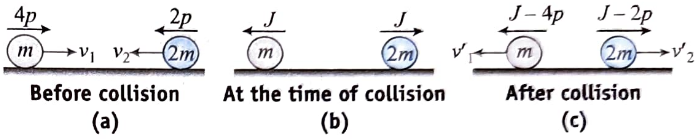
$$mg\sin 30° = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{gr}{2}}$$

Apply conservation of energy between A and B

$$\frac{1}{2}mv^2 + mg(r + r\sin 30°) = \frac{1}{2}mv_2^2$$

$\Rightarrow \quad e = 0.5$

8. Directions of linear impulses on the two colliding bodies at the time of collision are shown in Fig. (b).



Before collision (a) **At the time of collision** (b) **After collision** (c)

Linear impulse is equal to the change in linear momentum. Hence, momenta of the balls after collision are shown in Fig. (c).

Let v_1 and v_2 are their velocities before collision [in the directions shown in Fig. (a)] and v_1' and v_2' are the velocities after collision as shown in Fig. (c).

As $v = \dfrac{\text{linear momentum}}{\text{mass}}$ and $e = \dfrac{\text{relative speed of separation}}{\text{relative speed of approach}}$

$$e = \frac{v_1' + v_2'}{v_1 + v_2} = \frac{\dfrac{J-4p}{m} + \dfrac{J-2p}{2m}}{\dfrac{4p}{m} + \dfrac{2p}{2m}}$$

or $\quad e = \dfrac{3J - 10p}{10p} = \dfrac{3J}{10p} - 1$

For elastic collision, $e = 1$

$\Rightarrow \quad \dfrac{3J}{10p} = 2$ or $J = \dfrac{20}{3}p$

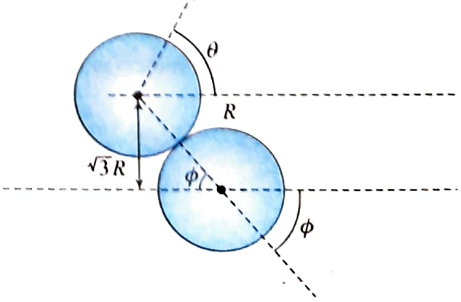
For perfectly inelastic collision, $e = 0$

$\Rightarrow \quad \dfrac{3J}{10p} = 1$ or $J = \dfrac{10}{3}p$

Exercise 2.4

1. (c, d) Linear momentum conservation along horizontal can be made only if there is no friction or non-impulsive friction.
2. (a, d) For elastic collision,

$$\theta + \phi = 90°$$

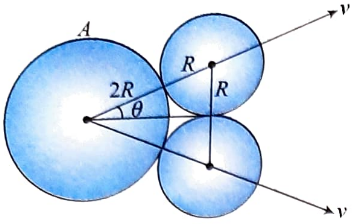


and from figure we use $\sin\phi = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \phi = 60°$ hence $\theta = 30°$

3. $\cos\theta = \dfrac{2\sqrt{2}R}{(2R+R)} = \dfrac{2\sqrt{2}}{3}$

Let initial velocity of A is u



Final velocity of small balls = v

Applying conservation of linear momentum

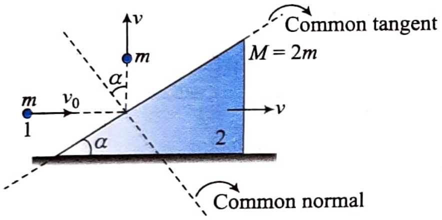
$$mu = 2mv\cos\theta$$

$$e = \frac{\text{separation velocity along line of impact}}{\text{approach velocity along line of impact}}$$

$$\Rightarrow \quad e = \frac{v}{u\cos\theta}$$

$$\Rightarrow \quad e = \frac{1}{2\cos^2\theta} = \frac{1}{2\times\frac{8}{9}} = \frac{9}{16}$$

4. In common tangent direction the speed of the particle will remain unchanged.



$$v_0\cos\alpha = v\sin\alpha \text{ or } v = v_0\cot\alpha$$

We can conserve linear momentum of the system 'particle + wedge' along horizontal direction.

$$mv_0 + 0 = 2mV \Rightarrow V = \frac{v_0}{2}$$

Coefficient of restitution, $e = \dfrac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$

These relative velocities are along common normal.

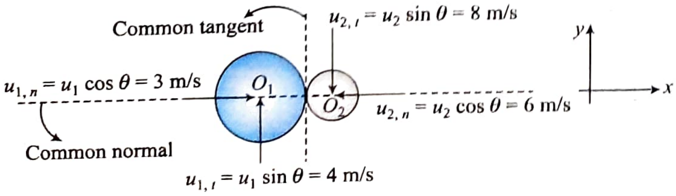
Hence, $e = \dfrac{V\sin\alpha + v\cos\alpha}{v_0\sin\alpha} = \dfrac{V}{v_0} + \dfrac{v}{v_0}\cot\alpha = \dfrac{1}{2} + \cot^2\alpha$

But $\tan\alpha = 2$ or $\cot\alpha = \dfrac{1}{2}$

$$\Rightarrow \quad e = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

5. In common tangent direction the velocities of the spheres will not change. Hence

$$v_{1,t} = u_{1,t} = 4\text{ m/s and } v_{2,t} = u_{2,t} = 8\text{ m/s}$$


Common tangent
$u_{2,t} = u_2 \sin\theta = 8$ m/s
y
x
$u_{1,n} = u_1\cos\theta = 3$ m/s
O_1
O_2
$u_{2,n} = u_2\cos\theta = 6$ m/s
Common normal
$u_{1,t} = u_1\sin\theta = 4$ m/s

In common normal direction we can conserve the linear momentum of the system

$$2m\times 3 - m\times 6 = 2mv_{1,n} + mv_{2,n}$$

or $\quad 2v_{1,n} + v_{2,n} = 0 \quad$ ...(i)

Restitution equation $e = \dfrac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$

$$\frac{5}{9} = \frac{v_{2,n} - v_{1,n}}{(3+6)}$$

or $\quad v_{2,n} - v_{1,n} = 5 \quad$ ...(ii)

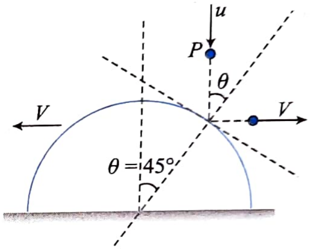
From Eqs. (i) and (ii),

$$v_{1,n} = -\frac{5}{3}\text{ m/s and } v_{2,n} = \frac{10}{3}\text{ m/s}$$

Hence final velocity of ball (1), $\vec{v}_1 = \left(-\dfrac{5}{3}\hat{i} + 4\hat{j}\right)$ m/s

and final velocity of ball (2), $\vec{v}_2 = \left(\dfrac{10}{3}\hat{i} - 8\hat{j}\right)$ m/s

6. Let the velocity of hemisphere and the particle just after collision be V and v respectively. Linear momentum of the system will be conserved in horizontal direction,



$mV = mv$ or $v = V \quad$ ...(i)

The coefficient of restitution equation

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} \text{ or } 1 = \frac{V\sin\theta + v\sin\theta}{u\cos\theta}$$

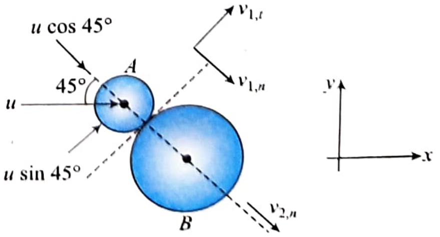
$V + v = u \quad$ ...(ii)

From (i) and (ii), $v = V = \dfrac{u}{2} = \sqrt{\dfrac{gh}{2}}$

7. Velocity of disc 'A' in tangent direction

$v_{1,t} = u\sin 45° = \dfrac{u}{\sqrt{2}} \quad$ ...(i)

The disc 'B' will move along common normal direction only.



And according to equation both the disc move perpendicular to each other. It means disc 'A' should move along common tangent direction.

Hence $v_A = \dfrac{u}{\sqrt{2}}$ along common tangent direction.

Now conserving linear momentum along common normal direction

$$m\cdot\frac{u}{\sqrt{2}}+0=m\times 0+2mv_{2,n}$$

or $$v_{2,n}=\frac{u}{2\sqrt{2}} \qquad \text{...(ii)}$$

or velocity of disc 'B' is $\frac{u}{2\sqrt{2}}$ along common normal.

Now coefficient of restitution

$$e=\frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}=\frac{v_{2,n}-0}{u\cos 45^\circ}=\frac{u/2\sqrt{2}}{u/\sqrt{2}}=\frac{1}{2}$$

8. (a) Velocity of centre of will be same before collision and after collision.

$$v_{cm}=\frac{m_1v_0+m_2\times 0}{m_1+m_2}=\frac{m_1v_0}{(m_1+2m_1)}=\frac{v_0}{3}$$

The velocities of centre of mass will remain unchanged after collision.

(b) Conserving linear momentum

In x-direction: $m_1v_0=m_2v_2\cos\theta$

or $$v_2\cos\theta=\frac{v_0}{2} \qquad \text{...(i)}$$

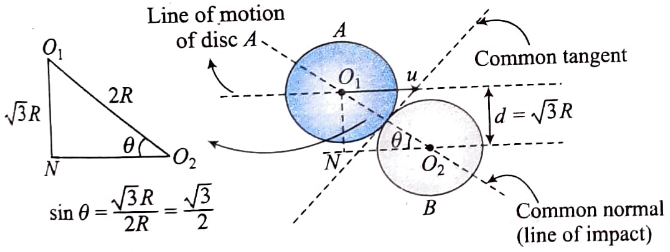
In y-direction: $0=m_2v_2\sin\theta-m_1\dfrac{v_0}{2}$

or $$v_2\sin\theta=\frac{v_0}{4} \qquad \text{...(ii)}$$

Squaring and adding (i) and (ii), we get

$$v_2^2=\left(\frac{v_0}{2}\right)^2+\left(\frac{v_0}{4}\right)^2 \Rightarrow v_2=\frac{\sqrt{5}}{4}v_0$$

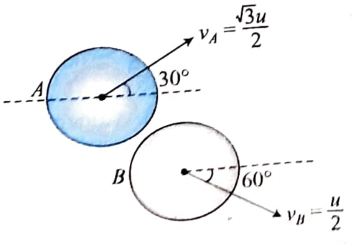
9.



$$\sin\theta=\frac{\sqrt{3}R}{2R}=\frac{\sqrt{3}}{2}$$

$$\theta=\frac{\pi}{3}\text{ or }60^\circ$$

As collision is elastic and balls are identical the velocities of balls will be interchanged along common normal direction. Hence velocity of ball 'A' along common normal will be zero and velocity of ball 'B' along common normal will be $u\cos\theta$ or $u\cos 60^\circ$ or $\frac{u}{2}$.



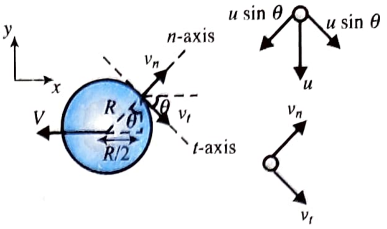
Along common tangent the velocities of the balls will not change. Hence ball 'A' will move in common tangent direction (30° above x-axis) with speed $u\sin 60^\circ$ or $\frac{u\sqrt{3}}{2}$.

10. Collision takes place between smooth bodies; therefore, tangential velocity of particle is unchanged.

$$v_t=u\sin\theta \text{ and } u=\sqrt{2gh}$$

where $\sin\theta=\dfrac{R/2}{R}=\dfrac{1}{2}$ or $\theta=30^\circ$

$$v_t=\frac{u}{2} \qquad \text{...(i)}$$



From conservation of momentum along x-axis, $p_i=p_f$

$$0+0=-MV+mv_n\sin\theta+mv_t\cos\theta \qquad \text{...(ii)}$$

From the definition of coefficient of restitution,

$$e=\frac{V\sin\theta-(-v_n)}{u\cos\theta-0}=0$$

[$e=0$, for a perfectly inelastic collision]

or, $V\sin\theta=-v_n$

$$v_n=-\frac{V}{2} \qquad \text{...(iii)}$$

On substituting expressions for v_n and v_t in Eq. (ii), we obtain

$$MV=m\left(-\frac{V}{2}\right)\frac{1}{2}+m\left(\frac{u}{2}\right)\frac{\sqrt{3}}{2}$$

$$\left(M+\frac{m}{4}\right)V=m\frac{u\sqrt{3}}{4}$$

where $u=\sqrt{2gh}$ or, $V=\dfrac{m\sqrt{6gh}}{(4M+m)}$

$$v=\sqrt{v_n^2+v_t^2}=\sqrt{\frac{6m^2gh}{4(4M+m)^2}+\frac{gh}{2}}$$

Exercise 2.5

1. (c) The horizontal component of velocity of sand just before falling on the cart is $v_s=0$.

The horizontal speed of cart = v_c (constant)

The rate of mass falling on car = μ (constant).

Horizontal force exerted by falling sand on cart = μv_{rel}

$$=\mu(v_c-v_s)=\mu v_c$$

∴ μ and v_c are constant, the horizontal force is constant.

2. $$F=v_{rel}\left(\frac{dm}{dt}\right)$$

$$=(v_{rel})(\text{mass of each shot})(\text{No. of shots/second})$$

$$=(300)(40\times 10^{-3})(40)\text{N}=48\text{ N}$$

3. Equation of motion we use for mass and container,

For mass $m_1g-T=m_1a$ ...(i)

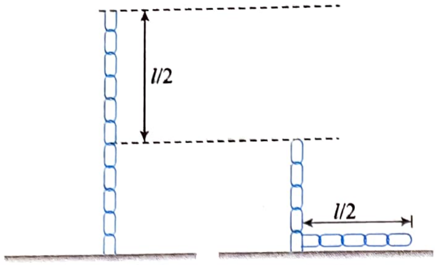
For container $T+bv_0-(m_0-bt)g=(m_0-bt)a$ ...(ii)

Adding above two equations, we get

$$m_1g+bv_0-m_0g+btg=(m_0+m_1-bt)a$$

$$\Rightarrow \quad a=\frac{(m_1-m_0+bt)g+bv_0}{m_0+m_1-bt}$$

4. Weight of the portion of the chain lying on the table, $W=\dfrac{mg}{2}$ (downwards)



Initially Finally

Using $v=\sqrt{2gh}$

Thrust force $F_{\text{thrust}}=v_{\text{rel}}\left(\dfrac{dm}{dt}\right)$

Here $v_{\text{rel}}=v$ and $\dfrac{dm}{dt}=\lambda v$

$$F_{\text{thrust}}=\lambda v^2$$

(where $\lambda=\dfrac{m}{l}$, is mass per unit length of chain)

$$v^2=2g\times\frac{l}{2}=gl$$

$$F_{\text{thrust}}=\left(\frac{m}{l}\right)(gl)=mg \text{ (downwards)}$$

$$F=W+F_{\text{thrust}}=\frac{mg}{2}+mg=\frac{3}{2}mg$$

5. The water leaks with zero relative velocity. Hence no reaction force is exerted on the container.

Then, $F_{\text{ext}}=Ma$

$$0=(M-\mu t)a \qquad (\because F_{\text{ext}}=0)$$

Since $M-\mu t\neq 0, a=0$

$$v=v_0 \text{ and } a=0$$

6. $-\dfrac{dm}{dt}v_{\text{rel}}=m\dfrac{dv}{dt}$

$$-\int_{m_0}^{M}\frac{dm}{m}v_{\text{rel}}=\int_{v_0}^{v}dv$$

$$v=v_0-v_{\text{rel}}\ln\frac{M}{m_0}$$

7. (i) The external force F_{ext} must overcome the impact force F_{thrust}

Then, $F_{\text{ext}}=F_{\text{thrust}}=v_{\text{rel}}\dfrac{dm}{dt}=\mu v$

(ii) The power delivered by the external force. $P=Fv=\mu v^2$

(iii) The rate of change of kinetic energy

$$\frac{dk}{dt}=\frac{d}{dt}\left(\frac{1}{2}mv^2\right)=\frac{v^2}{2}\frac{dm}{dt}=\frac{\mu v^2}{2}$$

The energy is lost during mechanical collision of sand with cart.

8. (i) Let velocity of the block at any time be v.

The thrust force on the block is

$$F_{\text{thrust}}=\rho A v_{\text{rel}}^2=(u-v)^2A\rho \qquad \text{...(i)}$$

The power due to impact force is

$$P=F_{\text{thrust}}.v=A\rho v_{\text{rel}}^2\cdot v=A\rho(u-v)^2v \qquad \text{...(ii)}$$

For maximum power, $\dfrac{dP}{dv}=0$

or $A\rho\{-v\times 2(u-v)+(u-v)^2\}=0\Rightarrow v=\dfrac{u}{3}$

The power applied on the block due to the thrust imparted by water jet on the block will be maximum when the velocity of the block reaches one third of water jet velocity.

Putting $v=\dfrac{u}{3}$, in Eqs. (i) and (ii) we get

$$F_{\text{thrust}}=\frac{4}{9}A\rho u^2 \text{ and } P_{\max}=\frac{4}{27}A\rho u^3$$

(ii) The thrust force on the block is

$$F_{\text{thrust}}=\rho A v_{\text{rel}}^2=(u-v)^2A\rho$$

$$(u-v)^2A\rho=\frac{Mdv}{dt}$$

$$\int_0^v\frac{dv}{(u-v)^2}=\frac{A\rho}{M}\int_0^t dt$$

which gives, $v=\dfrac{u}{\left(1+\dfrac{M}{A\rho ut}\right)}$

Exercises

Single Correct Answer Type

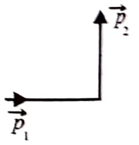
1. (1) $m_1u_1=m_2v_2$

$$\frac{1}{2}m_2v_2^2=\frac{1}{2}\left[\frac{1}{2}m_1u_1^2\right]$$

$$\Rightarrow (m_2v_2)v_2=\frac{1}{2}(m_1u_1)u_1 \Rightarrow \quad v_2=\frac{u_1}{2}$$

$$e=\frac{v_2-v_1}{u_1-u_2}=\frac{v_2-0}{u_1-0}=\frac{v_2}{u_1} \quad\Rightarrow\quad e=\frac{1}{2}$$

2. (4) $\vec{p}_1=2\times 3\hat{i}=6\hat{i}, \vec{p}_2=6\hat{j}$



$$\vec{I}=\vec{p}_2-\vec{p}_1=6\hat{j}-6\hat{i}$$

or $I=\sqrt{6^2+6^2}=6\sqrt{2}$ N s

3. (1) Initial momentum: $3\times 4+4\times(-3)=0$

Final momentum will also be zero. But both stick together, hence both come at rest.

4. (3) Component of initial momentum along the wall is

$$p_{iy}=mv_0\sin 37°\,\hat{j}$$

Component of final momentum along the wall is

$$p_{fy}=\frac{mv_0}{2}\sin 53°\,\hat{j}$$

So, component of impulse along the wall is $J_y=p_{fy}-p_{iy}$

$$=\frac{mv_0}{2}\times\frac{4}{5}-\frac{mv_0}{1}\times\frac{3}{5}\hat{j}=-\frac{mv_0}{5}\hat{j}$$

5. (3) Impulse = area of trapezium $=\frac{1}{2}\left(T+\frac{T}{2}\right)F_0=\dfrac{3TF_0}{4}$

According to impulse–momentum theorem,

Impulse = Change in momentum

$$\Rightarrow \quad \frac{3TF_0}{4} = mu \Rightarrow F_0 = \frac{4mu}{3T}$$

6. (4) As the collision is elastic, we can find

$$v_1 = \frac{1-n}{1+n}u,\ v_2 = \frac{2u}{1+n}$$

m
nm
u
$u_2 = 0$
m
nm
v_1
v_2

Before collision **After collision**

Hence, the required fraction is

$$\frac{\frac{1}{2}nmv_2^2}{\frac{1}{2}mu^2} = n\left(\frac{v_2}{u}\right)^2 = \frac{4n}{(1+n)^2}$$

7. (2) $\dfrac{\frac{1}{2}mv_1^2}{\frac{1}{2}nmv_2^2} = \dfrac{v_1^2}{nv_2^2} = \dfrac{(1-n)^2}{4n}$

8. (2) Force on table due to collision of balls:

$$F_{\text{dynamic}} = \frac{dp}{dt} = 2 \times 20 \times 20 \times 10^{-3} \times 5 \times 0.5 = 2\text{ N}$$

Net force on one leg $= \frac{1}{4}(2 + 0.2 \times 10) = 1$ N

9. (4) By conservation of linear momentum along the string

$$mu = (m + m + 3\,m)\,v \text{ or } v = \frac{u}{5}$$

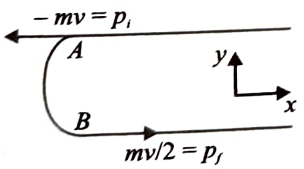
and impulse on block $A = 3\,m\,(v - 0) = \dfrac{3\,mu}{5}$

10. (3) $F_x = \dfrac{\Delta p_x}{\Delta t} = \dfrac{mv_{2x} - mv_{1x}}{\Delta t} = \dfrac{m(v_{2x} - v_{1x})}{\Delta t}$

$$= \frac{(150\times10^{-3})[30 - (-\,15 \sin 30°)]}{0.01}$$

$$= \frac{(150\times10^{-3})(30+7.5)}{0.01} = 562.5\text{ N}$$

11. (2) The momentum of the bead at A is $p_i = -\,mv\,\hat{i}$



The momentum of the bead at B is $\vec{p}_f = \left(\frac{mv}{2}\right)\hat{i}$

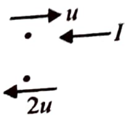
Therefore, the magnitude of the change in momentum between A and B is

$$\Delta p = \left|\vec{p}_f - \vec{p}_i\right| = \left(\frac{3}{2}\right)mv$$

Average force exerted by the bead on the wire is

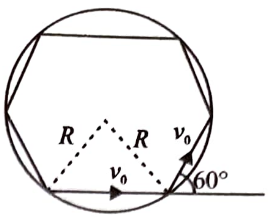
$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{3mv}{2T}$$

12. (2) $I = m\,[2u - (-\,u)] = 3mu$



$$W = \frac{1}{2}m\,[(2u)^2 - u^2] = \frac{3mu^2}{2} = \frac{Iu}{2}$$

13. (1)



I = change in momentum

$$= \sqrt{(mv_0)^2 + (mv_0)^2 - 2(mv_0)^2 \cos 60°}$$

$$= mv_0\sqrt{2 - 2\cos(\pi/3)} = 2\,mv_0 \sin\frac{\pi}{6}$$

14. (1) Velocity of light ball, when it is descended by height $2l$

$$v = \sqrt{2g \times 2l} = 2\sqrt{gl}$$

From conservation of linear momentum.

$$m \times 2\sqrt{gl} = 2mv' + mv'$$

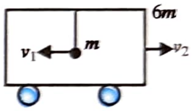
$$2m\sqrt{gl} = 3mv'$$

$$\frac{2}{3}\sqrt{gl} = v'$$

15. (3) When the rod becomes vertical

$$mv_1 = 6mv_2 \Rightarrow v_1 = 6v_2 \quad \text{...(i)}$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}6mv_2^2 = mgl \quad \text{...(ii)}$$



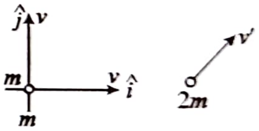
Solve (i) and (ii) to get

$$v_{\text{rel}} = v_1 + v_2 = \sqrt{\frac{7}{3}gl}$$

16. (2) Conservation of linear momentum yields

$$mv\hat{i} + mv\hat{j} = (2m)\vec{v}'$$

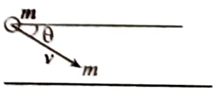
$$\Rightarrow \vec{v}' = \frac{v}{2}(i + j) \Rightarrow v' = \frac{v}{\sqrt{2}}$$



$$\eta = 1 - \frac{KE_f}{KE_i} = 1 - \frac{(1/2)(2m)v'^2}{(1/2)mv^2 + (1/2)mv^2}$$

$$\Rightarrow \eta = 1 - \left(\frac{v'}{v}\right)^2 = 1 - \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

17. (4) The impact takes place along the line joining them. Since the particle at rest is constrained to move on the horizontal surface, we cannot conserve the momentum along the line joining them. Since no net force is acting on the system of particles parallel to the surface, conservation of linear momentum along that direction yields



$$mv\cos\theta = (m + m)\,v' \Rightarrow v' = \frac{v}{2}\cos 60° = \frac{v}{4}$$

18. (3) By formula derived earlier,

$$\Delta\text{KE} = \frac{m_1 m_2 v_r^2}{2(m_1 + m_2)}(1 - e^2)$$

$$\frac{\Delta\text{KE}}{\text{KE}} = \frac{m_2}{m_1 + m_2}(1 - e^2) \text{ where KE} = \frac{1}{2}m_1 v^2;$$

$m_1 = m_2 = m$

$$\Rightarrow \frac{25}{100} = \frac{1}{4} = \frac{1}{2}(1-e^2) \quad (\because \Delta\text{KE of 25\% of KE of } m_1)$$

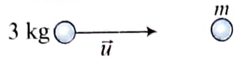
$$e = \frac{1}{\sqrt{2}}$$

19. (1) Conservation of linear momentum:

$3u = (3u/2) + mv$

$\Rightarrow mv = (3u/2)$ ...(i)

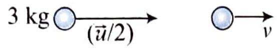
Just before collision



Newton's experimental law:

$u = -e(u/2 - v)$

Just after collision



Putting $e = 1$ for elastic collision, we obtain

$$v = \frac{3u}{2} \qquad \text{...(ii)}$$

Using (i) and (ii), we obtain $m = 1$ kg.

20. (3) As $|\vec{p}_1| = |\vec{p}_2| = |\vec{p}| = p_0$ (say), $\vec{p}$ is final momentum

From conservation of linear momentum

$$\vec{p}_1 + \vec{p}_2 = \vec{p} \quad \therefore \quad p = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\theta}$$

or $p = \sqrt{p^2 + p^2 + 2p^2\cos\theta}$

$$\therefore \quad \cos\theta = \frac{-1}{2} \Rightarrow \theta = 120°$$

21. (3) The impact force $F = (\Delta p/\Delta t)$ where Δp = change of momentum of water of mass Δm striking the pendulum with a speed v during time Δt. Since water falls dead after collision with the ping-pong ball

$$\Delta p = \Delta m\, v \Rightarrow F = v\frac{\Delta m}{\Delta t}$$

where $\frac{\Delta m}{\Delta t}$ = rate of flow of water in the nozzle.

Since the ball is in equilibrium,

$F - mg = 0 \Rightarrow F = mg$

$$\Rightarrow \frac{v\Delta m}{\Delta t} = mg \Rightarrow \frac{\Delta m}{\Delta t} = \frac{mg}{v}$$

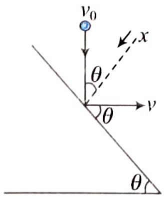
22. (2) Writing coefficient of restitution equation

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0 - v\sin\theta}{-v_0\cos\theta - 0} = \frac{v}{v_0}\tan\theta \qquad \text{...(i)}$$

The velocity of ball will remain unchanged in the direction of sloping surface (common tangent)

$v_0 \sin\theta = v\cos\theta$

$$\Rightarrow \frac{v}{v_0} = \tan\theta \qquad \text{...(ii)}$$



$\Rightarrow e = \tan^2\theta$

23. (3) Let final velocity of combined system is v, then apply conservation of linear momentum.

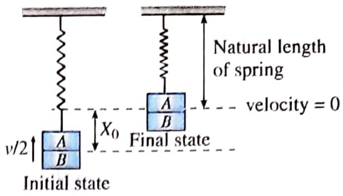
$3 \times 1 + 1 \times 1 - 2 \times 2 = 6v$

$v = 0$

Impulse experienced by A = change in momentum of A

$J = 3[0 - 1] = -3$ Ns

24. (2) The initial extension in spring is $x_0 = \frac{mg}{k}$. Just after collision of B with A the speed of combined mass is $v/2$. For the spring to just attain natural length the combined mass must rise up by $x_0 = \frac{mg}{k}$ (see the figure) and comes to rest.



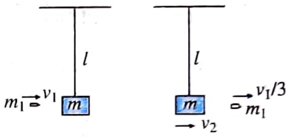
Applying conservation of energy between initial and final state:

$$\frac{1}{2}2m\left(\frac{v}{2}\right)^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = 2mg\left(\frac{mg}{k}\right)$$

Solving we get $v = \sqrt{\frac{6mg^2}{k}}$

25. (1) From conservation of momentum:

$$mv_2 + m_1v_1/3 = m_1v_1 \Rightarrow v_2 = \frac{2m_1v_1}{3m}$$



Now $v_2 = \sqrt{5gl}$

$$\frac{2m_1v_1}{3m} = \sqrt{5gl} \Rightarrow v_1 = \frac{3m}{2m_1}\sqrt{5gl}$$

26. (2) Conserving the momentum of two balls before and after collision in horizontal direction

$1 \times 6 = (1 + 1) \times v$ or $v = 3\ \text{ms}^{-1}$

Now at the instant of maximum deflection ball will be moving horizontal with same speed as of trolley (i.e., velocity of ball w.r.t. trolley will be 0)

Let it be v_0

$2 \times 3 = (4 + 2) \times v_0$

Now conserving energy,

$$\frac{1}{2} \times 2 \times 3^2 - 2 \times 10 \times 1.5(1 - \cos\theta) = \frac{1}{2} \times 2 \times v_o^2 + \frac{1}{2} \times 4 \times v_o^2$$

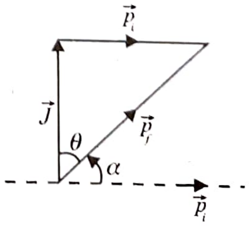
$\Rightarrow \cos\theta = 0.8 \Rightarrow \theta = 37°$

27. (3) From impulse–momentum theorem,

$$\vec{J} = \vec{p}_f - \vec{p}_i \Rightarrow \vec{p}_f = \vec{J} + \vec{p}_i$$

Here $p_i = 3000 \times 300 = 9 \times 10^5$ kg m/s

$J = 4000 \times 225 = 9 \times 10^5$ kg m/s



$$\tan\theta = \frac{p_i}{J} = 1 \Rightarrow \theta = 45°$$

So, the angle by which the space probe rotates is

$$\alpha = \frac{\pi}{2} - \theta = 45°$$

28. (3) For elastic collision, $e = 1$. Let speed of the ball be v towards left after collision, then w.r.t. wall, incident velocity = reflected velocity. We get $3 + 3 = v - 3 \Rightarrow v = 9$ m/s.

29. (3) When two identical balls undergo elastic oblique collisions with one of them stationary, then after collision they will move at right angles to each other.

Alternative method I: Just like in previous question, you can proceed using centre of mass frame of reference.

Alternative method II: Using basic equations of conservation of momentum, conservation of energy and coefficient of restitution, you can have the desired result.

30. (4) In case I there is no horizontal external impulse on ball and mass system while in case II friction is impulsive as normal with ground is impulsive. Hence, both statements are incorrect.

31. (2) Apply conservation of momentum after piercing M_1:

$$mu = M_1v + mu_1 \quad \text{...(i)}$$

After it enters into M_2, finally bullet also moves with velocity v, so

$$mu_1 = (M_2 + m)v \quad \text{...(ii)}$$

From (i) and (ii), we get

$$mu = (M_1 + M_2 + m)v \quad \text{...(iii)}$$

$$\text{Required percentage loss } = \frac{u - u_1}{u} \times 100 = \frac{m(u - u_1)}{mu} \times 100$$

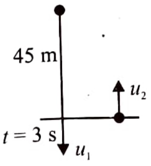
$$= \frac{M_1v}{(M_1 + M_2 + m)v} \times 100 = \frac{1}{1 + 2.98 + 0.02} \times 100 = 25\%$$

32. (1) $u_1 = \frac{p}{m}$, $u_2 = 0$, $v_1 = \frac{(p - J)}{m}$, $v_2 = \frac{J}{m}$

$$\text{Now apply } e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\frac{J}{m} - \frac{p - J}{m}}{p/m} = \frac{J - p + J}{p} = \frac{2J - p}{p}$$

33. (2) $\hat{j}$ component, i.e., component of velocity parallel to wall remains unchanged while $\hat{i}$ component will become $(-1/2)(2\hat{i})$ or $-\hat{i}$. Therefore, velocity vector of the sphere after it hits the wall is $-\hat{i} + 2\hat{j}$.

34. (3) Time taken to fall through 45 m $= \sqrt{\frac{2 \times 45}{10}} = 3$ s



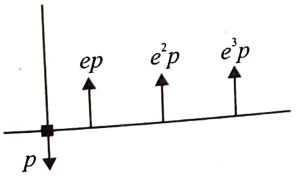
$$u_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 45} = 30 \text{ m/s}$$

$$u_2 = eu_1 = \frac{2}{3} \times 30 = 20 \text{ m/s}$$

Distance travelled in the fourth second = distance travelled in 1 s after rebounding

$$= 20 \times 1 - \frac{1}{2} \times 10\,(1)^2 = 15 \text{ m}$$

35. (4)



$$\Delta p = (p + ep) + (ep + e^2p) + (e^2p + e^3p) + \cdots$$

$$= p(1 + e)\,[1 + e + e^2 + \cdots] = \frac{p(1 + e)}{1 - e}$$

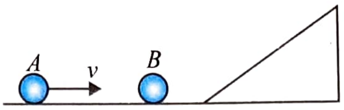
36. (1) $\frac{h_1}{h_0} = e^2 \Rightarrow \frac{80}{100} = e^2 \Rightarrow e = \sqrt{0.8}$

Velocity before the first impact $u = \sqrt{2gh}$

Velocity after the third impact $v_3 = e^3\sqrt{2gh}$

$$\text{KE} = \frac{1}{2}mv_3^2 = e^6\,mgh = (0.8)^3\,mgh$$

37. (2) $v = 16$ m/s



Let velocity of B after collision be v_2 and that of A be v_1. Then

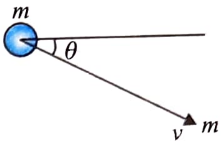
$$v_2 = \sqrt{2gs} = \sqrt{2 \times 10 \times 5} = 10 \text{ m/s}$$

$$mv_1 + mv_2 = mv$$

$$v_1 + 10 = 16 \Rightarrow v_1 = 6 \text{ m/s}$$

$$e = \frac{v_2 - v_1}{v} = \frac{10 - 6}{16} = \frac{1}{4}$$

38. (4) The impact takes place along the line joining them. Since the particle at rest is constrained to move on the horizontal surface, we cannot conserve the momentum along the line joining them. Since no net force is acting on the system of particles parallel to the surface, conservation of linear momentum along that direction yields the following result.



$$mv\cos\theta = (m + m)v' \Rightarrow v' = \frac{v}{2}\cos 60° = \frac{v}{4}$$

39. (1) As coefficient of restitution equation is valid in all frames of reference, so in the frame of lift

$$e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}} = \frac{v_f}{v_i} = \frac{\sqrt{2gh_2}}{\sqrt{2gh_1}} \quad \text{...(i)}$$

$$h_2 = e^2h_1$$

Impulse delivered by the elevator to ball is

$$J = |m\vec{v}_f - m\vec{v}_i| = (m\sqrt{2gh_2} + \sqrt{2gh_1})$$

If the elevator is accelerating, then replacing g with $g + g/4$, we get $h_2 = e^2h_1$.

40. (2) $v_0 = \sqrt{2gh}$ before collision.

Apply conservation principle of momentum,

$$mv_0 = (m + m)u$$

$$\therefore\; u = \frac{v_0}{2}$$

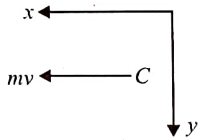
Applying conservation principle of energy,

$$\frac{1}{2}(2m)u^2 = 2mgh'$$

$$\therefore\; h' = \frac{u^2}{2g} \Rightarrow h' = \frac{v_0^2}{4g}$$

$$h' = \frac{2gh}{4g} \Rightarrow h' = \frac{h}{2}$$

41. (3) The velocity of the particle when it reaches point A from B is $v = \sqrt{(2gh)}$ (directed along BA)



$$(p_i)x = mv\cos\theta \quad \text{...(i)}$$

$$(p_i)y = mv\sin\theta \quad \text{...(ii)}$$

When the particle reaches point C, the momentum is as shown in figure.

$$(p_f)_x = mv \quad \text{...(iii)}$$

$$(p_f)_y = 0 \quad \text{...(iv)}$$

Therefore, $(\Delta p)_x = (p_f)_x - (p_i)_x = mv - mv\cos\theta$ ...(v)

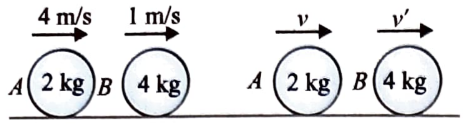
$$(\Delta p)_y = (p_f)y - (p_i)_y = -\,mv\sin\theta$$

Magnitude of change in momentum is

$$\sqrt{[mv(1-\cos\theta)]^2+(mv\sin\theta)^2}=2m(\sqrt{2gh})\sin\left(\frac{\theta}{2}\right)$$

42. (4) First consider the collision of balls A and B. Let the velocities of these two balls after their collision be v and v'.

Momentum after impact = Momentum before impact



$\therefore\ 2v + 4v' = 2 \times 4 + 4 \times 1$

$\therefore\ 2v + 4v' = 12$

$\Rightarrow v + 2v' = 6$ ...(i)

Relative velocity after impact = – e × relative velocity before impact

$v - v' = -1\,(4-1)$

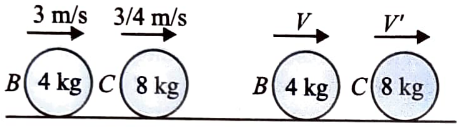
$v - v' = (-3)$ ...(ii)

Subtracting Eq. (ii) from Eq. (i), we get

$3v' = 9$ or $v' = 3$ m/s

Substituting in Eq. (ii), we get

$v - 3 = -3 \Rightarrow v = 0$



Hence, after the collision ball A is brought to rest, while ball B will move with a velocity of 3 m/s. Now consider the collision of balls B and C. Let the velocities of these balls after collision be V and V', respectively.

Total momentum after impact = Total momentum before impact

$4V + 8V' = 4 \times 3 + 8 \times \frac{3}{4} = 18$

$V + 2V' = \frac{9}{2}$ ...(iii)

Relative velocity after impact = – e × relative velocity before impact

$V - V' = (-1)\,(3 - 3/4)$

$V - V' = -9/4$ ...(iv)

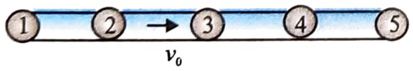
Subtracting Eq. (iv) from Eq. (iii), we get

$3V' = 27/4 \Rightarrow V' = 9/4$ m/s

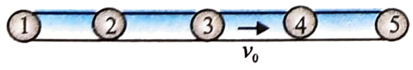
$\therefore\ V = 0$

$v_A = 0,\ v_B = 0$ and $v_C = \frac{9}{4}$ m/s

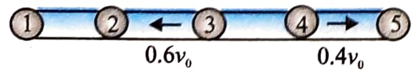
43. (4) The situation after various collisions is as shown in figure.



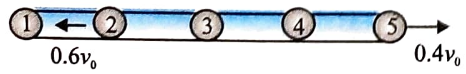
Initial given situation



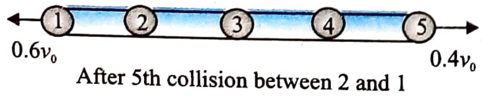
After 1st collision between 2 and 3



After 2nd collision between 3 and 4



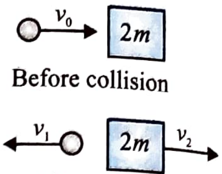
After 3rd and 4th collision between 4 and 5 and 3 and 2



After 5th collision between 2 and 1

After first collision, the momenta of 1 and 2 will be exchanged (property of elastic collision) and hence second ball starts to move towards 3 with velocity v_0.

44. (1) Velocity of the ball just before collision is $v_0 = \sqrt{2gL}$



Before collision

After collision

Applying momentum conservation along horizontal direction (because momentum is conserved in collision along the line of impact), we get

$mv_0 = -mv_1 + 2\,mv_2$

Applying coefficient of restitution equation, we get

$$e = \frac{2}{3} = \frac{v_1 + v_2}{v_0}$$

Solving the above two equations, we get

$$v_1 = \frac{v_0}{9} \text{ and } v_2 = \frac{5v_0}{9}$$

As the block moves a distance of $3L/2$ before coming to rest, so from work–energy theorem,

$$0 - \frac{1}{2}(2m)v_2^2 = -\mu \times 2\,mg\,\frac{3L}{2} \Rightarrow v_2^2 = 3\mu gL$$

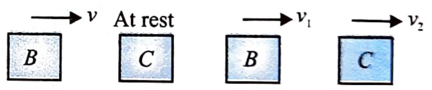
$$\frac{25}{81} \times 2\,gL = 3\,\mu gL \Rightarrow \mu = \frac{50}{243}$$

For ball, KE is converted to gravitational potential energy after collision, so

$$0 - \frac{mv_1^2}{2} = -\,mg \times L\,(1 - \cos\theta)$$

$$\cos\theta = \frac{80}{81}$$

45. (4) For collision of B and C



Before collision

After collision

$2mv = 2mv_1 + mv_2$

$$\frac{1}{2} = \frac{v_2 - v_1}{v} \Rightarrow 2v_2 - 2v_1 = v$$

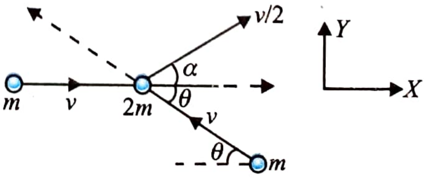
Solving above equations, $v_1 = \frac{v}{2}$ and $v_2 = v$

Now for blocks A and B plus spring system, using reduced mass concept, and applying work–energy theorem, let maximum compression in spring be x_0 and at the time of maximum compression relative velocity of blocks be zero. Reduced mass is given by

$$\mu = \frac{m \times 2m}{3m} = \frac{2m}{3}$$

$$0 - \frac{\mu \times (v - v/2)^2}{2} = -\frac{kx_0^2}{2} \Rightarrow x_0 = \left(\sqrt{\frac{m}{6k}}\right)v$$

46. (1) Let angle between initial velocities be $\pi - \theta$ and the situation is as shown in figure. Conserve the momentum along X- and Y-directions.



For X-axis: $mv - mv\cos\theta = 2m\frac{v}{2}\cos\alpha$,

For Y-axis: $mv\sin\theta = 2m\frac{v}{2}\sin\alpha$,

Solving above equations, $\theta = 60°$

Required angle $= \pi - 60° = 120°$

47. (1) Since, A and B have same mass. So, after elastic collision, they interchange their velocity.

$\therefore\ v_B = v$

After collision between B and C,

$$v'_B = \left(\frac{m-4m}{m+4m}\right)v + \left(\frac{2\times 4m\times 0}{m+4m}\right) = -\frac{3}{5}v$$

Again, collision takes place between A and B. So, velocity will be interchanged.

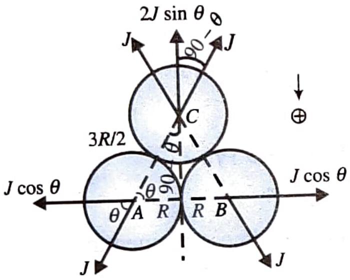
$\therefore\ v'_A = (v'_B) = \frac{3}{5}v$ in leftward direction.

48. (3) $J\cos\theta = Mv$ ...(i)

$-J\sin\theta = 0 - Mv_0$

$J\sin\theta = Mv_0$ ...(ii)

From Eqs. (i) and (ii), $\tan\theta = \frac{v_0}{v}$ ...(iii)



From ΔAOC, $\tan\theta = \frac{\sqrt{5}\,R/2}{R/2} = \sqrt{5}$

$\therefore\ \sqrt{5} = \frac{v_0}{v} \Rightarrow v = \frac{v_0}{\sqrt{5}}$

49. (3) $J\cos\theta = Mv$ ...(i)

$-J\sin\theta = \frac{Mv_0}{2} - Mv_0$

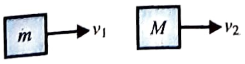
$-J\sin\theta = -\frac{Mv_0}{2}$ ...(ii)

From Eqs. (i) and (ii), $\tan\theta = \frac{v_0}{2v}$

50. (3) Let v_1, v_2 and v_3 be velocities of blocks 1, 2 and 3 after suffering collision each.

$mv = mv_1 + Mv_2$ and $v_1 - v_2 = -v$

Solving, we get $v_1 = \frac{m-M}{m+M} < 0$



$\therefore\ m < M$

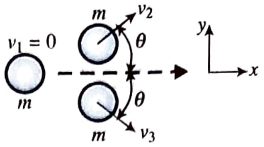
$\therefore\ |v_1| = \frac{M-m}{m+M}v$ ...(i)

and $v_1 = \frac{2m}{m+M}\times v_2 = \frac{4Mmv}{(m+M)^2}$ ...(ii)

$\therefore\ \frac{M-m}{M+m}v = \frac{4Mmv}{(M+m)^2}$

$$M^2 - m^2 = 4Mm \Rightarrow \frac{M}{m} = 2 + \sqrt{5}$$

51. (4) After collision by momentum conservation:



Along y-axis: $0 = 0 + mv_2\sin\theta - mv_3\sin\theta \Rightarrow v_2 = v_3$

Along x-axis:

$mv = 0 + mv_2\cos\theta + mv_3\cos\theta$

$mv = 2mv_2\cos\theta$

$v_2 = \frac{v}{2}\frac{1}{\cos\theta}$ so $v_2 = v_3 > \frac{v}{2}$ $\because \cos\theta < 1$

52. (1) $R = \frac{2u\cos\theta\cdot u\sin\theta}{g}$

After impact horizontal component remains the same ($= u\cos\theta$)

The vertical component becomes $V = e(u\sin\theta)$

$\therefore$ New range, $R' = \frac{2u\cos\theta\cdot eu\sin\theta}{g}$ or $R' = eR$

53. (2) Collision takes place along the normal. Therefore the magnitude normal component (v_y) of the velocity of the glass ball is changed to $v'_y = -ev_y$ just after the collision where as the horizontal component (v_x) of its velocity remains constant due to the absence of any horizontal force.

$\Rightarrow$ The velocity of the ball just after the impact $= v' = \vec{v}'_x + \vec{v}'_y$

$\Rightarrow \vec{v}' = v'_x\hat{i} + v'_y\hat{j}$

where $v'_x = a$ and $v'_y = eb$ $\quad(\because \vec{v} = a\hat{i} - b\hat{j})$

$\Rightarrow \vec{v}' = a\hat{i} + eb\hat{j}$

Therefore the magnitude of the velocity $\vec{v}' = |\vec{v}'| = \sqrt{a^2 + e^2b^2}$

and the direction is given as $\theta = \tan^{-1}\left(\frac{v'_x}{v'_y}\right) = \tan^{-1}\left(\frac{a}{eb}\right)$ to the normal (vertical).

54. (2) Conservation of linear momentum just before and after yields

$m_1v\hat{i} = m_2v_2\hat{i} + m_1v_1(-\hat{i})$

$\Rightarrow m_1v = m_2v_2 - m_1v_1$ ...(i)

$e = -\frac{v_1 + v_2}{0 - v} = 1$

Therefore, $v = v_1 + v_2$ ...(ii)

[for elastic collision, $e = 1$].

Eliminating v_2 from (i) and (ii), we obtain

$m_1v = m_2(v - v_1) - m_1v_1$

$\Rightarrow v_1 = \frac{m_2 - m_1}{m_1 + m_2}v$

Since $v_1 > 0$, $\frac{m_2 - m_1}{m_1 + m_2} > 1$

$\Rightarrow m_2 - m_1 > 0 \Rightarrow m_1 < m_2$

55. (3) Conservation of linear momentum of the system of bodies just before and after collision yields

$$mv_0 = (m+m)v \Rightarrow v = \frac{v_0}{2}$$

∴ The tension in the string $T = \frac{(2m)v^2}{l} + (2m)g$

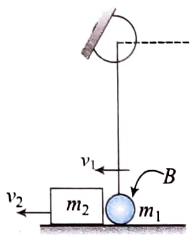
$$T = \frac{2m\,v_0^2}{4l} + 2mg = \frac{m(2gl)}{2l} + 2mg = 3mg$$

Initially the tension = $T_0 = mg$

⇒ Increase in tension = $2mg$

56. (2) Let the ball strike the block with a speed u_1. Since the initial speed (speed before collision) of the block = $u_2 = 0$ for the perfectly elastic collision.

$$m_1u_1 = m_1v_1 + m_2v_2$$



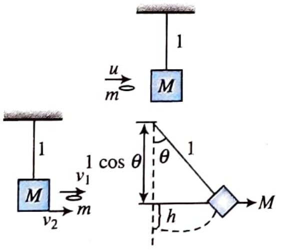
$$e = 1 = \frac{v_1}{v_2 - v_1} \Rightarrow v_2 - v_1 = u_1$$

$$0.5\,u_1 = 0.5\,v_1 + 2.5\,v_2$$

$$\Rightarrow v_2 = \frac{u_1}{3} = \frac{\sqrt{2gl}}{3} = \frac{20}{3}\,\text{m s}^{-1}$$

57. (3) Conservation of momentum just before and after the collision yields

$$mu = mv_1 + Mv_2 \qquad \text{...(i)}$$



Conservation of energy of the block between the point 1 and 2 after the bullet pieces

It yields ΔKE + ΔPE = 0

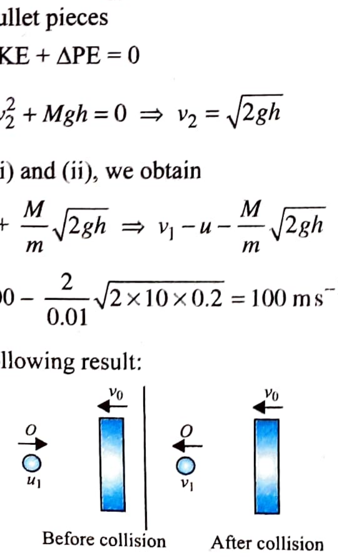
$$-\frac{1}{2}Mv_2^2 + Mgh = 0 \Rightarrow v_2 = \sqrt{2gh} \qquad \text{...(ii)}$$

By using (i) and (ii), we obtain

$$u = v_1 + \frac{M}{m}\sqrt{2gh} \Rightarrow v_1 - u - \frac{M}{m}\sqrt{2gh}$$

$$\Rightarrow v_1 = 500 - \frac{2}{0.01}\sqrt{2\times10\times0.2} = 100\text{ m s}^{-1}$$

58. (3) Use the following result:



$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow 1 = \frac{-v_0 - (-v_1)}{u_1 - (-v_0)} \Rightarrow v_1 = u_1 + 2v_0$$

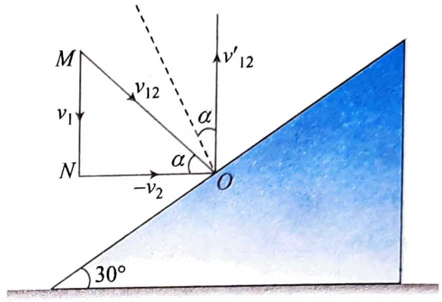
Now velocity after first collision

$$V' = V + 2(2V) = 5V$$

Velocity after second collision

$$V'' = V' + 2(2V) = 5V + 4V = 9V$$

59. (2) In the figure $\vec{v}_{12}$ = velocity of ball w.r.t. wedge before collision and $\vec{v}'_{12}$ = velocity of ball w.r.t. wedge after collision, which must be in vertically upward direction as shown. In elastic collision $\vec{v}_{12}$ and $\vec{v}'_{12}$ will make equal angle (say α) with the normal to the plane.



We can show that α = 30°

∴ $\angle MON = 30°$

Now $\frac{v_1}{v_2} = \tan 30° = \frac{1}{\sqrt{3}}$

Multiple Correct Answers Type

1. (2), (3), (4)

In an elastic collision, linear momentum of the system is always conserved i.e., in all the three stages of collision. But the kinetic energy of the system is not always conserved. It is conserved only before and after the collision. During collision it gets converted into potential energy.

2. (1), (3), (4)

In an SHM, the maximum velocity is at the equilibrium position.

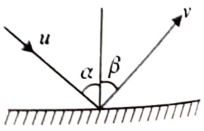
3. (2), (3)

When the second body is at rest, velocities after the elastic collision are given by

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 \text{ and } v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$

It is obvious that both bodies move after the collision. If $m_1 = m_2$, then $v_1 = 0$ and $v_2 = u_1$. Further, v_2 cannot be zero. So, options (2) and (3) are correct.

4. (2), (3)



Let the ball strike the floor at angle α with the vertical and velocity at the instant of striking the floor be u and let the ball rebound with velocity v at angle β with the vertical as shown in figure.

Since the floor is smooth, therefore horizontal component of velocity remains unchanged. Hence

$$v \sin\beta = u \sin\alpha \qquad \text{...(i)}$$

Since, the floor is inelastic, therefore normal component of velocity just after the collision is less than that just before the collision. Hence,

$$v \cos\beta < u \cos\alpha \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), tan β > tan α or β > α

Hence, option (3) is correct.

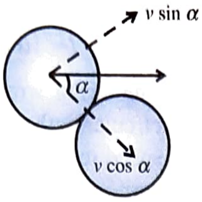
Since the ball has a horizontal component of velocity, therefore, at highest point, its kinetic energy is not equal to zero.

It means, at highest point, potential energy of the ball is less than the kinetic energy just after rebound. Hence, option (1) is incorrect.

Since the floor is inelastic, therefore there is some loss of energy during the collision. Hence, the total energy of the ball does not remain conserved. So, option (2) is also correct.

5. (1), (3)

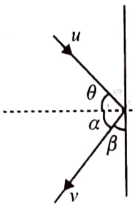
As masses are equal and collision elastic Therefore, velocity along line of impact gets exchange and velocity perpendicular to line of impact remains unchanged.



If $\alpha = 45°$, then their speed will be same after collision, hence KE will be same.

6. (1), (3)

If wall is smooth $v \cos \alpha = eu \cos \theta$



$$v \sin \alpha = u \sin \theta$$

$$\cot \alpha = eu \cot \theta$$

$$\alpha > \theta$$

$$\alpha + \beta > \theta + \beta$$

$$90° > \theta + \beta$$

If wall is rough, then $v \cos \beta < u \sin \theta$ ...(i)

$$v \sin \beta = eu \cos \theta \quad \text{...(ii)}$$

From (i) and (ii), $e < \tan \beta / \cot \theta$

7. (2), (4)

In the first case: $(m + m)v_1 = mv_0 \Rightarrow v_1 = \frac{v_0}{2}$

In the second case:

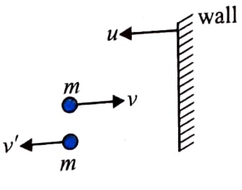
$$-\int T\,dt = (m + m)v - mv_0 \quad \text{...(i)}$$

$$\int T\,dt = 2\,mv \quad \text{...(ii)}$$

From Eqs. (i) and (ii), $4mv = mv_0$

$\Rightarrow v = \frac{v_0}{4}$ and required ratio = 2

8. (2), (3)



In an elastic collision: $v_{sep} = v_{app}$

or $v' - u = v + u$ or $v' = v + 2u$

Change in momentum of ball is

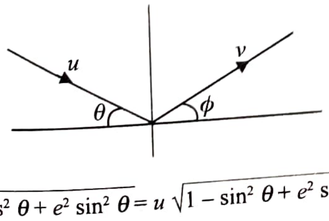
$$|p_f - p_i| = |m(-v') - mv|$$
$$= m(v' + v) = 2m(u + v)$$

Average force is $\frac{\Delta p}{\Delta t} = \frac{2m(u + v)}{\Delta t}$

Change in KE, $K_f - K_i = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2 = 2m\,u\,(u + v)$

9. (1), (2, (4)

$v \sin \varphi = eu \sin \theta$, $v \cos \varphi = u \cos \theta$



$$v = u\sqrt{\cos^2 \theta + e^2 \sin^2 \theta} = u\sqrt{1 - \sin^2 \theta + e^2 \sin^2 \theta}$$

$$= u\sqrt{1 - (1 - e^2) \sin^2 \theta}$$

$$I = m(v \sin \phi + u \sin \theta) = mu \sin \theta (1 + e)$$

$$\text{Ratio of KE} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \cos^2 \theta + e^2 \sin^2 \theta$$

10. (1), (2), (4)

In completely inelastic collision, bodies coalesce

$$m_1u_1 - m_2u_2 = (m_1 + m_2)V$$

$$2 \times 3 - 1 \times 4 = (2 + 1)V; \; V = \frac{2}{3} \text{ m/s}$$

Net momentum of system $P = m_1u_1 - m_2u_2$

$$2 \times 3 - 1 \times 4 = 2 \text{ kg m/s}$$

$$\text{Loss in KE} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)V^2$$

$$= \frac{1}{2} \times 2 \times (3)^2 + \frac{1}{2} \times (1) \times (4)^2 - \frac{1}{2} \times (2 + 1) \times \left(\frac{2}{3}\right)^2$$

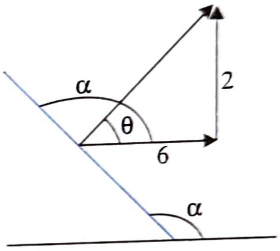
$$= 9 + 8 - \frac{2}{3} = 17 - \frac{2}{3} = \frac{49}{3}\text{J}$$

11. (1), (3)

Impulse is change in momentum. Hence,

$$\text{impulse} = 2(\vec{v}_2 - \vec{v}_1) = 2(3\hat{i} + \hat{j})$$

As impulse is in the normal direction of colliding surface



$$\tan \theta = \frac{1}{3}$$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right)$$

$$\therefore \; \alpha = 90° + \tan^{-1}\left(\frac{1}{3}\right)$$

12. (3), (4)

In option (1), neither momentum nor kinetic energy is conserved so collision is not elastic. Hence, option (1) is not correct.

Same difficulty lies with option (2). So, only options (3) and (4) are correct.

13. (1), (3)

Since both masses have equal kinetic energy, so

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_2(\alpha u_1)^2$$

$$\Rightarrow m_2 = \frac{m_1}{\alpha^2} \quad \text{...(i)}$$

By law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 + \frac{m_1}{\alpha^2}(\alpha u_1) = 0 + \left(\frac{m_1}{\alpha^2}\right)v_2$$

$$v_2 = \alpha u_1(1 + \alpha) \quad \text{...(ii)}$$

Further by law of conservation of energy, we have

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}\left(\frac{m_1}{\alpha^2}\right)(\alpha u_1)^2 = 0 + \frac{1}{2}\left(\frac{m_1}{\alpha^2}\right)v_2^2$$

$$m_1u_1^2 = \frac{1}{2}\left(\frac{m_1}{\alpha^2}\right)v_2^2$$

$$v_2 = \sqrt{2}(\alpha u_1) \quad \text{...(iii)}$$

From Eqs. (ii) and (iii), we get

$$\frac{v_2}{\alpha u_1} = \sqrt{2} = 1 + \alpha \Rightarrow \alpha = \sqrt{2} - 1$$

$$\frac{u_2}{u_1} = \sqrt{2} - 1$$

$$\frac{u_1}{u_2} = \sqrt{2} + 1 \text{ [on rationaliting]}$$

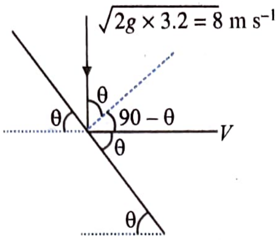
$$\frac{m_2}{m_1} = \frac{1}{\alpha^2} = \left(\frac{1}{\sqrt{2}-1}\right)^2 = (\sqrt{2}+1)^2$$

$$\frac{m_2}{m_1} = 3 + 2\sqrt{2}$$

14. (1), (2)

$$8 \sin\theta = v\cos\theta$$

$$\frac{8\cos\theta}{2} = v\sin\theta$$



$$2\tan\theta = \cot\theta$$

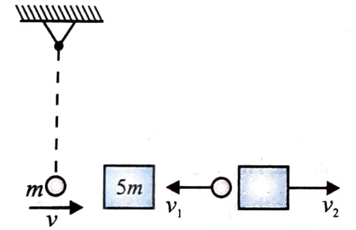
$$\Rightarrow \tan\theta = \frac{1}{\sqrt{2}}$$

$$v = \frac{8}{\sqrt{2}} = 4\sqrt{2} \text{ ms}^{-1}$$

$$\Delta K = \frac{1}{2} \times 1\left[\left(4\sqrt{2}\right)^2 - 8^2\right] = -16 \text{ J}$$

15. (1), (4)

The velocity of bob just before the impact is $v = \sqrt{2gl}$ along the horizontal direction.



Before collision After collision

From momentum conservation,

$$mv = -mv_1 + 5mv_2$$

From coefficient of restitution equation,

$$1 = \frac{v_1 + v_2}{v} \Rightarrow v_1 + v_2 = v$$

Solving above equations, we get $v_1 = \frac{2v}{3}$

$$v_2 = \frac{v}{3}$$

For tension in string, $T - mg = \frac{mv_1^2}{l}$

$$\Rightarrow T = \frac{17}{9}mg$$

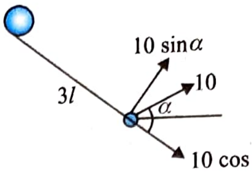
Let the maximum height attained by the bob be h, then

$$\frac{mv_1^2}{2} = mgh$$

$$\Rightarrow h = \frac{4l}{9}$$

16. (1), (2), (4)

The impulse exerted by the string on the cylinder is equal and opposite to the impulse exerted by the cylinder on the string.



$$|\vec{J}| = 40000 \times 0.25 \times 10^{-3} = 10 \text{ N s along the string.}$$

For marble, $J = 0 - (-2 \times 10\cos\alpha)$

As final velocity of marble along the string is zero

Putting $J = 10$ Ns, we get $\alpha = 60°$.

The string wraps up around the cylinder and marble moves in a circular path of varying radius with speed $10\sin\alpha = 5\sqrt{3}$ m/s. The speed remains constant as only the tension force is present which is perpendicular to the velocity of the marble.

For string to break, the impulsive tension has to be $\geq 2 \times 10^5$ N

$$\therefore\ 40000 \times 0.25 \times 10^{-3} \geq 2 \times 10^5 \times \Delta t$$

$$\Rightarrow t \leq 0.05 \text{ ms}$$

17. (1), (2), (3)

i. Force acting on the horizontal direction is zero. Since block and wedge both are initially at rest. So, velocity of COM (B plus W) does not move horizontally

ii. $v_{cm} = \dfrac{mv - (-mv)}{M + m}$ $\left[\because v_{cm} = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\right]$

$$v_{cm} = \frac{2mv}{M + m}$$

iii. When B reaches its highest position of W, the speed W is equal to velocity of COM i.e., $v = \dfrac{2mv}{M + m}$

18. (1), (3)

From conservation of linear momentum

$$mv_0 = (m + 2m)v \Rightarrow v = \frac{v_0}{3}$$

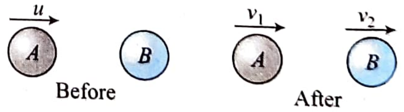
From energy conservation

$$\frac{1}{2}mv_0^2 = \frac{1}{2}(m + 2m)v^2 + \frac{1}{2}kx_0^2$$

$$k = \frac{2mv_0^2}{3x_0^2}$$

So, choices (1) and (3) are correct and choices (2) and (4) are wrong.

19. (1), (4)



Conserving linear momentum:

$$mu + 2m \times 0 = mv_1 + 2mv_2$$

or $v_1 + 2v_2 = u$ ...(i)

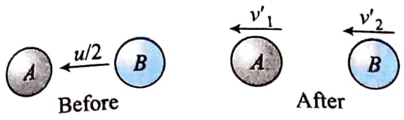
Coefficient of restitution equation

$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow \frac{1}{2} = \frac{v_2 - v_1}{u - 0}$$

or $v_2 - v_1 = \dfrac{u}{2}$ ...(ii)

From (i) and (ii); $v_1 = 0, v_2 = \dfrac{u}{2}$

Now ball '*B*' will collide with the wall elastically and its direction of motion will change. The ball '*B*' will collide with ball '*A*' which is at rest.



Again applying conservation of linear momentum

$$m\times 0+2m\times\frac{u}{2}=mv_1'+2mv_2'$$

or $v_1'+2v_2'=u$ ...(iii)

and restitution equation, $\frac{1}{2}=\frac{v_2'-v_1'}{0-\frac{u}{2}}$

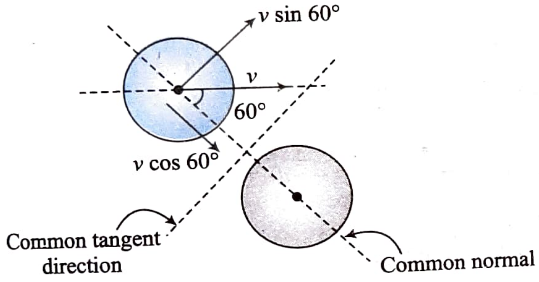
or $v_2'-v_1'=-\frac{u}{4}$ ...(iv)

from Eqs. (i) and (ii), $v_1'=\frac{u}{2}$ and $v_2'=\frac{u}{4}$

i.e., $v_A=\frac{u}{2}, v_B=\frac{u}{4}$

20. (3), (4)

In elastic collision of identical bodies velocity interchanged in common normal directional and velocity of the bodies remains unchanged along common tangent directions.

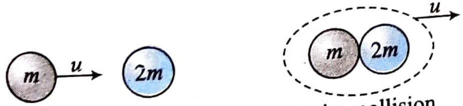


Component of velocity of *A* along common normal is *v* cos 60° and this velocity of *A* after collision with *B* is interchanged. Hence *A* moves along *v* sin 60° which is normal to common normal.

21. (1), (2), (3), (4)

At the time of collision both particles have common velocity and hence the system has minimum kinetic energy.

Applying conservation of linear momentum



Just before collision During collision

$$m\times u+2m\times 0=(m+2m)v\ \Rightarrow v=\frac{u}{3}$$

Initial kinetic energy $=\frac{1}{2}u^2=6\,\text{J}$

Kinetic energy during collision, $K_{\text{colliison}}=\frac{1}{2}(3m)\left(\frac{u}{3}\right)^2$

$$\Rightarrow K_{\text{collision}}=\frac{1}{3}\cdot\frac{1}{2}u^2=2\,\text{J}$$

Elastic potential energy during collision,

$$U_{\text{collision}}=6-2=4\,\text{J}$$

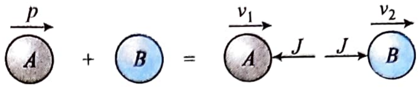
Total energy remains constant and hence KE of system first decreases & then increases.

22. (1), (3)

As all the collisions are elastic and head on and the masses of the colliding bodies are equal, the velocities after collision are interchanged.

For I$^{\text{st}}$ bead: $W=\Delta K,\ Fd=\frac{1}{2}mu^2\Rightarrow u=\sqrt{\frac{2Fd}{m}}$

23. (2), (3)



Applying impulse momentum equation

For *A*: $p-J=mv_1$ ...(i)

For *B*: $J=mv_2$ ...(ii)

from (i) and (ii),

$$e=\frac{v_2-v_1}{u_1-u_2}=\frac{\left[\frac{J}{m}-\left(\frac{p-J}{m}\right)\right]}{\left(\frac{p}{m}-0\right)}=\frac{2J}{p}-1$$

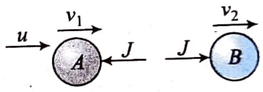
24. (1), (2), (4)

If two identical bodies collide head on elastically their velocities interchanged.

Hence (1) is correct.

Impulse exist in action and reaction pair of equal magnitude during collision irrespective of type of collision.

Hence option (2) is correct.



Applying impulse momentum equation

$m_1u-J=m_1v_1$ ...(i)

and $J=m_2v_2$ ...(ii)

From (i) and (ii) we get, $v_1=u-\left(\frac{m_2}{m_1}\right)v_2$

As $m_1>>m_2$, hence v_1 is positive or in same direction as initial velocity of motion of m_1. Hence option (4) is correct.

25. (2), (3)

Applying conservation of linear momentum

$$m_1u_1+m_2u_2=m_1v_1+m_2v_2$$

$1\times 21+2\times(-4)=1\times 1+2\times v_2$ (same direction as initial)

$\Rightarrow v_2=6$ m/s

$$e=\left(\frac{v_2-v_1}{u_1-u_2}\right)=\left(\frac{6-1}{21-(-4)}\right)=\frac{5}{25}=0.2$$

Loss in kinetic energy,

$$\Delta K=\frac{1}{2}\times1\times(21)^2+\frac{1}{2}\times2\times(4)^2-\frac{1}{2}\times1\times1-\frac{1}{2}\times2\times6^2=200\,\text{J}$$

Impulse of force between two balls

$$J=m\Delta v=1\times(21-1)=20\ \text{Ns}$$

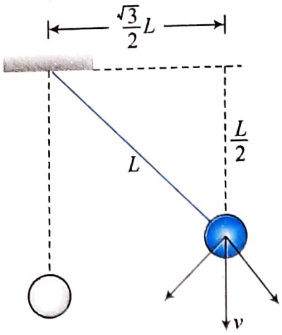
26. (1), (2), (3)

Let *v* be the velocity of the bob when string gains full length.

By conservation of energy,

$$\frac{1}{2}mv^2=mg\frac{L}{2}\ \Rightarrow\ v=\sqrt{gL}$$

After jerk $v \sin\theta$ becomes zero, hence impulse applied by string $J = mv\sin\theta$



$$\Rightarrow \quad J = 0.2\sqrt{gL} \times \frac{1}{2} = \frac{\sqrt{gL}}{10}$$

Velocity of ball after jerk, will perpendicular to string.

$$= v\cos\theta = \sqrt{gL} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3gL}}{2}$$

27. (1), (3), (4)

In completely inelastic collision, bodies coalesce

$$m_1u_1 - m_2u_2 = (m_1 + m_2)V$$

$$2\times3 - 1\times4 = (2+1)V \Rightarrow V = \frac{2}{3}\text{ m/s}$$

Net momentum of system

$$p = m_1u_1 - m_2u_2$$

$$= 2\times3 - 1\times4 = 2\text{ kg m/s}$$

$$\text{Loss in KE} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}\ m_2u_2^2 - \frac{1}{2}(m_1+m_2)V^2$$

$$= \frac{1}{2}\times2\times(3)^2 + \frac{1}{2}\times(1)\times(4)^2 - \frac{1}{2}\times(2+1)\times\left(\frac{2}{3}\right)^2$$

$$= 9 + 8 - \frac{2}{3} = 17 - \frac{2}{3} = \frac{49}{3}\text{J}$$

28. (2), (3), (4)

The acceleration of the system, $a = \frac{3-2}{3+2}g = 2\text{ m/s}^2$

The velocity of block A, just before collision

$$v^2 = u^2 + 2as \Rightarrow v = \sqrt{2\times2\times1} = 2\text{ m/s}$$

Impulse A exerts $J = 3\times2 = 6$ Ns

Let the maximum height reached by block B after string slacks be s.

Then for block B,

$$0 - (2)^2 = 2(-10)s \Rightarrow 0.2\text{ m}$$

Hence maximum height reached by block B should be 2.2 m.

29. (2), (3, (4)

(1) Just before collision, both P & Q arrive at equilibrium position

$$\therefore \quad v_P = \omega\frac{A}{2} = \sqrt{\frac{k}{m}}\frac{A}{2}$$

(2) Speed of Q just before collision is $v_Q = \omega A = \sqrt{\frac{k}{m}}A$

(3) The block shall meet after time $t = \frac{T}{4}$, where T is time period of either isolated spring block system,

$$t = \frac{T}{4} = \frac{1}{4}2\pi\sqrt{\frac{m}{k}} = \frac{\pi}{2}\sqrt{\frac{m}{k}}$$

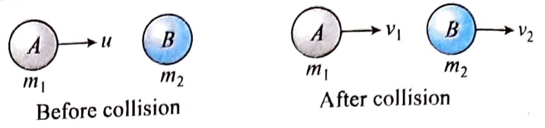
(4) By conservation of linear momentum

$$\frac{A}{2}\sqrt{\frac{k}{m}} = 2mv \Rightarrow v = \frac{A}{4}\sqrt{\frac{k}{m}}$$

For new system,

$$A'\omega' = \frac{A}{4}\sqrt{\frac{k}{m}} \Rightarrow A' = \frac{A}{4}$$

30. (1), (2)



From conservation of linear momentum, $m_1u = m_1v_1 + m_2v_2$

and by definition of 'e', $e = 1 = \frac{v_2 - v_1}{u - 0} \Rightarrow v_2 = v_1 + u$

$$\text{Therefore } v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u = \left[\frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}\right]u$$

$$\text{and } v_2 = \frac{2m_1u}{m_1 + m_2} = \frac{2u}{\left[1 + \frac{m_2}{m_1}\right]}$$

For (1): $v_1 = -\frac{u}{3}$ and $v_2 = \frac{2}{3}u$

For (2): $v_1 = -\frac{u}{2}$ and $v_2 = \frac{u}{2}$

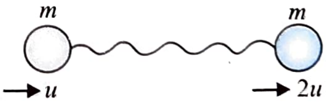
For (3): $v_1 = \frac{u}{3}$ and $v_2 = -\frac{4}{3}u$

For (4): $v_1 = \frac{u}{2}$ and $v_2 = \frac{3}{2}u$

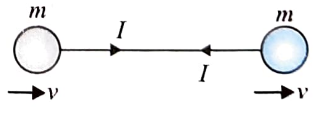
Linked Comprehension Type

For Problems 1–3

1. **(4)** Since the collision is elastic, there is no loss in KE.
2. **(1)** After the first collision, the velocities are as shown in figure:



Now after some time, the string will become tight. Let at this point the common velocity of the balls be v.



$$2mv = mu + 2mu \Rightarrow v = 3u/2$$

$$\text{KE}_i = \frac{1}{2}\ mu^2 + \frac{1}{2}m\ (2u)^2 = \frac{5mu^2}{2}$$

$$\text{KE}_f = \frac{1}{2}\ 2m\ v^2 = \frac{1}{2}\ 2m\left(\frac{3u}{2}\right)^2 = \frac{9mu^2}{4}$$

$$\text{KE}_{\text{Loss}} = \text{KE}_i - \text{KE}_f = \frac{5mu^2}{2} - \frac{9mu^2}{4} = \frac{mu^2}{4}$$

3. **(1)** For left ball: $I = mv - mu = m\frac{3u}{2} - mu = \frac{mu}{2}$

For Problems 4–7

4. (2) Using conservation of linear momentum in horizontal direction just before and just after collision,

$$mu = (m + M)\,V \Rightarrow V = \frac{mu}{(M+m)}$$

5. (3) Using conservation of mechanical energy,

$$\frac{1}{2}(M+m)\,V^2 = (m+M)gh$$

$$V^2 = 2gh \Rightarrow \left[\frac{mu}{(M+m)}\right]^2 = 2gh$$

$$u^2 = \left(\frac{M+m}{m}\right)^2 2gh$$

$$\Rightarrow u = \frac{[M+m]}{m}\sqrt{2gh}$$

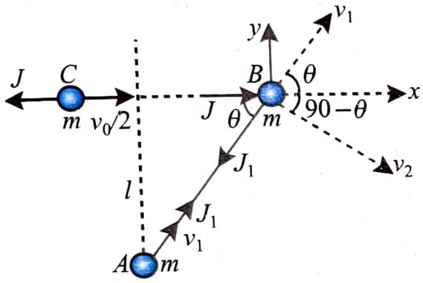
6. (4) As bullet and block coalesce after collision, hence collision is perfectly inelastic.

7. (2) Energy dissipated in collision = initial energy – final energy

$$= \frac{1}{2}mu^2 - (m+M)gh$$

For Problems 8–12

8. (4) Let after collision, the velocities are as shown in the figure.



Here $\theta = 60°$. Let impulse between C and B during collision be J. For the system of A and B:

$$J\cos\theta = 2mv_1 \qquad \text{...(i)}$$

$$J\sin\theta = mv_2 \qquad \text{...(ii)}$$

$$\Rightarrow v_1 = \frac{J}{m}\frac{\cos\theta}{2},\quad v_2 = \frac{J}{m}\sin\theta$$

Velocity of B:

$$\vec{v}_B = v_1\cos\theta\hat{i} + v_1\sin\theta\hat{j} + v_2\sin\theta\hat{i} - v_2\cos\theta\hat{j}$$

$$= (v_1\cos\theta + v_2\sin\theta)\hat{i} + (v_1\sin\theta - v_2\cos\theta)\hat{j}$$

$$= \frac{J}{m}\left[\left(\frac{\cos^2\theta}{2} + \sin^2\theta\right)\hat{i} + \left(\frac{\cos\theta\sin\theta}{2} - \cos\theta\sin\theta\right)\hat{j}\right]$$

$$= \frac{J}{m}\left[\frac{7}{8}\hat{i} - \frac{\sqrt{3}}{8}\hat{j}\right]$$

From here we can find direction of unit vector in the direction of $\vec{v}_B$.

9. (2) Velocity of A: $\vec{v}_1 = v_1\cos\theta\hat{i} + v_1\sin\theta\hat{j}$

$$\Rightarrow \vec{v}_1 = \cos\theta\hat{i} + \sin\theta\hat{j} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$$

10. (1) For C: $J = mv_0 - \dfrac{mv_0}{2} \quad\Rightarrow\quad J = \dfrac{mv_0}{2}$

Impulse on A due to string:

$$J_1 = mv_1 = \frac{J}{2}\cos\theta \quad\Rightarrow\quad J_1 = \frac{mv_0}{2}\frac{\cos\theta}{2} = \frac{mv_0}{8}$$

11. (2) From Eqs. (i) and (iii), we get

$$\frac{mv_0}{2}\cos\theta = 2mv_1 \quad\Rightarrow v_1 = \frac{v_0}{8}$$

12. (1) From Eqs. (ii) and (iii), we get

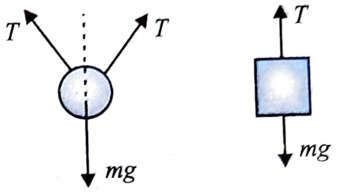
$$\frac{mv_0}{2}\sin\theta = mv_2 \quad\Rightarrow v_2 = \frac{\sqrt{3}\,v_0}{4}$$

$$\text{Loss in KE} = \frac{1}{2}mv_0^2 - \left[\frac{1}{2}m\left(\frac{v_0}{2}\right)^2 + \frac{1}{2}mv_1^2 + \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2\right]$$

Put the values to get loss in KE $= \dfrac{17}{64}mv_0^2$

For Problems 13–15

13. (1)



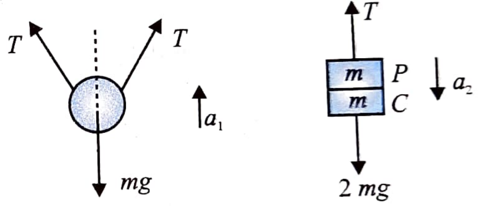
$$2T\sin\theta = mg,\ T = mg$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

14. (2) $2T\sin\theta - mg = ma_1$, $\theta = 30°$

$$2\,mg - T = 2\,ma_2$$

Simplify to get: $a_1 + 2a_2 = g$



From constraints, we have initially:

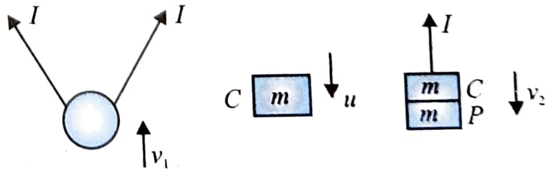
$$a_2 = 2a_1\sin\theta \Rightarrow a_2 = a_1$$

15. (4) From constraints, we have initially:

$$v_2 = 2v_1\sin\theta \Rightarrow v_2 = v_1$$

Let I be the impulse generated in the string,

$$2I\sin\theta = mv_1 \Rightarrow I = mv_1$$



$$I = -2mv_2 - (-mu) \Rightarrow mv_1 + 2mv_2 = mu$$

$$\Rightarrow v_1 + 2v_1 = u \Rightarrow v_1 = \frac{u}{3}$$

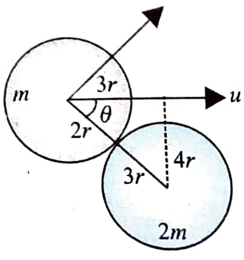
For Problems 16–18

16. (4), 17. (1), 18. (2)

Tangential component = $u\sin\theta = 4\,u/5$

Along normal velocity of m before collision:

$$u_1 = u\cos\theta = \frac{3u}{5}$$



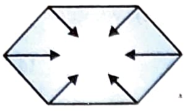
Along normal: $mv_1 + 2mv_2 = mu_1$

$$\frac{v_2 - v_1}{u_1} = 1$$

Solve to get $v_1 = -\dfrac{u}{5}$, $v_2 = \dfrac{2u}{5}$

For Problems 19–21

19. (3) The vector sum of momenta for first six collisions is zero as shown.



20. (2) Since an impulse acts on the ball during each collision, so linear momentum does not remain constant.

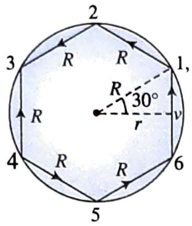
The impulsive force is directed towards the centre of hexagon, therefore the lever arm of this impulsive force is zero.

As a result, it does not produce any torque and angular momentum remains constant. Further, kinetic energy also remains same for elastic collision.

21. (2) $v = \omega r$

Time between first and seventh collision:

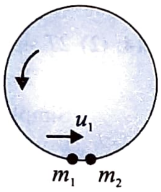
$$t = \frac{6R}{v} = \frac{6R}{\omega r} = \frac{6R}{\omega R \cos 30^\circ} = 4\sqrt{3}\ \text{s}$$



For Problems 22–24

22. (3) Just before collision, velocity of m_1

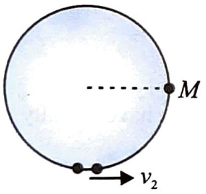
$$u_1 = \sqrt{2g2a} = 2\sqrt{ga}$$



Just after collision, m_1 is brought to rest and let velocity of m_2 be v_2.

From conservation of linear momentum,

$$m_2 v_2 = m_1 u_1 \qquad \text{...(i)}$$



Now $v_2 = \sqrt{2ga}$ so that m_2 can rise up to point M.

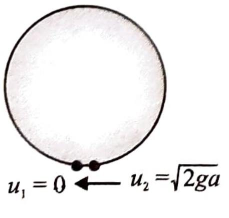
Putting the value of u_1 and v_2 in Eq. (i), we get

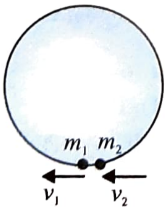
$$m_2\sqrt{2ga} = m_1 2\sqrt{ga} \Rightarrow \frac{m_1}{m_2} = \frac{1}{\sqrt{2}}$$

23. (4) $e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\sqrt{2ga} - 0}{2\sqrt{ga} - 0} = \frac{1}{\sqrt{2}}$

24. (1) $m_1 v_1 + m_2 v_2 = m_2 u_2$

$$\Rightarrow m_1 v_1 + \sqrt{2} m_1 v_2 = \sqrt{2} m_1 u_2$$





$$\Rightarrow v_1 + \sqrt{2} v_2 = \sqrt{2} u_2 \qquad \text{...(i)}$$

$$e = \frac{1}{\sqrt{2}} = \frac{v_1 - v_2}{u_2 - u_1} \Rightarrow \frac{1}{\sqrt{2}} = \frac{v_1 - v_2}{u_2}$$

$$\Rightarrow v_1 - v_2 = \frac{u_2}{\sqrt{2}} \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get $v_1 = u_2 = \sqrt{2ga}$

$$v_2 = \left(\frac{\sqrt{2} - 1}{\sqrt{2}}\right) u_2 = (\sqrt{2} - 1)\sqrt{ga}$$

Both are positive, so both move in same direction.

Height raised by $m_1 = \frac{v_1^2}{2g} = a$, same as that of m_2

For Problems 25–27

25. (1), 26. (3), 27. (2)

u = velocity of sphere A before impact. As the spheres are identical, the triangle ABC formed by joining their centres is equilateral. The spheres B and C will move in direction AB and AC after impact making an angle of 30° with the original lines of motion of ball A.

Let v be the speed of the balls B and C after impact.

Momentum conservation gives

$$\left(\frac{m}{2}\right) u = mv \cos 30^\circ + mv \cos 30^\circ$$

$$u = 2\sqrt{3}\, v \Rightarrow v = \frac{u}{2\sqrt{3}} \qquad \text{...(i)}$$

From Newton's experimental law, for an oblique collision, we have to take components along normal, i.e., along AB for balls A and B.

$$v_B - v_A = -e(u_B - u_A)$$

$$v - 0 = -e(0 - u\cos 30^\circ)$$

$$v = eu\cos 30^\circ \qquad \text{...(ii)}$$

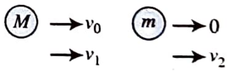
Combining Eqs. (i) and (ii), we get $e = 1/3$.

$$\text{Loss in KE} = \frac{1}{2}\frac{m}{2}u^2 - 2\left(\frac{1}{2}mv^2\right)$$

$$= \frac{1}{4}mu^2 - m\left(\frac{u}{2\sqrt{3}}\right)^2 = \frac{1}{6}mu^2$$

For Problems 28–30

28. (3), 29. (4), 30. (2)



$$Mv_0 = Mv_1 + mv_2 \qquad \text{...(i)}$$

$$\frac{v_2 - v_1}{v_0} = 1 \qquad \text{...(ii)}$$

Solve to get: $v_1 = \frac{M - m}{M + m} v_0$ and $v_2 = \frac{2M}{m + M} v_0$

$$v_{2,1} = v_2 - v_1 = v_0 \quad \therefore\ t_2 - t_1 = \frac{L}{v_0}$$

For Problems 31–33

31. (1), 32. (3), 33. (2)

Let velocity of ball A at highest point be v then first considering its FBD at this point

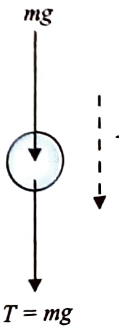
$$mg + T = \frac{mv^2}{r}$$

where $T = mg$

$$\Rightarrow v^2 = 2rg$$

$$\Rightarrow v = \sqrt{2rg} = 2\sqrt{6}\ \text{m/s}$$

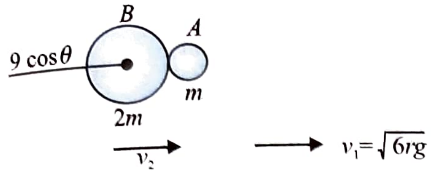
If velocity of ball A just after the collision is v_1 then according to law of conservation of energy, its KE at A = (KE + PE) at highest point



$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 + mg\,(2r)$$

$$v_1 = \sqrt{6rg} \text{ or } v_1 = 6\sqrt{2} \text{ m/s}$$

If angle of projection of ball B is θ, then horizontal component of its velocity will be $9 \cos \theta$ and it remains constant.



Now considering collision of balls B and A, let velocities of balls A and B, just after the collision, be v_1 and v_2, respectively as shown in figure.

Then according to the law of conservation of momentum,

$$mv_1 + 2mv_2 = 2m\,(9 \cos \theta) \quad \text{...(i)}$$

Since collision is elastic, therefore, $e = 1$

$$\frac{v_1 - v_2}{9 \cos \theta} = 1 \text{ or } v_1 - v_2 = 9 \cos \theta \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\cos \theta = \frac{1}{\sqrt{2}} \text{ or } \theta = 45°$$

$$v_2 = \frac{3}{\sqrt{2}} \text{ ms}^{-1}$$

Height of initial position of ball A (lowest position) is equal to maximum height ascended by the ball B. It is equal to

$$H = \frac{(9 \sin \theta)^2}{2g} = \frac{81}{40}\text{m}$$

Height of point of suspension from the ground is

$$H + r = \frac{129}{40}\text{m}$$

For Problems 34–35

34. (2), 35. (3)

When ball is released, it moves along a vertical circle with centre at A. Kinetic energy of ball just before collision is equal to loss of its potential energy from point of release to the point of collision. Therefore, velocity v_1 of ball, just before collision, is given by

$$\frac{1}{2}mv_1^2 = mg\,(l - l \cos \theta) \Rightarrow v_1 = 5 \text{ m/s}$$

After collision, the block starts to move towards right. But it is retarded by force of friction and ultimately it comes to rest. According to law of conservation of energy,

Kinetic energy of the block just after collision = Work done by it against friction

Therefore, its velocity v_2 just after collision is given by

$$\frac{1}{2}mv_2^2 = \mu mgs$$

where $m = 0.5$ and $s = 2.50$ m.

$\therefore\ v_2 = 5$ m/s

Coefficient of restitution, $e = -\dfrac{v_2 - v_1}{u_2 - u_1}$

where $u_2 = 0$, $u_1 = 5$ m/s and $v_2 = 5$ m/s.

$$\therefore\ e = -\frac{5 - v_1}{0 - 5} \quad \text{or } v_1 = (5 - 5e) \quad \text{...(i)}$$

Applying law of conservation of momentum,

$$mu_1 + mu_2 = mv_1 + mv_2$$

$$5m + (m \times 0) = mv_1 + 5m \quad \text{or} \quad v_1 = 0$$

Substituting in Eq. (i), we get $e = 1$.

For Problems 36–38

36. (2), 37. (3), 38. (1)

When shell strikes the ball and gets stuck with it, combined body of mass $2m$ starts to move to the right. Let velocity of the combined body (just after collision) be v_1. According to law of conservation of momentum,

$$(m + m)\,v_1 = mv_0 \text{ or } v_1 = \frac{v_0}{2} = 3 \text{ m/s}$$

As soon as the combined body starts to move rightwards, thread becomes inclined to the vertical. Horizontal component of its tension retards the combined body while trolley accelerates rightwards due to the same component of tension.

Inclination of thread with the vertical continues to increase till velocities of both (combined body and trolley) become identical or combined body comes to rest relative to the trolley. Let velocity at that instant of maximum inclination of thread be v.

According to law of conservation of momentum,

$$(2m + M)v = 2\,mv_1 \text{ or } v = 1 \text{ m/s}$$

During collision of ball and shell, a part of energy is lost. But after that there is no loss of energy. Hence, after collision, kinetic energy lost is used up in increasing gravitational potential energy of the combined body.

If maximum inclination of threads with the vertical is θ, then according to law of conservation of energy,

$$\frac{1}{2}(2m)v_1^2 - \frac{1}{2}(2m + M)\,v^2 = 2\,mg\,(l - l \cos \theta)$$

$$\cos \theta = 0.8 \text{ or } \theta = 37°$$

For Problems 39–40

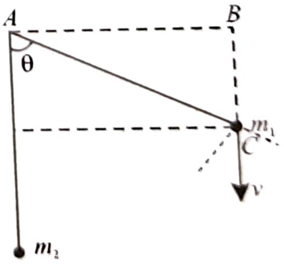
39. (4), 40. (1)

Ball of mass m_1 falls freely till thread becomes taut. At that instant inclination θ of the thread with the vertical be given by

$$\sin \theta = \frac{\left(\frac{\sqrt{3}}{2}l\right)}{l} = \frac{\sqrt{3}}{2} \text{ or } \theta = 60°$$

Ball of mass m_1 falls freely through height

$$l \cos \theta = \frac{l}{2}$$



Velocity of this ball at this instant is $v = \sqrt{2g \times \frac{l}{2}} = \sqrt{gl}$

It can be resolved into two components.

i. $v \cos \theta$, along the thread. But thread is inextensible, hence this component decreases to zero (due to tension developed in the thread).
ii. $v \sin \theta$, perpendicular to the thread. Due to this component ball starts to move along a circle whose centre is at A.

According to law of conservation of energy,

Kinetic energy of ball m_1 just before collision = Its kinetic energy at AC + Further loss of its potential energy

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1(\sqrt{gl}\sin\theta)^2 + m_1g(l - l\cos\theta)$$

$u_1 = 4$ m/s

According to law of conservation of momentum,

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

and coefficient of restitution, $e = -\frac{v_2 - v_1}{u_2 - u_1} = 1$

Substituting $u_1 = 4$ m/s, $u_2 = 0$, $m_1 = 0.1$ kg and $m_2 = 0.3$ kg, $v_2 = 2$ m/s, the height to which it rises is

$$h = \frac{v_2^2}{2g} = 0.20 \text{ m}$$

For Problems 41–44

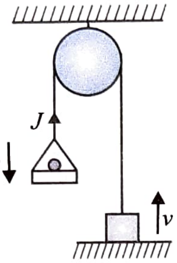
41. (4), 42. (2), 43. (4), 44. (3)

Due to collision, an impulsive tension occurs in the string.

For ball + pan system, $-J = (m + m_0)v - m_0v_0$

For block, $J = Mv \Rightarrow v = 2$ m/s

As the string is taut and the mass of the block is greater than the mass of ball + pan, so the block and ball + pan are under deceleration, given by



Just after collision

$$a = \frac{3g - 2g}{5} = 2 \text{ m/s}^2$$

So, block will again come to ground after time t, given by

$$0 = 2t - \frac{1}{2} \times 2t^2 \Rightarrow t = 2\text{s}$$

At this instant, the pan + ball system is moving with $v = 2$ m/s and the string gets slacked, so the system is moving under gravity.

To determine the speed of the system at $t = 2.6$ s, first find the time after which string is again jerked. That will happen when pan + ball system crosses its $t = 2$ s instant during its downward journey, i.e., after its time of flight of motion under gravity.

$$T = \frac{2 \times 2}{10} = 0.4 \text{ s}$$

i.e., string will again jerk at $t = 2.4$ s.

For $t > 2.4$ s, the system is again under a retardation of 2 m/s². Before this, find the velocity of various components of system just after jerk. We have

$$-J' = 2v' - 2 \times 2 \text{ and } J = 3v'$$

$$5v = 4 \Rightarrow v = \frac{4}{5} = 0.8 \text{ m/s}$$

With this velocity, the ball + pan system moves down for 0.2 s under a retardation of 2 m/s².

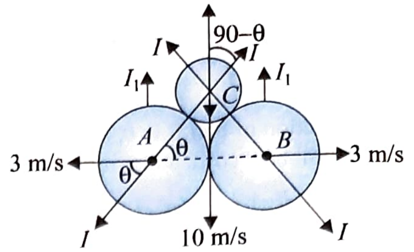
$$v = 0.8 - 2 \times 0.2 = 0.4 \text{ m/s (downwards)}$$

The maximum height attained by the block after the second jerk is given by $0 = (0.8)^2 - 2 \times 2H \Rightarrow H = 0.16$ m.

For Problems 45–47

45. (1) First calculate, $\theta = 45°$. As the balls A and B are constrained to move horizontally (immediately after collision), if 'I' be the impulse imparted by ball 'C' to each of A and B, the impulse received by ball C from them would be $2I\sin\theta$.

Now, each of ball B and C received impulse 'I' as shown in figure, but moves horizontally as its vertical component gets balanced by impulse imparted to ball B and C by the respective strings and hence,



$$I\cos\theta = M_AV_A = M_BV_B$$

$$\Rightarrow I = \frac{M_AV_A}{\cos\theta} \qquad (I = \text{magnitude of impulse})$$

Now, for ball C, if its final velocity is V_C' downwards, we have

$$M_CV_C' = M_CV_C - 2I\sin\theta$$

$$\Rightarrow V_C' = V_C - 2\frac{M_A}{M_C}V_A \qquad (\because \theta = 45°)$$

$= -2$ m/s (–ve sign indicates that it is directed upwards)

46. (4) Impulse provided by each string,

$$I_1 = I\sin\theta = 6 \text{ Ns}$$

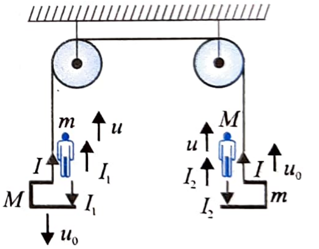
47. (4) $e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$

$$\Rightarrow e = \frac{3\cos\theta + V_C'\sin\theta}{10\sin\theta} \Rightarrow e = \frac{3+2}{10} = \frac{1}{2}$$

For Problems 48–50

48. (3) $I_1 = mu$, $I_2 = Mu$,

$$I_1 - I = Mu_0,\ I - I_2 = mu_0$$



Solve to get: $u_0 = \frac{(m - M)u}{m + M}$

Relative velocity of left man w.r.t. his trolley:

$$u + u_0 = u + \frac{(m - M)u}{m + M} = \frac{2mu}{m + M}$$

49. (2) $I = I_1 - Mu_0 = mu - Mu_0 = mu - \frac{M(m - M)}{m + M}u = \frac{(m^2 + M^2)u}{M + m}$

50. (1) Time taken for men to come to rest, $t_1 = \frac{u}{g}$

Acceleration of trolleys (after the men have jumped),

$$a = \frac{(m - M)}{m + M}g$$

Time taken by trolleys to come to rest, $t_1 = \frac{u_0}{a} = \frac{u}{g}$

Matrix Match Type

1. i. → d.; ii. → a.; iii. → c.; iv. → b.

From theory.

i. Momentum = mv

ii. Impulse = change in momentum = $m(v_2 - v_1)$

iii. In elastic collision, momentum and KE both are conserved.

iv. In inelastic collision, momentum is conserved but KE is not conserved.

2. i. → **a., d.; ii. → c., d.; iii. → c.; iv. → a., b., c.**

In inelastic collision, kinetic energy is always lost. In elastic collision, kinetic energy remains constant.

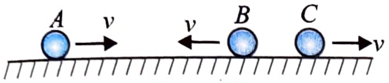
If total work done is zero, then kinetic energy will remain constant. If non-conservative forces are not present, then kinetic energy may decrease, increase or remain constant, depending upon other forces.

3. i. → **a., d.; ii. → c., d.; iii. → c., d.; iv. → c., d.**

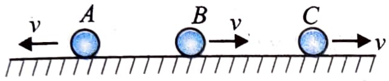
In elastic collision, kinetic energy, momentum and total energy all remain conserved. In inelastic collision, kinetic energy is lost but other quantities remain conserved.

4. i. → **a., b.; c.; ii. → a., b., c.; iii. → a., b.; iv. → a., b.**

$$v_{CM} = \frac{mv + mv + m(-v)}{3m} = \frac{v}{3}$$



After first collision



If A and B are connected, then they will act as a mass of $2m$.

If A and B are connected by an ideal string, then finally A and B will be at rest.

5. i. → **c., d.; ii. → c.; iii. → a.; iv. → a., b., c., d.**

i. $8u_1 + 2(-3) = 8v_1 + 2v_2$

$$\frac{v_2 - v_1}{u_1 - (-3)} = e = 0.5$$

Solving, we get $v_1 = \dfrac{7u_1 - 9}{10}$, $v_2 = \dfrac{12u_1 + 6}{10}$

v_2 is always positive.

For $v_1 > 0$ $u_1 > \dfrac{9}{7}$ m/s

ii. $8u_1 + 2(-3) = 8v_1 + 2v_2$

$$\frac{v_2 - v_1}{u_1 - (-3)} = e = 1$$

For maximum energy to transfer to m_2, $v_1 = 0$

Solving, we get $u_1 = 2$ m/s.

iii. For this $v_2 = 0$

iv. Depends upon e.

6. i. → **c.; ii. → d.; iii. → a.; iv. → b.**

$M_A = 1$ kg, $M_B = 2$ kg

For bullet, $\int F\,dt = m(v_f - v_i)$

$= 0.25[(-100) - (-200)] = 25$ kg m/s ...(i)

This is also impulse by particle on A.

For block A, $\int (F - T)\,dt = M_A(v - 0)$ ...(ii)

For block B, $\int T\,dt = M_B(v - 0)$ ...(iii)

From Eqs. (ii) and (iii), we get

$\int F\,dt = (M_A + M_B)v \Rightarrow 25 = 3v$ or $v = \dfrac{25}{3}$ m/s

$$\int T\,dt = M_B v = \frac{2 \times 25}{3} = \frac{50}{3}$$

Now applying the law of conservation of energy,

$$M_B gh = +\frac{1}{2}(M_A + M_B)v^2$$

$$\Rightarrow h = \frac{1}{2}\frac{(M_A + M_B)v^2}{M_B g} = 5.21 \text{ m}$$

7. i. → a., b.; ii. → a., b.; iii. → d.; iv. → c.

$x_1 = u\cos\theta\, t_1$; $x_2 = u\cos\theta\, t_2$

$$\sqrt{\frac{2h}{g}} = \frac{T}{2} = \frac{t_1 + t_2}{2};\ \sqrt{\frac{2h}{2g}} = \frac{t_2 - t_1}{2}$$

Dividing, $\dfrac{t_2 + t_1}{t_2 - t_1} = \sqrt{2}$

Also, $\dfrac{x_2 - x_1}{x_2 + x_1} = \dfrac{t_2 - t_1}{t_2 + t_1}$

$$T = \frac{2u\sin\theta}{g};\ t_1 + t_2 = \frac{2u\sin\theta}{g}$$

$$\frac{1}{2} = \frac{u\sin\theta}{g(t_1 + t_2)}$$

Also, $x_1 + x_2 = u\cos\theta\,(t_1 + t_2)$

$$\frac{u\cos\theta\,(t_1 + t_2)}{x_1 + x_2} = 1$$

8. (2) For (i): During collision forces are action-reaction pair. So they are in same magnitude.

For (iii): $e = -\left(\dfrac{v_2 - v_1}{u_2 - u_1}\right) \Rightarrow \left|\dfrac{v_2 - v_1}{u_2 - u_1}\right| < 1$

$|v_2 - v_1| < |u_2 - u_1|$ so (d) is correct not (a).

9. (1) For both collisions to be perfectly elastic, using conservation of linear momentum;

$mv_1 + mv_2 + 2mv_3 = mu$

$\Rightarrow v_1 + v_2 + 2v_3 = u$ ...(i)

and $v_1 - v_2 = u$ ...(ii)

and $v_2 - v_3 = u$ ...(iii)

Solving Eqs. (i), (ii) and (iii)

$$v_1 = \frac{3u}{2},\ v_2 = \frac{u}{2} \text{ and } v_3 = -\frac{u}{2}$$

$\therefore$ $v_1 > v_2 = v_3$ (speed)

For both collisions with $e = \dfrac{1}{2}$

$v_1 + v_2 + 2v_3 = u$ ...(iv)

and $v_1 - v_2 = \dfrac{u}{2}$ ...(v)

and $v_2 - v_3 = \dfrac{u}{2}$ ...(vi)

Solving Eqs. (iv), (v) and (vi)

$$v_1 = \frac{7u}{8};\ v_2 = \frac{3u}{8} \text{ and } v_3 = -\frac{u}{8}$$

$\therefore$ $v_1 > v_2 > v_3$

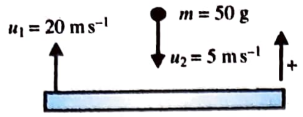
For perfectly inelastic collision between B and C they will move together after collision and A will move away. Thus; $v_1 > v_2 = v_3$

Numerical Value Type

1. **(5)** Speed of the nth ball = $\sqrt{(5gr)}$

Since $e = 1$, speed of 1st ball= speed of nth ball = $\sqrt{(5gr)} = 5\ \text{m s}^{-1}$

2. **(45)** The situation is analysed in the figure. We can consider mass of platform to be very large compared to that of ball, so we have



$$\vec{v}_1 = \vec{u}_1 \quad e = \left(\frac{v_2 - v_1}{u_2 - u_1}\right) = \left(\frac{v_2 - u_1}{u_1 - (-u_2)}\right) \Rightarrow \vec{v}_2 = 2\vec{u}_1 - \vec{u}_2$$

Thus rebound velocity of ball is

$$\vec{v}_2 = 2 \times 20 - (-5) = 45\ \text{m s}^{-1} \text{ upward}$$

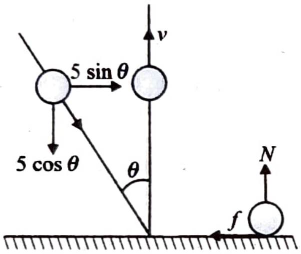
3. **(1)** From impulse–momentum theorem,

$$\int N\,dt = m(v + 5\cos\theta) \qquad \text{...(i)}$$

$$\int f\,dt = m \times 5\sin\theta$$

$$\mu \int N\,dt = m \times 5\sin\theta \qquad \text{...(ii)}$$

$$\Rightarrow \mu m(v + 5\cos\theta) = m \times 5\sin\theta$$



According to Newton's law of restitution,

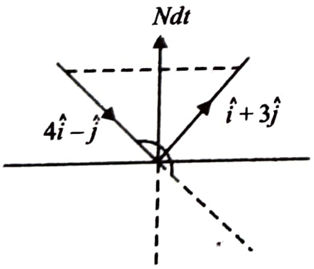
$$v = e \times 5\cos\theta$$

Solve to get $\mu = 1$

4. **(9)** $\int N dt = 1[(\hat{i} + 3\hat{j}) - (4\hat{i} - \hat{j})] = -3\hat{i} + 4\hat{j}$

Component of $4\hat{i} - \hat{j}$ along $-3\hat{i} + 4\hat{j}$

$$= \frac{-12 - 4}{25}(-3\hat{i} + 4\hat{j}) = -\frac{16}{25}(-3\hat{i} + 4\hat{j})$$



Speed of approach $= \frac{16}{25}\sqrt{25} = \frac{16}{5}$

Component of $\hat{i} + 3\hat{j}$ along $-3\hat{i} + 4\hat{j}$ is $\frac{-3 + 12}{25}(-3\hat{i} + 4\hat{j})$

Speed of separation = 9/5

Speed of separation = $e \times$ speed of approach

$$e = \frac{9}{16} \text{ or } n = 9$$

5. **(3)** For first case: $W = \Delta\text{KE} \Rightarrow -Rd = -\frac{1}{2}mu^2$

$$Rd = \frac{1}{2}mu^2 \qquad \text{...(i)}$$

For second case: $mu = (M + m)\,v$, v is common velocity

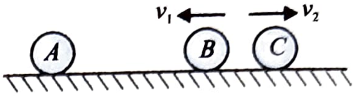
$$\Rightarrow v = \frac{m}{(M + m)}u$$

$$-Rd' = \frac{1}{2}(M + m)v^2 - \frac{1}{2}mu^2 = \frac{1}{2}\frac{m^2u^2}{(M + m)} - \frac{1}{2}mu^2$$

$$\Rightarrow Rd' = \frac{1}{2}\frac{Mm}{(M + m)}u^2 \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), $\frac{Rd}{Rd'} = \frac{M + m}{M} \Rightarrow d' = 3$ cm

6. **(2)** For the first collision, $e = 1$, $v = v_1 + v_2$



$$\Rightarrow v_2 = v - v_1 \qquad \text{...(i)}$$

By momentum conservation

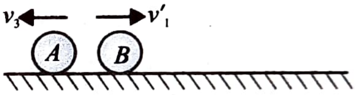
$$m_Bv = -m_Bv_1 + m_Cv_2$$

$$m_Bv = -m_Bv_1 + 4\,m_Bv_2$$

$$v_2 = \frac{v_1 + v}{4} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), $v_1 = \frac{3}{5}v$ and $v_2 = \frac{2}{5}v$

For the second collision, $e = 1$



$$v_1 = v'_1 + v_3 \Rightarrow v_3 = v_1 - v'_1 \qquad \text{...(iii)}$$

By momentum conservation, $-m_B v_1 = m_B v'_1 - m_A v_3$

or $-m_B v_1 = m_B v'_1 - 4\,m_B v_3$ $\qquad (\because m_A = 4\,m_B)$

$$v_3 = \frac{v'_1 + v_1}{4} \qquad \text{...(iv)}$$

From Eqs. (iii) and (iv), $v'_1 = \frac{3}{5}v_1 = \frac{3}{5}\left(\frac{3}{5}v\right) = \frac{9}{25}v$

Clearly, $\frac{9}{25}v < \frac{2}{5}v$

Therefore 'B' cannot collide with 'C' for the second time. Hence, the total number of collisions is 2.

7. **(2)** $\vec{F}_1 = -4\hat{i}$ $\qquad 0 < t \le 1$ s

$= -2\hat{i}$ $\qquad 1 \le t \le 3$ s

$\vec{F}_2 = -\hat{j}$ $\qquad 0 < t \le 2$ s

$= -(2t - 3)\hat{j}$ $\qquad 2 \le t \le 3$ s

Initial velocity of the particle is $\vec{u} = 10\hat{i}$

From impulse momentum theorem, $\int d\vec{p} = \int \vec{F}\,dt$

$$m\vec{v} - m\vec{u} = \int_2^3 (\vec{F}_1 + \vec{F}_2)dt$$

(where $\vec{v}$ is the required velocity)

$$1 \times \vec{v} - 1 \times 10\hat{i} = -8\hat{i} - 4\hat{j} \text{ or } \vec{v} = 2\hat{i} - 4\hat{j}$$

or $v = \sqrt{2^2 + 4^2} = 2\sqrt{5}$ m/s

8. **(38)** The collision does not alter the vertical component of the velocity of either ball, so $t_1 + t_2 = T$.

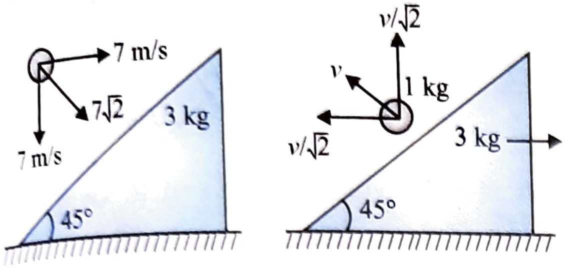
$$\Rightarrow \frac{50}{u\cos 30^\circ} + \frac{50}{0.7u\cos 30^\circ} = \frac{2u\sin 30^\circ}{10} \Rightarrow u \approx 38 \text{ m/s}$$

9. **(4)** $v_x = 7$ m/s, velocity along y-axis of the ball just before collision.

$$v_y = \sqrt{2 \times 9.8 \times 2.5} = 7 \text{ m/s}$$

As $v_x = v_y$, so it strikes the plane of incline perpendicularly.

Let the ball rebound with velocity V and let v_1 be the velocity of the wedge.



Applying the principle conservation of momentum in horizontal direction, we get

$$1 \times 7 = -1 \times \frac{v}{\sqrt{2}} + 3v_1$$

$$7\sqrt{2} = 3\sqrt{2}\, v_1 - v \qquad \text{...(i)}$$

Applying the equation for coefficient of restitution, we get

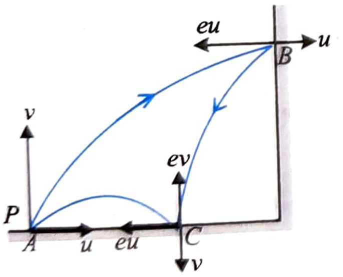
$$e = 1 = \frac{v_1/\sqrt{2} + v}{7\sqrt{2}},\ 7\sqrt{2} = \frac{v_1}{\sqrt{2}} + v \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), $v_1 = 4$ m/s and $v = 5\sqrt{2}$ m/s.

10. **(1)** u time $(AB) = eu$ (time BC) + eu (time CA)

$$u\frac{v}{g} = eu\left(\frac{v}{g}\right) + eu \times 2\frac{ev}{g} \Rightarrow e = \frac{1}{2}$$

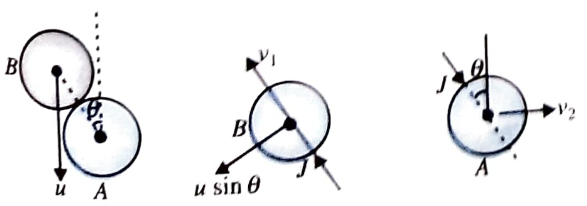
$$n = 1$$



11. **(1250)** $v_2 = \sqrt{5gl} = \sqrt{5 \times 10 \times 1.5} = 5\sqrt{3}$ m/s

For A: $J\sin\theta = mv_2 = 5\sqrt{3}\,\text{m}$

$$\Rightarrow J \times \frac{1}{2} = 5\sqrt{3}\,\text{m} \Rightarrow J = 10\sqrt{3}\,\text{m}$$



For B: $J = m(v_1 + u\cos\theta)$

$$\Rightarrow 10\sqrt{3}m = m\left(v_1 + \frac{u\sqrt{3}}{2}\right) \qquad \text{...(i)}$$

$$\Rightarrow 20\sqrt{3} = 2v_1 + u\sqrt{3}$$

By Newton's experimental law:

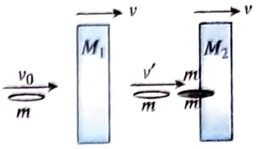
$$e = 1 = \frac{v_2\sin\theta + v_1}{u\cos\theta} \Rightarrow \frac{u\sqrt{3}}{2} = \frac{v_2}{2} + v_1$$

$$\Rightarrow u\sqrt{3} = 5\sqrt{3} + 2v_1 \qquad \text{...(ii)}$$

From (i) and (ii), $u = 12.5$ m/s $= 1250$ cm/s

12. **(33.3)** Conservation of linear momentum of the system yields

$$mv_0 = M_1v + mv' = (M_2 + m)v$$



$$\Rightarrow v = \frac{mv_0}{M_2 + m}$$

and $mv' = mv_0 - M_1v$

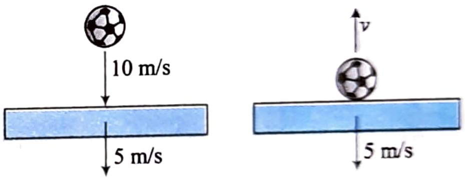
$$= mv_0 - \frac{M_1mv_0}{M_2 + m}$$

$$= mv_0\left[1 - \frac{M_1}{M_2 + m}\right]$$

$$\Rightarrow v' = v_0 - \frac{v_0M_1}{M_2 + m}$$

Now find $\dfrac{|v' - v_0|}{v_0} \times 100 = \dfrac{M_1}{(M_2 + m)} \times 100 = 33.3\%$

13. **(1)** By definition of e: $e = \dfrac{v_2 - v_1}{u_1 - u_2}$



Just before collision Just after collision

We have $0.8 = \dfrac{v + 5}{10 - 5} \Rightarrow v = 1$ m/s

14. **(2)** In the one dimensional elastic collision with one body at rest, the body moving initially comes to rest and the one which was at rest earlier starts moving with the velocity that first body had before collision.

so, if m and V_0 be the mass and velocity of body,

the change in momentum $= mV_0$

$$\Rightarrow \int F dt = mV_0 \Rightarrow F = \frac{2mV_0}{\Delta t} = 2\text{N}$$

15. **(4)** For first collision with plate A, final velocity of ball

$$v_1 = ev_0 = e\sqrt{2gh_0} \qquad \text{...(i)}$$

For second collision, $mv = 4mv' \Rightarrow v' = \dfrac{v}{4}$

$$\Rightarrow e\sqrt{\frac{2gh_0}{4}} = \sqrt{2gh_2} \Rightarrow e = \frac{2}{3}$$

Height attained after first collision

$$\therefore h_1 = e^2h_0 = \frac{2}{3} \times \frac{2}{3} \times 9 = 4\text{ m}$$

16. **(1.8)** Since wall is smooth, vertical component of velocity will not change and hence total time of flight will not change.

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 25}{10} \times \frac{3}{5} = 3\text{ s}$$

Ball hits the wall with a horizontal velocity $u \cos\theta$ and let us assume that the ball returns with a horizontal velocity v_x.

Using $e = \dfrac{\text{velocity of separation}}{\text{velocity of approach}}$

Here $e = 1$; $u\cos\theta + \dfrac{u}{5} = v_x - \dfrac{u}{5}$

Solving, we get $v_x = u\cos\theta + \dfrac{2u}{5}$

Let the time taken by the ball from point of projection to the hitting with wall be t.

$$\therefore \quad u\cos\theta \cdot t = \left(u\cos\theta + \frac{2u}{5}\right)(T-t)$$

$$\Rightarrow \quad 25\times\frac{4}{5}t = \left(25\times\frac{4}{5}+\frac{2\times 25}{5}\right)(3-t)$$

$$\Rightarrow \quad 20t = (20+10)(3-t)$$

$$\Rightarrow \quad 50t = 90$$

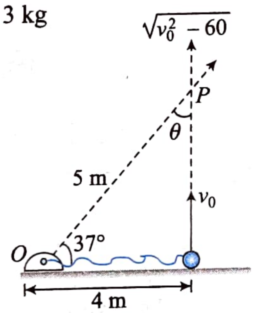
$$t = \frac{9}{5} = 1.8\text{ s}$$

17. (3) $\Delta\bar{p}$ (change in momentum per collision) $= 2m\sqrt{2gh}$

$$= 2\times 15\times 10^{-3}(10) = 0.3\,\text{kg}\,\text{ms}^{-1}$$

Then in $\Delta\bar{p}_{100}$ (in 1 second) = 30 kg ms^{-1}. Force exerted = 30 N So weight required = 30 N. So mass = 3 kg

18. (8) $J = 5\sqrt{v_0^2 - 60}\left(\dfrac{3}{5}\right) = 6$



$$v_0^2 = 64$$

$$v_0 = 8\,\text{m/s}$$

19. (8) In frame of elevator, during collision,

$$v_{\text{approach}} = \sqrt{2a_{\text{rel}}h_1}$$

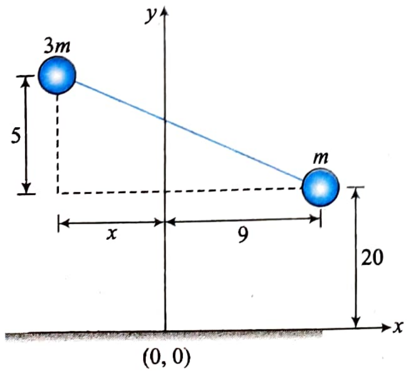
where a_{rel} is relative acceleration of ball w.r.t. elevator

$$a_{\text{rel}} = g + \frac{g}{4} = \frac{5g}{4}$$

$$v_{\text{separation}} = ev_{\text{approach}}$$

$$h_2 = \frac{v_{\text{separation}}^2}{2a_{\text{rel}}} = e^2h_1$$

20. (13) The external force on two body system acts along y-axis. The initial momentum of the two body system is zero. Hence the CM of two body system always moves along y-axis.

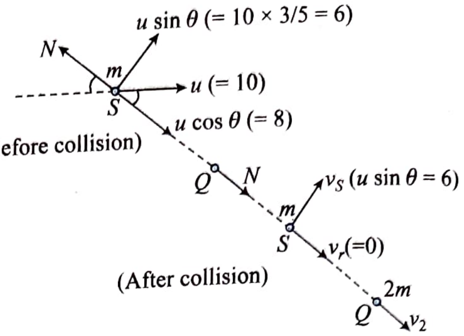


$\because$ Centre of mass of two body system lies along y-axis, the x-coordinate of centre of mass of two body system is $x_{\text{cm}} = 0$

$$\therefore \quad 3m\times x = m\times 9 \Rightarrow x = 3$$

$$\therefore \quad \text{Length of string is } \sqrt{5^2+(9+3)^2} = 13\text{ cm}$$

21. (0.50)



Impulsive force N on Q acts along the direction SQ it means direction of line of impact is towards SQ.

So $e = (v_2 - v_1)/u\cos\theta$

$$v_2 - 0 = 8e \Rightarrow v_2 = 8e \quad \text{...(i)}$$

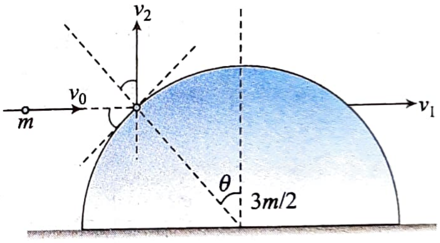
conservation of linear momentum in SQ direction gives:

$$mu\cos\theta + 2m(0) = mv_1 + 2mv_2$$

$$\therefore \quad 8 = 2v_2 \Rightarrow v_2 = 4\text{ m/s} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), $e = 1/2$

22. (60)



$$mv_0 = m(0) + (3m/2)v_1$$

$$(2/3)v_0 = v_1 \quad \text{...(i)}$$

$$e = 1 = (v_2\cos\theta + v_1\sin\theta)/(v_0\sin\theta) \quad \text{...(ii)}$$

For m: $(v_2\sin\theta = v_0\cos\theta)$ ...(iii)

Solving we get the answer $\theta = 60°$

23. (3) As $v_2 = \dfrac{2m_1}{m_1+m_2}v_1$

$$\therefore \quad v_3 = \frac{2m_2}{m_2+m_3}v_2 = \frac{4m_1m_2v}{(m_1+m_2)(m_2+m_3)}$$

For maximum v_3, $\dfrac{dv_3}{dm_2} = 0 \Rightarrow m_2 = \sqrt{m_1m_3} = 3$ kg

Archives

JEE Main

Single Correct Answer Type

1. (3) As rubber ball is freely falling, the ball is dropped ($u = 0$). If we take downwards direction negative, the velocity at any time t can be given by the relation $v = -gt$. As when coming down, v increases, makes collision, the value of v becomes +ve, the speed decreases, momentarity comes to zero and then moves in downwards direction and speed increases. The change from $+v$ to $-v$ is almost instantaneous.

Further $h = -\frac{1}{2}gt^2$ is a parabola which opens downwards.

Therefore option (3) is correct.

2. (1)

If it is a completely inelastic collision, then

$$m_1v_1 + m_2v_2 = m_1v + m_2v$$

$$\Rightarrow\ v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$\text{KE} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$$

As $\vec{p}_1$ and $\vec{p}_2$ both simultaneously cannot be zero, total KE cannot be lost.

3. (2) Here, mass of the body, $m = 0.4$ kg

Since position–time (x–t) graph is a straight line, so motion is uniform. Because of impulse direction of velocity change as it can be seen from the slopes of the graph.

From graph,

Initial velocity, $u = \dfrac{(2-0)}{(2-0)} = 1\ \text{ms}^{-1}$

Final velocity, $v = \dfrac{(0-2)}{(4-2)} = -1\ \text{ms}^{-1}$

Initial momentum, $p_i = mu = 0.4 \times 1 = 0.4$ Ns

Final momentum, $p_f = mv = 0.4 \times (-1) = -0.4$ Ns

Impulse = Change in momentum = $p_f - p_i$

$= (-0.4 - 90.4)$ Ns $= -\,0.8$ Ns

$|\text{Impulse}| = 0.8$ Ns

4. (3) Loss of energy is maximum when collision is inelastic as in an inelastic collision there will be maximum deformation.

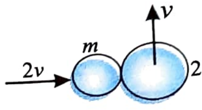
KE in CM frame is $\frac{1}{2}\left(\frac{Mm}{M+m}\right)v_{\text{rel}}^2$

$$K_i = \frac{1}{2}\left(\frac{Mm}{M+m}\right)V^2;\ K_f = 0 \qquad (\because V_{\text{ed}} = 0)$$

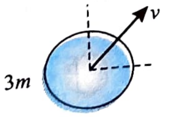
Hence, loss in energy is $\frac{1}{2}\left(\frac{Mm}{M+m}\right)V^2$

$$\Rightarrow\ f = \frac{M}{M+m}$$

5. (3) Applying conservation of linear momentum just before and just after collision.



Just before collision



Just after collision

$$m2v\hat{i} + 2mv\hat{j} = (m + 2m)\vec{v}'$$

$$\vec{v}' = \frac{2}{3}v(\hat{i} + \hat{j})$$

Hence $|\vec{v}'| = \dfrac{2\sqrt{2}}{3}v$

Initial kinetic energy of the masses just before collision,

$$K_{\text{initial}} = \frac{1}{2}m(2v)^2 + \frac{1}{2}2mv^2 = 3mv^2$$

Final kinetic energy of the system just after collision,

$$K_{\text{final}} = \frac{1}{2}(3m)\left(\frac{2\sqrt{2}}{3}v\right)^2 = \frac{4}{3}mv^2$$

Loss in kinetic energy,

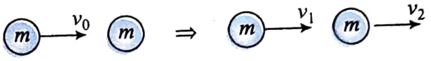
$$\Delta K = K_{\text{initial}} - K_{\text{final}}$$

$$= 3mv^2 - \frac{4}{3}mv^2 = \frac{5}{3}mv^2$$

Hence, percentage loss in kinetic energy

$$\frac{\Delta K}{K}\% = \frac{\frac{5}{3}mv^2}{3mv^2} \times 100 = 56\%$$

6. (3) Initial



$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \qquad \text{...(i)}$$

From momentum conservation

$$mv_0 = m(v_1 + v_2) \qquad \text{...(ii)}$$

$$(v_1 + v_2)^2 = v_0^2 \Rightarrow v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$$

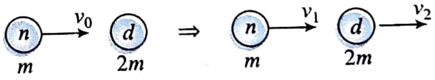
$$2v_1v_2 = -\frac{v_0^2}{2}$$

$$(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2 = \frac{3}{2}v_0^2 + \frac{v_0^2}{2} = 2v_0^2$$

The relative velocity between the two particles, after collision,

$$v_1 - v_2 = \sqrt{2}v_0$$

7. (2) Let initial speed of neutron is v_0



By momentum conservation,

$$mv_0 = mv_1 + 2mv_2 \Rightarrow v_1 + 2v_2 = v_0 \qquad \text{...(i)}$$

Coefficient of restitution,

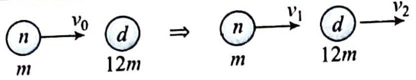
$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow 1 = \frac{v_2 - v_1}{v_0 - 0}$$

$$v_2 - v_1 = v_0 \qquad \text{...(ii)}$$

From (i) and (ii), $v_2 = \dfrac{2v_0}{3}$ and $v_1 = -\dfrac{v_0}{3}$

$$\text{Fractional loss, } P_d = \frac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{v_0}{3}\right)^2}{\frac{1}{2}mv_0^2}$$

$$\Rightarrow\ P_d = \frac{8}{9} \approx 0.89$$



Using conservation of linear momentum

$$mv_0 = mv_1 + 12mv_2 \Rightarrow v_1 + 12v_2 = v_0 \qquad \text{...(i)}$$

Coefficient of restitution $e = \dfrac{v_2 - v_1}{u_1 - u_2} \Rightarrow 1 = \dfrac{v_2 - v_1}{v_0 - 0}$

$$v_2 - v_1 = v_0 \qquad \text{...(ii)}$$

From (i) and (ii), $v_2 = \dfrac{2v_0}{13}$ and $v_1 = -\dfrac{11v_0}{13}$

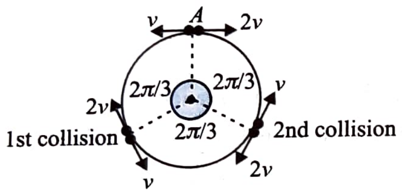
Fractional loss fraction loss of energy,

$$P_c = \frac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{11v_0}{13}\right)^2}{\frac{1}{2}mv_0^2} \Rightarrow P_c = \frac{48}{169} \approx 0.28$$

JEE Advanced

Single Correct Answer Type

1. (3) Velocity will exchange after each collision



2. (4) $R = u\sqrt{\dfrac{2h}{g}}$

$$\Rightarrow 20 = V_1\sqrt{\frac{2\times5}{10}} \text{ and } 100 = V_2\sqrt{\frac{2\times5}{10}}$$

$\Rightarrow V_1 = 20$ m/s, $V_2 = 100$ m/s

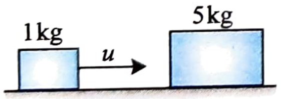
Applying momentum conservation just before and just after the collision $(0.01)(V) = (0.2)(20) + (0.01)(100)$

$V = 500$ m/s

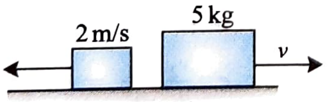
Multiple Correct Answers Type

1. (1), (3)

By conservation of linear momentum



before collision



after collision

$$u = 5v - 2 \qquad \text{...(i)}$$

By Newton's experimental law of collision

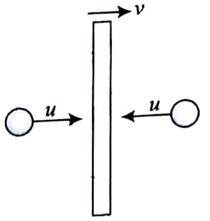
$$u = v + 2 \qquad \text{...(ii)}$$

Using Eqs. (i) and (ii), we have $v = 1$ m/s and $u = 3$ m/s

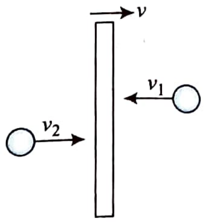
Kinetic energy of the centre of mass $= \dfrac{1}{2}m_{\text{system}}v_{\text{CM}}^2 = 0.75$ J

2. (1), (3), (4)

Let mass of a gas molecule is m. Assuming collision of the molecule with plate perfectly elastic ($e = 1$)



Just before collision



Just after collision

For solution from leading face:

Using $e = \left(\dfrac{v_2 - v_1}{u_1 - u_2}\right)_n \Rightarrow 1 = \dfrac{v - v_1}{-u - v}$

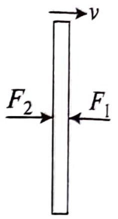
$\Rightarrow \quad v_1 = (u + 2v)$

Hence change in velocity of the molecule colliding from leading face

$$\Delta v_1 = (u + 2v) - (-u) = 2(u + v)$$

Hence force applied by gas molecules on leading face

$$F_1 = \frac{dp_1}{dt} = \rho A(u + v)\cdot 2(u + v)$$



$\Rightarrow \quad F_1 = 2\rho A(u + v)^2$

For collision from tailing face:

$$1 = \frac{v - v_2}{u - v} \Rightarrow v_2 = 2v - u$$

Hence change in velocity

$$\Delta v_2 = (2v - u) - u = 2v - 2u = 2(v - u)$$

Hence $F_2 = \rho A(v - u)\cdot 2(v - u) = 2\rho A(v - u)^2$

Hence force difference $= F_1 - F_2 =$ resistive force

$$F_{\text{resis}} = F_1 - F_2 = 8\rho Auv \Rightarrow F_{\text{resis}} \propto uv \text{ or } F_{\text{resis}} \propto v$$

Hence option (4) is correct.

Hence difference pressure $P = \dfrac{F_{\text{resistance}}}{A} = 8\rho uv$

Hence option (1) is correct.

The equation of motion of plate

$$F - F_{\text{resistance}} = m\frac{dv}{dt}$$

$$F - 8\rho Auv = m\frac{dv}{dt}$$

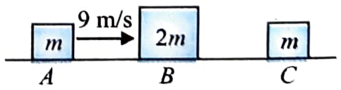
After long time v will be sufficient so that $F = 8\rho Auv$

After that v = constant i.e., plate will move with constant velocity.

Hence option (3) is correct.

Numerical Value Type

1. (4) After the first collision,



$$mv_A = mv_A' + 2mv_B$$

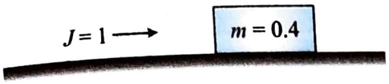
$$-1 = \frac{v_B - v_A}{0 - v_A} \Rightarrow v_B = 6 \text{ m/s}$$

After the second collision,

$$2mv_B = (2m + m)v_C$$

$\Rightarrow v_C = (2/3)\,v_B \Rightarrow v_C = 4$ m/s

2. (6.30)



Given: Impulse $J = 1$ Ns, mass of the block $m = 0.4$ kg

Velocity of the block, $v_0 = \frac{J}{m} = 2.5$ m/s

As, $v = v_0 e^{-t/\tau}$

We can write

$$\frac{dx}{dt} = v_0 e^{-t/\tau} \text{ or } dx = v_0 e^{-\tau/2} \cdot dt$$

Integrating both sides $\int_0^x dx = v_0 \int_0^\tau e^{-t/\tau} dt$

As we know $\int e^{-x} dx = \frac{e^{-x}}{-1}$

$$x = v_0 \left| \frac{e^{-t/\tau}}{-\frac{1}{\tau}} \right|_0^\tau$$

$x = 2.5(-4)(e^{-1} - e^{0})$ or $x = 25(-4)(0.37 - 1)$

Hence, $x = 6.30$

Chapter 3

Concept Application Exercises

Exercise 3.1

1. The angular velocity of B with respect to A,

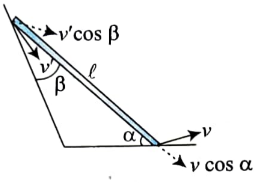
$$\omega = \omega_{B,A} = \frac{v_{B,A}}{l} = \frac{0.20 + 0.30}{1} = 0.50 \text{ rad/s}$$

From right hand rule, it is along + z direction, hence in vector form $\vec{\omega} = 0.50\,\hat{k}$ rad/s

2. $v_P = v_c - R\omega = 10 - \frac{20}{100} \times 10 = 8$ m/s

$$v_Q = v_C + R\omega = 10 + 2 = 12 \text{ m/s}$$

3. Let the velocity at Q be v'. As rod is a rigid body,



v′cos β
v′
β
ℓ
α
v
v cos α

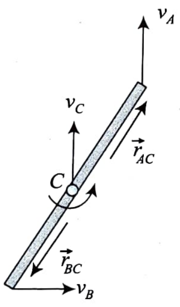
$$v' \cos\beta = v\cos\alpha$$

or $$v' = \frac{v\cos\alpha}{\cos\beta}$$

$$\omega = \frac{v'\sin\beta + v\sin\alpha}{l} = \frac{1}{l}\left(\frac{v\cos\alpha\sin\beta}{\cos\beta} + v\sin\alpha\right)$$

$$= \frac{v\sin(\alpha+\beta)}{l\cos\beta}$$

4. Let us assume that, the mid-point of the rod has velocity $\vec{v}_C$. Then, we have



v_A
v_C
$\vec{r}_{AC}$
C
$\vec{r}_{BC}$
v_B

$$\vec{v}_A = \vec{v}_{AC} + \vec{v}_C \qquad \text{...(i)}$$

$$\vec{v}_B = \vec{v}_{BC} + \vec{v}_C \qquad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we have

$$\vec{v}_A + \vec{v}_B = \vec{v}_{AC} + \vec{v}_{BC} + 2\vec{v}_C$$

Substituting $\vec{v}_{AC} = +\vec{\omega}_{AC} \times \vec{r}_{AC}$

and $\vec{v}_{BC} = +\vec{\omega}_{BC} \times \vec{r}_{BC}$, we get

$$\vec{v}_A + \vec{v}_B = \vec{\omega}_{AC} \times \vec{r}_{AC} + \vec{\omega}_{BC} \times \vec{r}_{BC} + 2\vec{v}_C$$

Substituting $\vec{\omega}_{AC} = \vec{\omega}_{BC} = \vec{\omega}$ (angular velocity of the rod), we have,

$$\vec{v}_A + \vec{v}_B = \vec{\omega} \times (\vec{r}_{AC} + \vec{r}_{BC}) + 2\vec{v}_C$$

Since C is the mid-point of the rod, $\vec{r}_{AC} = -\vec{r}_{BC}$. Then we have,

$$\vec{v}_C = \frac{\vec{v}_A + \vec{v}_B}{2}$$

This tells us that when any two points A and B, (say;) of a rigid body have velocities $\vec{v}_A$ and $\vec{v}_B$ respectively, the mid-point C (but not necessarily the centre of mass) of the line joining the points will have a velocity $\vec{v}_C = \frac{\vec{v}_A + \vec{v}_B}{2}$.

5. (a) Two particles of a rigid body must have same velocity component along the line joining them.

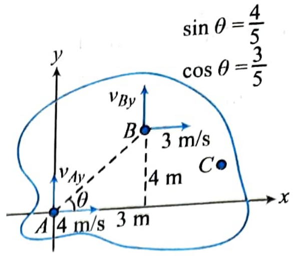
Velocity of 'A' along AB = velocity of 'B' along AB

$$4\cos\theta + v_{Ay}\sin\theta = 3\cos\theta + v_{By}\sin\theta$$

$$4 \times \frac{3}{5} + v_{Ay} \times \frac{4}{5} = 3 \times \frac{3}{5} + v_{By} \times \frac{4}{5}$$

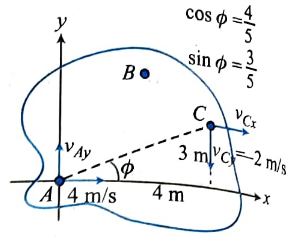
$$12 + 4v_{Ay} = 9 + 4v_{By}$$

$$\frac{3}{4} = v_{By} - v_{Ay} \qquad \text{...(i)}$$



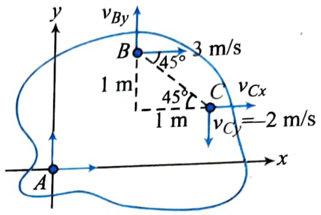
$\sin\theta = \frac{4}{5}$
$\cos\theta = \frac{3}{5}$
y
v_{By}
B
3 m/s
v_{Ay}
4 m
C
θ
A
4 m/s
3 m
x

(a)



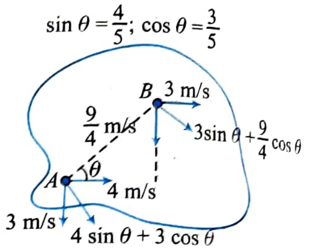
$\cos\phi = \frac{4}{5}$
$\sin\phi = \frac{3}{5}$
y
B
C
v_{Cx}
v_{Ay}
3 m
$v_{Cy} = -2$ m/s
ϕ
A
4 m/s
4 m
x

(b)



y
v_{By}
B
45°
3 m/s
1 m
45°
C
v_{Cx}
1 m
$v_{Cy} = -2$ m/s
A
x

(c)



$\sin\theta = \frac{4}{5}$; $\cos\theta = \frac{3}{5}$
B
3 m/s
$\frac{9}{4}$ m/s
$3\sin\theta + \frac{9}{4}\cos\theta$
θ
A
4 m/s
3 m/s
$4\sin\theta + 3\cos\theta$

(d)

Similarly, for A and C

Velocity of 'A' along AC = velocity of 'C' along AC

$$4\cos\phi + v_{Ay}\sin\phi = v_{Cx}\cos\phi - v_{Cy}\sin\phi$$

$$4 \times \frac{4}{5} + v_{Ay} \times \frac{3}{5} = v_{Cx} \times \frac{4}{5} - 2 \times \frac{3}{5}$$

$$4v_{cx} - 3v_{Ay} = 22 \qquad \text{...(ii)}$$

and for B and C,

$$3\cos 45° - v_{By}\cos 45° = 2\cos 45° + v_{Cx}\cos 45°$$

$$v_{By} + v_{Cx} = 1 \qquad \text{...(iii)}$$

Solving we get

$$v_{Ay} = -3 \text{ m/s};\ v_{By} - \frac{9}{4} \text{ m/s};\ v_{Cx} = \frac{13}{4} \text{ m/s}$$

$$\therefore \qquad \vec{v}_A = 4\hat{i} - 3\hat{j};\ \vec{v}_B = 3\hat{i} - \frac{9}{4}\hat{j};\ \vec{v}_C = \frac{13}{4}\hat{i} - 2\hat{j}$$

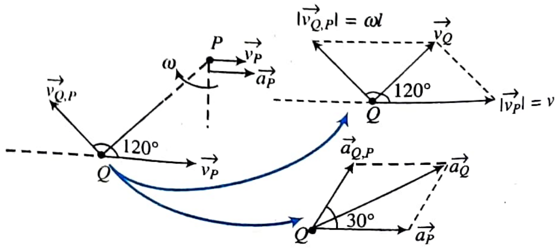
(b) Angular velocity $(\omega) = \dfrac{\text{Relative velocity of } A \text{ w.r.t. } B \perp \text{ to } AB}{AB}$

$$= \frac{\text{Velocity of } B \perp \text{to } AB - \text{velocity of } A \perp \text{to } AB}{AB}$$

$$\omega = \frac{\left(3 \times \frac{4}{5} + \frac{9}{4} \times \frac{3}{5}\right) - \left(4 \times \frac{4}{5} + 3 \times \frac{3}{5}\right)}{5}$$

$$= \frac{1}{4} \text{ rad/s}$$

6. $v_Q = |\vec{v}_{QP} + \vec{v}_P| = \sqrt{l^2\omega^2 + v^2 + 2vl\omega\cos 120°}$



$|\vec{v}_{Q,P}| = \omega l$
$\vec{v}_Q$
P
$\vec{v}_P$
ω
$\vec{a}_P$
$\vec{v}_{Q,P}$
120°
$|\vec{v}_P| = v$
Q
120°
Q
$\vec{v}_P$
$\vec{a}_{Q,P}$
$\vec{a}_Q$
30°
Q
$\vec{a}_P$

$$= \sqrt{(1\times 2)^2 + 1^2 + 2(1)(2)\times\left(\frac{-1}{2}\right)} = \sqrt{3}\ \text{m s}^{-1}$$

$$a_Q = |\vec{a}_{QP} + \vec{a}_P| = \sqrt{a_{QP}^2 + a_P^2 + 2a_{QP}a_P\cos 30^\circ}$$

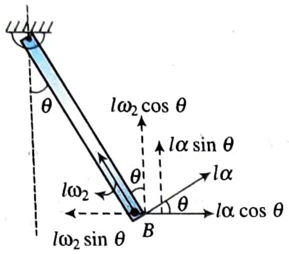
where $a_{QP} = l\omega^2 = 1(2)^2 = 4\ \text{m s}^{-2}$ and $a_P = 1\ \text{m s}^{-2}$

$$a_Q = \sqrt{4^2 + 1^2 + 2.4\times 1\times\frac{\sqrt{3}}{2}} = \sqrt{17+4\sqrt{3}}\ \text{m s}^{-2}$$

7. $\vec{a}_B = \vec{a}_{BA} + \vec{a}_A = \vec{a}_{BA_t} + \vec{a}_{BA_r} + \vec{a}_A$

where $a_A = 0,\ a_{BA_t} = l\alpha$

and $a_{BA_r} = l\omega^2$



Then resolving the vectors in x and y directions, we have

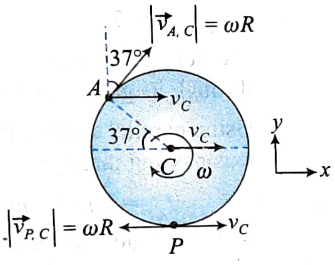
$$a_{B_x} = l\alpha\cos\theta - l\omega^2\sin\theta$$

and $$a_{B_y} = l\alpha\sin\theta + l\omega^2\cos\theta$$

Exercise 3.2

1. In rolling, the angular velocity $\vec{\omega}$ and velocity of the center of a rolling body are related as

$$v_C = \omega R \Rightarrow \omega = \frac{v_C}{R} = \frac{25}{5} = 5\ \text{rad/s}$$



If the velocity of point P is zero, the rotation of the cylinder should be in clockwise sense.

Velocity of point A, $\vec{v}_A = \vec{v}_{A,C} + \vec{v}_C$

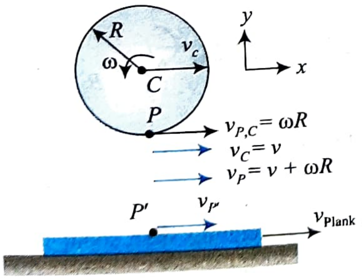
$$\Rightarrow \vec{v}_A = (\omega R\sin 37^\circ\hat{i} + \omega R\cos 37^\circ\hat{j}) + v_C\hat{i}$$

$$= (v_C + \omega R\sin 37^\circ)\hat{i} + \omega R\cos 37^\circ\hat{j}$$

$$= \left(25 + 5\times 5\times\frac{3}{5}\right)\hat{i} + 5\times 5\times\frac{4}{5}\hat{j} = (40\hat{i} + 20\hat{j})\ \text{m/s}$$

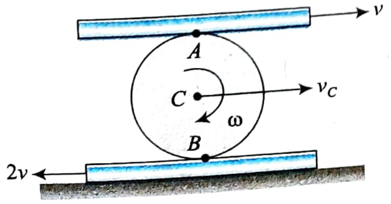
2. As disc is rolling, there should not be relative sliding between plank and disc at point of contact.

$\vec{v}_P = \vec{v}_{P'}$ = velocity of plank.



$$\vec{v}_P = \vec{v}_{P,C} + \vec{v}_C = \omega R + v_C = 2\times\frac{1}{2} + 2 = 3\ \text{m/s}$$

3. As the disc is rolling points A and B which are on the disc and in contact with the planks will not slide. The velocities of points A and B should be the same as the planks. Let the disc rotate with an angular velocity ω and let the velocity of centre of the mass of the disc be v_c.



Velocity of point A, v_A, $v_C + \omega R = v$ ...(i)

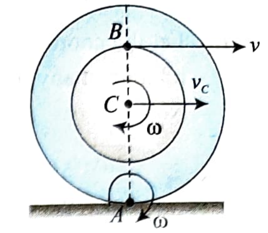
Velocity of point B, v_B, $\omega R - v_C = 2v$ ...(ii)

From Eqs. (i) and (ii), $\omega = 3v/2R$ and $v_C = -v/2$

Hence, velocity of centre of disc will be $v/2$ towards the left and the angular velocity will be $3v/2R$ the in the clockwise sense.

4. As the cotton reel is rolling, lowest point of the reel contact with ground should be ICR.

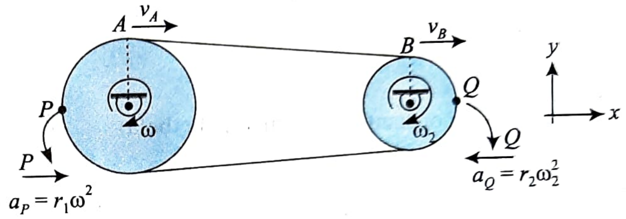
$$v_B = v_P = v = \omega(R + r)$$



$$\Rightarrow \omega = \frac{v}{(R+r)} \text{ (in clockwise sense)}$$

$$v_c = \omega R = \frac{vR}{(R+r)}$$

5. (i) As belt is not sliding $v_A = \omega r_1 = v_B$



(ii) $v_B = \omega_2 r_2 \Rightarrow \omega_2 = \dfrac{\omega r_1}{r_2}$

(iii) $\vec{a}_P = r_1\omega^2(\hat{i})$ and $\vec{a}_Q = r_2\omega_2^2(-\hat{i})$

$$\Rightarrow \vec{a}_Q = r_2\left(\frac{\omega r_1}{r_2}\right)^2(-\hat{i}) = -\frac{\omega^2 r_1^2}{r_2}(\hat{i})$$

Hence, acceleration of point P w.r.t. point Q

$$\vec{a}_{P,Q} = \vec{a}_P - \vec{a}_Q = r_1\omega^2\left(1 + \frac{r_1}{r_2}\right)\hat{i}$$

6. The acceleration of P is $\vec{a}_P = \vec{a}_{PC} + \vec{a}_{CO} + \vec{a}_O$ ...(i)

$a_O = 0$ ...(ii)

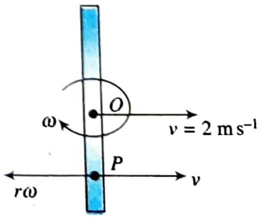
$$\vec{a}_P = \vec{a}_{PC} + \vec{a}_{CO}$$

$$\vec{a}_{PC} = r\omega_{PC}^2\,\hat{j} + 0 = \frac{v^2}{r}\hat{j} \quad \text{...(iii)}$$

$$a_{CO} = r_{CO}\omega_{CO}^2 = \frac{v^2}{R-r}\hat{j} \quad \text{...(iv)}$$

Using the above equations $a_P = v^2\left[\dfrac{1}{r} + \dfrac{1}{R-r}\right]\hat{j}$

7. Let P be a point passing through instantaneous axis of rotation. Therefore $v_P = 0$.



$$\Rightarrow |\vec{v}_{PO} + \vec{v}_O| = 0$$

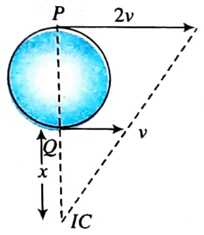
$$\Rightarrow v_{PO} - v_O = 0 \Rightarrow v_{PO} = v_O$$

where $v_O = v$ and $v_{PO} = (OP)\omega$

$\Rightarrow (OP)\ \omega = v$

$$\Rightarrow OP = v/\omega = \frac{2}{2.\pi} = \frac{1}{\pi}\ \text{m}$$

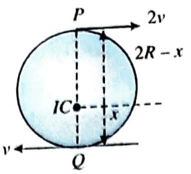
8. (a) $\dfrac{v_P}{v_Q} = \dfrac{2R + x}{x}$



$$\Rightarrow \frac{2v}{v} = \frac{2R + x}{x}$$

$$2x = 2R + x \quad \therefore x = 2R$$

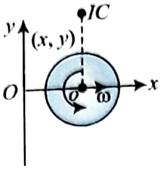
$$\therefore \quad \omega = \frac{v_Q}{x} = \frac{v}{2R}$$



(b) $\frac{v_P}{v_Q} = \frac{2R - x}{x}$

$$\frac{2v}{v} = \frac{2R - x}{x}$$

$\therefore\ x = 2R/3$ and $\omega = \frac{v_Q}{x} = \frac{3v}{2R}$



9. (a) Let the coordinates of instantaneous centre of rotation at any time t be (x, y).

Now, $v = \omega y$

$$\sqrt{2ax} = \omega y$$

$\therefore\ y^2 = \left(\frac{2a}{\omega^2}\right)x$ (parabola)

(b) $v = \omega y$

$$v = \frac{\alpha x y}{v}$$

$\therefore\ xy = \frac{v^2}{\alpha}$ = constant (rectangular hyperbola)

Exercise 3.3

1. MR^2 (infact MI of any part of mass M of a ring of radius R about an axis passing through the geometrical centre and perpendicular to the plane of the ring is MR^2).

2. $\frac{MR^2}{2}$. These cases are same as circular disc. The distributions with respect to axis of rotation is same pattern as circular disc.

3. Moment of inertia of the plate about axis 1 (by taking rods perpendicular to axis 1)

$$I_1 = \frac{Mb^2}{3}$$

Moment of inertia of the plate about axis 2 (by taking rods perpendicular to axis 2)

$$I_2 = \frac{Ml^2}{12}$$

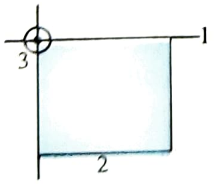
Moment of inertia of the plate about axis 3 (by taking rods perpendicular to axis 3)

$$I_3 = \frac{Mb^2}{12}$$

Moment of inertia of the plate about axis 4 (by taking rods perpendicular to axis 4)

$$I_4 = \frac{Ml^2}{3}$$

4. $I_3 = I_1 + I_2 = \frac{Ml^2}{3} + \frac{Ml^2}{3} = \frac{2Ml^2}{3}$



5. For calculation of the moment of inertia about centre, use parallel axis theorem

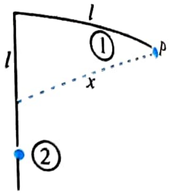
$$I = 2\left[\frac{Ml^2}{12} + m\left(\frac{b}{2}\right)^2\right] + 2\left[\frac{mb^2}{12} + m\left(\frac{l}{2}\right)^2\right]$$

$$I = \frac{2}{3}m(l^2 + b^2)$$

Hence, we can also use parallel axis theorem.

$$I_{PQ} = 2\left(\frac{mb}{3}\right)^2 + mb^2 = \frac{5}{3}mb^2$$

6. $I_P = I_{P_1} + I_{P_2}$



$$I_P = \frac{Ml^2}{3} + \left(\frac{Ml^2}{12} + Mx^2\right)$$

$$x^2 = l^2 + \left(\frac{l}{2}\right)^2 = \frac{5l^2}{4}$$

$$\Rightarrow\quad I_P = \frac{Ml^2}{3} + \left[\frac{Ml^2}{12} + M\left(\frac{5l^2}{4}\right)\right] = \frac{5}{3}Ml^2$$

7. $I = I_{\text{disc}} + I_{\text{ring}} + I_{\text{point masses}}$

$$= \frac{mR^2}{2} + mR^2 + 4\left[\frac{m}{4}\left(\frac{R}{2}\right)^2\right] = \frac{7mR^2}{4}$$

8. In the system shown in the figure, the total moment of inertia can be given as the sum of moment of inertia of the rod and that of the spherical ball.

Moment of inertia of the rod about the axis passing through an end is $I_{\text{rod}} = Ml^2/3$.

Moment of inertia of the spherical ball about the axis passing through its geometrical centre is $I = 2mr^2/5$.

Using parallel axis theorem, we get the moment of inertia about the axis passing through the end of rod as

$$I_{AA'} = \frac{2}{5}mr^2 + ml^2$$

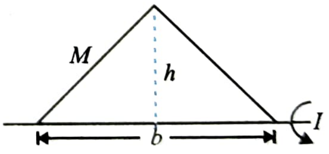
Thus total moment of inertia of the system can be given as

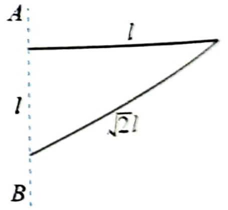
$$I_{\text{total}} = I_{\text{rod}} + I_{AA'}$$

$$= \frac{Ml^2}{3} + \left(\frac{2}{5}mr^2 + ml^2\right)$$

9. We know moment of inertia of the triangular lamina about one the side is

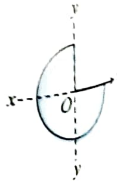
$$I = \frac{Mh^2}{6}$$





Here in the problem $I = \frac{Ml^2}{6}$

10. It is important to notice that the MI (Moment of inertia) of the remaining disc about the z-axis will be $I_{zz} = \frac{1}{2}MR^2$



But $I_{xx} = I_{yy}$,

$$\therefore\quad I_{xx} = \frac{1}{2}I_{zz} = \frac{1}{4}MR^2$$

11. $I_0 = I_{CM} + Md^2$

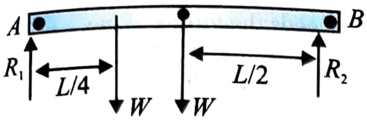
$M(x^2 + y^2) = I_0 - I_{CM}$ [= a constant]

$$\Rightarrow\ M(x^2 + y^2) = M\left(l^2 + \frac{l^2}{4}\right)$$

$$\Rightarrow\ x^2 + y^2 = \frac{5l^2}{4}$$

Exercise 3.4

1. $R_1 + R_2 = 2W$ ...(i)

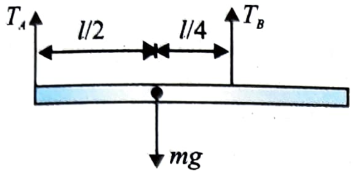


Balancing torque about A:

$$R_2L = W\frac{L}{4} + W\frac{L}{2}$$

$$R_2 = \frac{3W}{4} \text{ and } R_1 = \frac{5W}{4}$$

2.



$$T_A + T_B = mg \qquad \text{...(i)}$$

$$T_A\frac{l}{2} = T_B\frac{l}{4} \Rightarrow 2T_A = T_B \qquad \text{...(ii)}$$

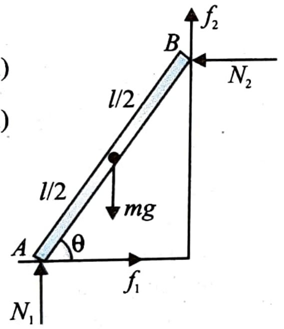
From Eqs. (i) and (ii),

$$T_A = \frac{mg}{3},\ T_B = \frac{2mg}{3}$$

3. $f_1 = \mu_1 N_1, f_2 = \mu_2 N_2$

$$N_2 = f_1 = \mu_1 N_1 \qquad \text{...(i)}$$

$$N_1 + f_2 = mg \qquad \text{...(ii)}$$



In limiting case $f_1 = \mu_1 N_1$ and $f_2 = \mu_2 N_2$

Substituting the values of f_1 and f_2 in Eqs. (i) and (ii), we get

$$N_2 = \frac{\mu_1 mg}{1 + \mu_1\mu_2}$$

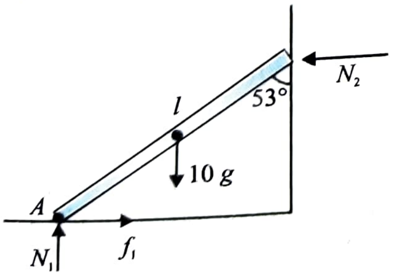
Taking torque about A and putting it to be zero,

$$mg\frac{l}{2}\cos\theta = f_2\, l\cos\theta + N_2\, l\sin\theta$$

$$mg\cos\theta = 2\mu_2 N_2\cos\theta + 2N_2\sin\theta$$

Put the value of N_2 and simplify to get $\tan\theta = \dfrac{1-\mu_1\mu_2}{2\mu_1}$

4. $N_1 = 10g = 100\text{N}$



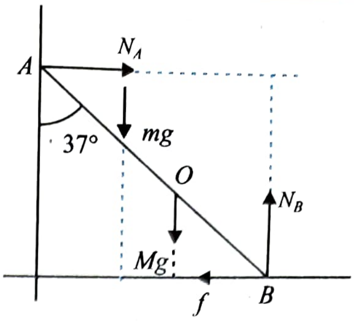
Balancing torque about A:

$$10g\frac{l}{2}\cos 37° = N_2\, l\cos 53°$$

$$N_2 = \frac{200}{3}\ \text{N}$$

Now $f_1 = N_2 = \dfrac{200}{3}$ N

5. If the electrician works safely, the ladder + man system should be in equilibrium



For equilibrium, $\Sigma F = 0$ and $\Sigma\,\tau = 0$

for $\quad \Sigma F_x = 0, N_A = f$

for $\quad \Sigma F_y = 0, N_B = mg + Mg = 60 \times 10 + 16 \times 10 = 760$ N

Taking torque about B,

$$N_A\, 10\cos 37° = mg\,8\sin 37° + Mg\,5\sin 37°$$

$$N_A = \frac{\tan 37°}{10}(65 \times 10 \times 8 + 16 \times 10 \times 5)$$

From (i), $f = N_A = 420$ N

For maximum value of coefficient of friction,

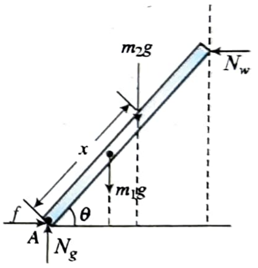
$$f \le \mu N_B$$

$$\mu \ge \frac{f}{N_B} = \frac{420}{760} = 0.55$$

6. (a) $\sum F_x = f - N_w = 0 \qquad \text{...(i)}$

$$\sum F_y = N_g - m_1g - m_2g = 0 \qquad \text{...(ii)}$$

$$\sum \tau_A = -m_1g\left(\frac{L}{2}\right)\cos\theta - m_2gx\cos\theta + N_wL\sin\theta = 0$$



From the torque equation, $N_w = \left[\frac{1}{2}m_1g + \left(\frac{x}{L}\right)m_2g\right]\cot\theta$

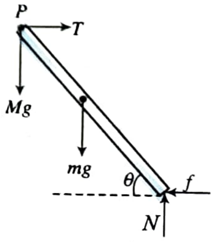
Then, from Eq. (i), $f = N_w = \left[\frac{1}{2}m_1g + \left(\frac{x}{L}\right)m_2g\right]\cot\theta$

and from Eq. (ii), $N_g = (m_1 + m_2)g$

(b) If the ladder is on the verge of slipping when $x = d$, then

$$\mu = \frac{f_{x=d}}{N_g} = \frac{\left(\frac{m_1}{2} + \frac{m_2d}{L}\right)\cot\theta}{m_1 + m_2}$$

7. (a) We can use $\Sigma F_x = \Sigma F_y = 0$ and $\Sigma\tau = 0$ with pivot point at the contact on the floor.



Then $\sum F_x = T - \mu_s N = 0$

$$\sum F_y = N - Mg - mg = 0$$

$$\sum \tau = Mg(L\cos\theta) + mg\left(\frac{L}{2}\cos\theta\right) - T(L\sin\theta) = 0$$

Solving the above equations gives

$$M = \frac{m}{2}\left(\frac{2\mu_s \sin\theta - \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)$$

This answer is the maximum value for M if $\mu_s < \cot\theta$. If $\mu_s \geq \cot\theta$, the mass M can increase without limit. It has no maximum value, and part (b) cannot be answered as stated either. In the case $\mu_s < \cot\theta$, we proceed.

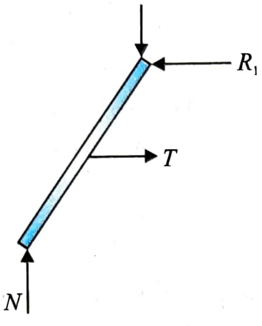
(b) At the floor, we have the normal force in the y-direction and frictional force in the x-direction. The reaction force then is

$$R = \sqrt{N^2 + (\mu_s N)^2} = (M+m)g\sqrt{1+\mu_s^2}$$

At point P, the force of the beam on the rope is

$$F = \sqrt{T^2 + (Mg)^2} = g\sqrt{M^2 + \mu_s^2 (M+m)^2}$$

8. The forces acting on different parts are shown in figure. Consider the vertical equilibrium of 'the ladder plus the person' system. The forces acting on this system are its weight (80 kg)g and the contact force $N + N = 2N$ due to the floor.



Thus, $2N = (80 \text{ kg})\, g$

or $N = (40 \text{ kg})\,(10 \text{ m/s}^2) = 400 \text{ N}$

Next consider the equilibrium of the left leg of the ladder. Taking torques of the forces acting on it about the upper end,

$$N(2 \text{ m}) \tan 30° = T(1 \text{ m})$$

or $T = N\dfrac{2}{\sqrt{3}} = (400 \text{ N}) \times \dfrac{2}{\sqrt{3}} = \dfrac{80}{\sqrt{3}}\text{N}$

9. When it is on the verge of slipping, the cylinder is in equilibrium.

$$\sum F_x = 0, \quad f_1 = N_2 = \mu_s N_1 \text{ and } f_2 = \mu_s N_2$$

$$\sum F_y = 0, \quad P + N_1 + f_2 = F_g$$

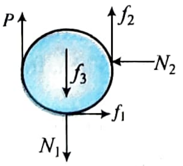
$$\sum \tau = 0, \quad P = f_1 + f_2$$

As P grows so do f_1 and f_2

Therefore, since $\mu_s = \dfrac{1}{2}$, $f_1 = \dfrac{N_1}{2}$ and $f_2 = \dfrac{N_2}{2} = \dfrac{N_1}{4}$

then $P + N_1 + \dfrac{N_1}{4} = F_g$ ...(i)

and $P = \dfrac{N_1}{2} + \dfrac{N_1}{4} = \dfrac{3}{4}N_1$ ...(ii)



so $P + \dfrac{5}{4}N_1 = F_g$

becomes $P + \dfrac{5}{4}\left(\dfrac{4}{3}P\right) = F_g$ or $\dfrac{8}{3}P = F_g$

Therefore $P = \dfrac{3}{8}F_g$

10. (a) If the acceleration is a, we have $P_x = ma$ and $P_y + N - F_g = 0$. Taking the origin at the centre of gravity, the torque equation gives

$$P_y(L-d) + P_x h - Nd = 0$$

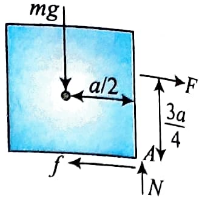
Solving these equations, we find $P_y = \dfrac{F_g}{L}\left(d - \dfrac{ah}{g}\right)$

(b) If $P_y = 0$, then

$$d = \frac{ah}{g} = \frac{(2.00 \text{ ms}^{-2})(1.50 \text{ m})}{10 \text{ ms}^{-2}} = 0.30 \text{ m}$$

Exercise 3.5

1. In the limiting case normal reaction will pass through A. The cube will tip about A if torque of F exceeds the torque of mg.

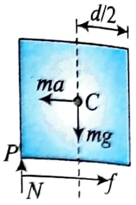


Hence, $F\left(\dfrac{3a}{4}\right) > \left(\dfrac{a}{2}\right)mg$ or $F > \dfrac{2}{3}mg$

Therefore, minimum value of F is $\dfrac{2}{3}mg$.

2. As there is no relative sliding between the blocks, the common acceleration of system $a = \dfrac{F}{2m} = \dfrac{kt}{2m}$

The force diagram of block B in reference frame attached to A is shown in figure. The normal force passes through left edge at the instant the block is about to topple.

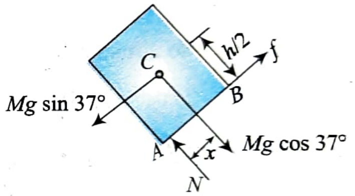


The block 'A' will be on verge of toppling about point P torque of pseudo force equals torque due to its weight.

$$ma\frac{h}{2} = mg\frac{d}{2}$$

$$m\frac{kt}{2m}\frac{h}{2} = mg\frac{d}{2} \qquad \therefore\ t = \frac{2dgm}{kh}$$

3. Here $\mu = 0.8$, $\dfrac{b}{h} = \dfrac{1}{3} = 0.33$; $\tan\theta = \dfrac{3}{4} = 0.75$



Since $\mu > b/h$ and $\mu > \tan\theta$, therefore the block has a greater tendency to topple over.

From the condition of translation equilibrium, we have

$$N = Mg\cos 37° = (50)(10)\left(\frac{4}{5}\right) \text{ or } N = 400 \text{ N} < f_{\max}$$

$$f_{\max} = \mu N = 0.8\,(400) = 320 \text{ N}$$

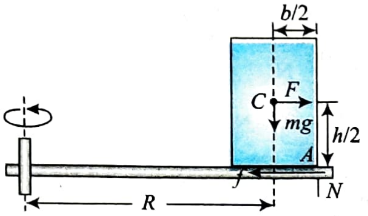
$$f = Mg\sin 37° = (50)(10)\left(\frac{3}{5}\right) = 300 \text{ N}$$

For rotational equilibrium, $\sum \tau_c = 0 \Rightarrow Nx = f\dfrac{h}{2}$

Thus, $x = \dfrac{f}{N}\dfrac{h}{2} = \left(\dfrac{300}{400}\right)\left(\dfrac{3}{2}\right) = \dfrac{9}{8} > \dfrac{b}{2}$

Since the value of x is more than $b/2$ which is not possible, therefore the block topples over.

4. An observer on the disc, will observe a pseudo force F acting on the cylinder (the centrifugal force). The block can fall off either by slipping away or toppling about point P, depending on whichever takes place first. In any case friction force f acts opposite to F. The critical angular speed ω_1, slipping occurs when $F = f$



or $m\omega_1^2 R = \mu mg \Rightarrow \omega_1 = \sqrt{\dfrac{\mu g}{R}}$

F tries to rotate the block about P, but the weight W opposes it. The rotation is possible if,

$$\tau_F \geq \tau_W$$

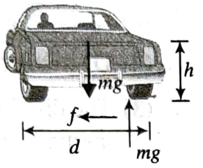
$$F\frac{h}{2} = mg\frac{b}{2} \Rightarrow F = mg\frac{b}{h}$$

$$m\omega_2^2 R = mg\frac{b}{h} \Rightarrow \omega_2 = \sqrt{\frac{bg}{hR}}$$

$\because \mu > \dfrac{b}{h}$ (given)

$\therefore \omega_1 > \omega_2$ and the block falls off by rolling over at $\omega_1 = \omega_2$.

5. When the car is on the point of rolling over, the normal force on its inside wheels is zero.



$$\sum F_y = ma_y: \quad N - mg = 0$$

$$\sum F_x = ma_x: \quad f = \frac{mv^2}{R}$$

Take torque about the centre of mass:

$$fh - N\frac{d}{2} = 0$$

Then by substitution,

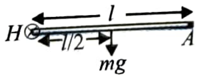
$$\frac{mv_{\max}^2}{R}h - \frac{mgd}{2} = 0 \Rightarrow v_{\max} = \sqrt{\frac{gdR}{2h}}$$

A wider wheelbase (larger d) and a lower centre of mass (smaller h) will reduce the risk of rollover.

Exercise 3.6

1. (a) $\tau_H = I_H\alpha$

$$mg\frac{l}{2} = \frac{ml^2}{3}\alpha \Rightarrow \alpha = \frac{3g}{2l}$$

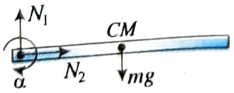


(b) $a_{tA} = \alpha l = \dfrac{3g}{2l}l = \dfrac{3g}{2}$

$a_{CA} = \omega^2 r = 0 \times l = 0$

($\omega = 0$ just after release)

2. Free-body diagram of the rod
In the previous problem



$$a = \frac{3g}{2l}$$

Hence, $a_{\text{CM}} = a_t = \dfrac{\alpha l}{2} = \dfrac{3g}{4}$

From the free-body diagram in the vertical direction

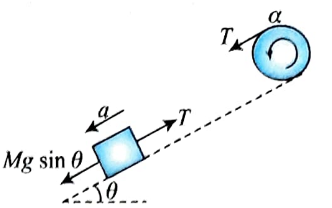
$$mg - N_1 = ma_{\text{CM}} = m\left(\frac{3g}{4}\right)$$

$$\Rightarrow N_1 = \frac{mg}{4}$$

In horizontal direction, $F_{ext} = ma_{\text{CM}} \Rightarrow N_2 = 0$

($\because a_{\text{CM}}$ in horizontal $= 0$ as $\omega = 0$ just after release).

3. Suppose the deceleration of the block is a. The linear deceleration of the rim of the wheel is also a. The angular deceleration of the wheel is $\alpha = a/r$. If the tension in the string is T, the equations of motion are as follows:



$$Mg\sin\theta - T = Ma \text{ and } Tr = I\alpha = \frac{Ia}{r}.$$

Eliminating T from these equations,

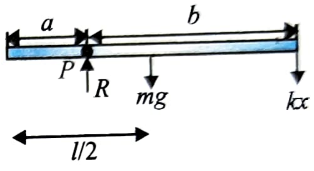
$$Mg\sin\theta - I\frac{a}{r^2} = Ma$$

$$\Rightarrow \quad a = \frac{Mgr^2\sin\theta}{(I + Mr^2)}$$

The initial velocity of the block up the incline is $v = \omega r$. Thus, the distance moved by the block before stopping is

$$x = \frac{v^2}{2a} = \frac{\omega^2 r^2(I + Mr^2)}{2Mgr^2\sin\theta} = \frac{(I + Mr^2)\,\omega^2}{2Mg\sin\theta}$$

4. Just after burning of a thread, torque about point P.



$$\tau_p = mg\left(\frac{l}{2} - a\right) + kxb$$

$$I_p\alpha = mg\left(\frac{l}{2} - a\right) + kxb$$

where, $I_p = \dfrac{ml^2}{12} + m\left(\dfrac{l}{2} - a\right)^2 = \dfrac{56}{300}$ kg m^2

$$\therefore \quad \frac{56}{300}\alpha = \frac{28}{5} \Rightarrow \alpha = 30 \text{ rad/s}$$

Equation of motion at this instant

$$mg + kx - R = ma_0 = m\alpha\left(\frac{l}{2} - a\right)$$

where a_0 is the acceleration.

$$20 + 6 - R = 2 \times 30 \times \frac{1}{10}$$

$$\Rightarrow \quad R = 20 \text{ N}$$

5. The acceleration whole system, $a_1 = \dfrac{F}{m_1 + m_2}$

The acceleration of the point K w.r.t. the axis of the cylinder

$$a_2 = \alpha R$$

where α is given by

$$FR = I\alpha \quad \text{or} \quad \alpha = \frac{FR}{m_1R^2/2} = \frac{2F}{m_1R}$$

$$\Rightarrow \quad a_2 = \frac{2F}{m_1}$$

$\therefore$ The acceleration of the point K w.r.t. ground

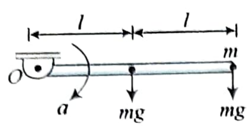
$$= a_1 + a_2 = \frac{F}{m_1 + m_2} + \frac{2F}{m_1} = F\frac{(3m_1 + 2m_2)}{m_1(m_1 + m_2)}$$

6. (i) $F_1 - F_2 = (m + M)a \Rightarrow a = \dfrac{F_1 - F_2}{m + M}$

(ii) $F_1 r = \dfrac{mr^2}{2}\alpha \Rightarrow \alpha = \dfrac{2F_1}{mr}$

7. (i) Torque equation: The internal forces and the reaction forces at O. Then the net torque about O is

$$\tau_0 = mgl + mg(2l) = 3mgl$$



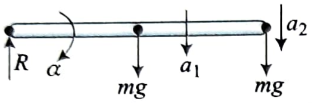
which produces an angular acceleration α. Using Newton's 2nd law of rotation $\tau = I\alpha$.

We have $3mgl = I_0\alpha$ where $I_0 = ml^2 + m(2l)^2 = 5ml^2$

This gives $\alpha = \dfrac{3g}{5l}$

(ii) Force equation: Referring FBD, we have

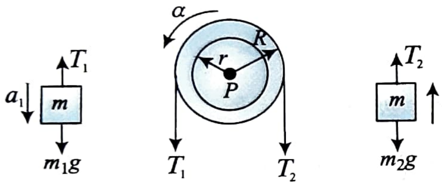
$$F_y = R - mg - mg = -ma_1 - ma_2$$



Substituting $a_1 = l\alpha$ and $a_2 = 2l\alpha$, we have $R = m(2g - 3l\alpha)$

Now substituting $\alpha = \dfrac{3g}{5l}$, we have $R = \dfrac{mg}{5}$

8. Torque equation: $T_1R - T_2r = I_C\alpha$ ...(i)



Writing force equation for block '1'

$$mg - T_1 = ma_1 = m(R\alpha) \Rightarrow T_1 = mg - m(R\alpha) \qquad \text{...(ii)}$$

Writing force equation for block '2'

$$T_2 - mg = ma_2 \Rightarrow T_2 = mg + m(r\alpha) \qquad \text{...(iii)}$$

From (i), (ii) and (iii)

$$(mg - mR\alpha)R - (mg + mr\alpha)r = I_c\alpha$$

$$mg(R - r) = [I_c + m(R^2 + r^2)]\alpha$$

$$\alpha = \frac{mg(R - r)}{I_c + m(R^2 + r^2)}$$

But $I_C = mk^2 \Rightarrow \alpha = \dfrac{mg(R - r)}{mk^2 + m(R^2 + r^2)} = \dfrac{g(R - r)}{k^2 + (R^2 + r^2)}$

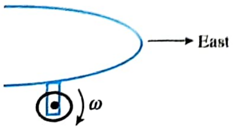
Hence $a_1 = \alpha R = \dfrac{gR(R - r)}{k^2 + (R^2 + r^2)}$

and $a_2 = \alpha r = \dfrac{gr(R - r)}{k^2 + (R^2 + r^2)}$

Exercises

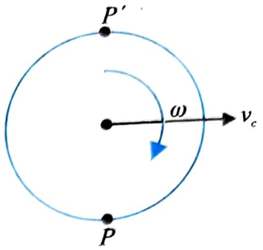
Single Correct Answer Type

1. (4) The wheel should be rotated as shown. It is clockwise, so the direction of angular acceleration is inward or in north direction.



2. (2) Farther the mass from axis, greater will be the moment of inertia.

3. (4) The instantaneous axis of rotation will fall on the outside of the sphere. Since in case of pure rotation, the instantaneous axis of rotation will be on the centre. In case of pure rolling, the instantaneous axis of rotation will be on P.



But if $v_C > \omega R$, then the instantaneous axis of rotation will fall out of the sphere.

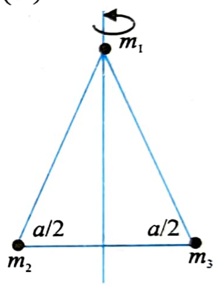
4. (3) $I_B = I_A + Md^2$

5. (4) Distribution of mass is farthest.

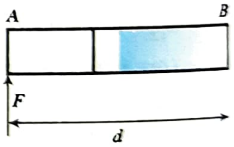
6. (3) The distribution of mass is nearest to axis xx, hence moment of inertia is least about the xx-axis.

7. (2) MOI is $\Sigma m_i r_i^2$. About BC masses are spread far away than about any other axis.

8. (2) $I = m_2\left(\dfrac{a}{2}\right)^2 + m_3\left(\dfrac{a}{2}\right)^2$



9. (2) $\tau = I\alpha$, $\alpha = \tau/I$
   (1) $\tau = Fd$, I large
   α minimum
   (2) $\tau = Fd$, I minimum
   α maximum
   (3) $\tau = \dfrac{Fd}{2}$
   (4) $\tau = \dfrac{Fd}{2}$



10. (3) Because the planes of two rings are mutually perpendicular and centres are coincident, hence an axis, which is passing through the centre of one of the rings and to its plane, will be along the diameter of other ring. Hence, moment of inertia of the system

$$= I_{CM} + I_{diameter} = mr^2 + \frac{mr^2}{2} = \frac{3}{2}mr^2.$$

11. (2) According to the theorem of parallel axis,

$$I = I_{CG} + M(2R)^2$$

where I_{CG} = MI about an axis through the centre of gravity.

$$I = \frac{2}{5}MR^2 + 4MR^2 = \frac{22}{5}MR^2$$

or $MK^2 = \frac{22}{5}MR^2 \Rightarrow K = \sqrt{\frac{22}{5}}\,R$

12. (2) Given that $I_Q = 4I_P$

Here I = mass × (radius)². If σ is the linear mass density (mass/length) of the wire, then

$$\sigma \times 2\pi(nr) \times n^2r^2 = 4(\sigma \times 2\pi r \times r^2)$$

$$\Rightarrow n^3 = 4 \quad \Rightarrow \quad n = (4)^{\frac{1}{3}}$$

13. (3) Let M be the mass of each disc. Let R_A and R_B be the radii of discs A and B, respectively. Then $M = \pi R_A^2 t d_A = \pi R_B^2 t d_B$. As $d_A > d_B$, so $R_A^2 < R_B^2$. Now,

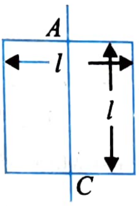
$$I_A = \frac{1}{2}MR_A^2, \quad I_B = \frac{1}{2}MR_B^2$$

$$\frac{I_A}{I_B} = \frac{R_A^2}{R_B^2} < 1, \text{ i.e., } I_A < I_B$$

14. (4) $I_{\text{median line}} = I_A + I_B + I_C + I_D$

$$= 2 \times \frac{Ml^2}{12} + 2M\left(\frac{l}{2}\right)^2$$

$$= \frac{Ml^2}{6} + \frac{Ml^2}{2} = \frac{2}{3}Ml^2$$



15. (1) Mass of each of the four parts $= \frac{M}{3}$

Mass of the plate including the cut piece $= \frac{4M}{3}$

Moment of inertia of the whole plate (including the cut piece) about the said axis $= \left(\frac{4M}{3}\right)\frac{l^2}{6}$

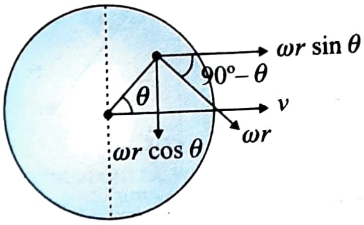
Now moment of inertia of the remaining portion should be 3/4 of the above $= Ml^2/6$.

16. (2) Here, $\omega = \frac{v}{R}$

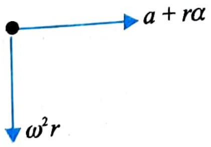
$$V_{P_x} = \left(v + \frac{v}{R}r\sin\theta\right)\hat{i}$$

$$V_{P_y} = -\left(\frac{v}{R}r\cos\theta\right)\hat{j}$$

$$\therefore \quad \vec{V}_P = \left(v + \frac{vr\sin\theta}{R}\right)\hat{i} - \frac{vr\cos\theta}{R}\hat{j}$$



17. (1) FBD of point P



In the earth frame, we have

So, resultant $= \sqrt{(a + r\alpha)^2 + (\omega^2 r)^2}$

18. (4) Moment of inertia of discs A and B about the axis through their centre of mass and perpendicular to the plane will be

$$I_{AA} = I_{BB} = \frac{1}{2}Mr^2$$

Now, moment of inertia of disc A about an axis through B, by theorem of parallel axis will be

$$I_{AA} = I_{BB} + M(2r)^2 = \frac{1}{2}Mr^2 + 4Mr^2 = \frac{9}{2}Mr^2$$

So $I = I_{BB} + I_{AB} = \frac{1}{2}Mr^2 + \frac{9}{2}Mr^2 = 5Mr^2$

19. (4) $\tau = (2mx^2)\alpha_1$

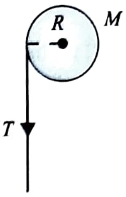
$$\tau = (2 \times m \times 4x^2)\alpha_2$$

Equating τ, $8mx^2\alpha_2 = 2mx^2\alpha_1$ or $\alpha_2 = \frac{\alpha_1}{4}$

20. (2) Mass of incomplete ring $= m - \frac{M}{2\pi} \times \frac{\pi}{6} = M - \frac{M}{12} = \frac{11M}{12}$

Moment of inertia of incomplete ring $= \left(\frac{11M}{12}\right)R^2 = \frac{11}{12}MR^2$

21. (3) Torque exerted on the disc



$$\tau = TR$$

Now $\tau = I\alpha$

$$\alpha = \frac{\tau}{I} = \frac{TR}{\frac{1}{2}MR^2}$$

$$= \frac{2TR}{MR^2} = \frac{2T}{MR}$$

22. (3) Tangential acceleration, $a = r\alpha = R\left(\frac{2T}{MR}\right) = \frac{2T}{M}$

23. (3) From the above problem, $T = \frac{Ma}{2}$.

Now, $mg - T = ma$

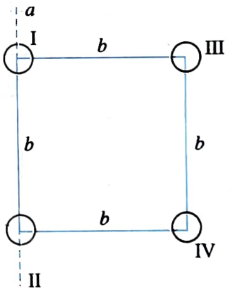
or $mg - M\frac{a}{2} = ma$

or $mg = \left(m + \frac{M}{2}\right)a$

$$a = \frac{2mg}{M + 2m}$$

24. (2) $T = \frac{Ma}{2} = \frac{Mmg}{M + 2m}$

25. (2) Combined moment of inertia of I and II $= 2 \times \frac{2}{5}mr^2 = \frac{4}{5}mr^2$



Combined moment of inertia of III and IV

$$= 2\left[\frac{2}{5}mr^2 + mb^2\right] = \frac{4}{5}mr^2 + 2mb^2$$

Combined moment of inertia of the system

$$= \frac{4}{5}mr^2 + \frac{4}{5}mr^2 + 2mb^2 = \frac{8}{5}mr^2 + 2mb^2$$

26. (3) The moment of inertia of the uniform rod about an axis through one end and perpendicular to length is

$$I = \frac{ML^2}{3}$$

Torque ($\tau = I\alpha$) acting on the centre of gravity of the rod is given by

$$\tau = Mg\left[\frac{L}{2}\sin\theta\right]$$

or $\frac{ML^2}{3}\alpha = Mg\frac{L}{2}\sin\theta$

$$\alpha = \frac{3g}{2L}\sin\theta$$

27. (4) Taking torque about the attachment point for W, we get

$$-T_1(0.4L) + T_2\,(0.3L) + 500\,(0.2L) = 0$$

$$T = 1000 \text{ N, where } T_1 = T_2 = T$$

$$\Sigma F_y = 0 \Rightarrow 2T - W - 500 = 0$$

$$\Rightarrow \quad W = 1500 \text{ N}$$

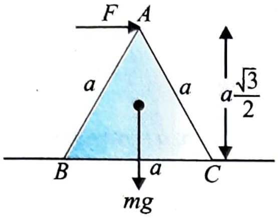
28. (3) Net torque about C is zero.

$$Fa + 100\,d = 0$$

$$F = -100\frac{d}{a} = -150$$

$$\vec{F} = -150\,\hat{i}$$

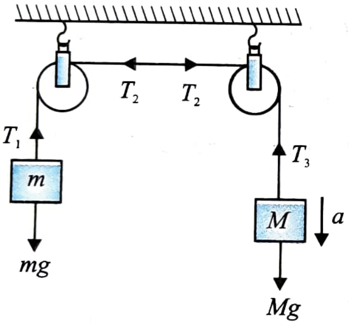
29. (1) The tendency of rotating will be about point C. For the minimum force, the torque of F about C has to be equal to the torque of mg about C.



$$F\left(a\frac{\sqrt{3}}{2}\right) = mg\left(\frac{a}{2}\right) \quad \Rightarrow \quad F = \frac{mg}{\sqrt{3}}$$

30. (1) $T_1 - mg = ma$ ...(i)

$r(T_1 - T_2) = I\alpha$ ...(ii)



$Mg - T_3 = Ma$ ...(iii)

$r(T_3 - T_2) = I\alpha$ ...(iv)

and $a = R\alpha$

From Eqs. (ii) and (iv), we get

$$T_3 - T_1 = \frac{2Ia}{R^2}$$

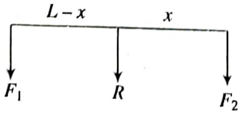
From Eqs. (i) and (iii), we get

$$(M - m)g = (M + m)a + T_3 - T_1$$

$$(M - m)g = (M - m)a + \frac{2Ia}{r^2}$$

$$\Rightarrow \quad a = \frac{(M - m)g}{\left(M + m + \frac{2I}{r^2}\right)}$$

31. (4) For $\tau = 0$



$$F_1(L - x) = F_2 x$$

$$F_2\left(\frac{3L}{4} - x\right) = F_1\left(x + \frac{L}{4}\right)$$

$$\Rightarrow \quad \frac{L - x}{x + (L/4)} = \frac{x}{(3L/4) - x}$$

$$\frac{3L}{4} = x + \frac{3x}{4} + \frac{x}{4} = 2x \Rightarrow 3L = 8x$$

So putting the value of x

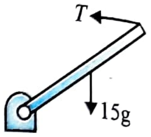
$$F_1\frac{5L}{8} = F_2\frac{3L}{8} \Rightarrow F_1 : F_2 = 3:5$$

32. (3) The force should pass through the centre of mass of the assembly.

33. (4) Given $\alpha_A = 2\alpha = 5 \text{ m/s}^2 \Rightarrow \alpha = \frac{5}{2} \text{ m/s}^2$

Hence, acceleration of B, $a_B = 1(\alpha) = \frac{5}{2} \text{ m/s}^2$

34. (3) The free-body diagram of the rod is shown in the figure. (Force exerted by the hinge on the rod are not shown.)



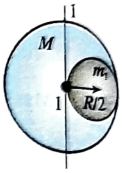
Take torque about the hinge,

$$T \times 3 - 15\,g \times 2 = 0$$

$$\Rightarrow \quad T = 100 \text{ N}$$

35. (3) Perpendicular distance of F_3 is greatest from O, hence torque of F_3 is greatest.

36. (3) Mass of cut disc: $m_1 = M/4$



Moment of Inertia of original disc about axis 1:

$$I = \frac{1}{4}MR^2$$

Moment of Inertia of small disc about axis 1:

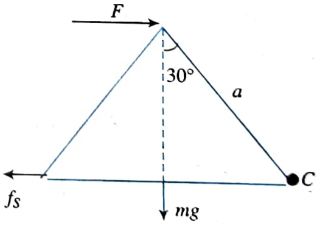
$$I' = \frac{1}{4}m_1\left(\frac{R}{2}\right)^2 + m_1\left(\frac{R}{2}\right)^2$$

$$= \frac{5}{16}m_1R^2 = \frac{5}{64}MR^2$$

Required moment of inertia:

$$I - I' = \frac{1}{4}MR^2 - \frac{5}{64}MR^2 = \frac{11}{64}MR^2$$

37. (3) About C, for toppling $\tau_F > \tau_{mg}$



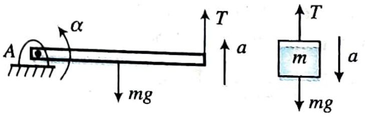
$$Fa\cos 30° > mg\frac{a}{2} \Rightarrow F > \frac{mg}{\sqrt{3}}$$

If no translation, then $f_s = F > \dfrac{mg}{\sqrt{3}}$

but $f_s \le \mu\, mg$

$$\Rightarrow \quad \frac{mg}{\sqrt{3}} \le \mu\, mg \Rightarrow \mu \ge \frac{1}{\sqrt{3}}$$

38. (3) $\alpha = \dfrac{a}{l}$ ...(i)



$mg - T = ma$ ...(ii)

$$Tl - mg\frac{l}{2} = \frac{ml^2}{3}\alpha \qquad \text{...(iii)}$$

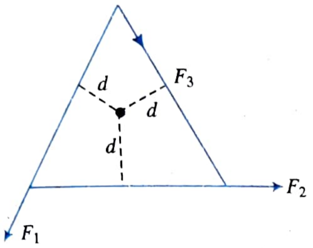
Solving the above equations: $a = \frac{3g}{8}$

39. (3) Clearly, the block shall topple about its edge through O. The torque FL of the applied force is clockwise. The torque $MgL/2$ of the weight is anticlockwise.

Applying condition for rotational equilibrium.

$$-FL + mg\,L/2 = 0 \text{ or } F = \frac{mg}{2}$$

40. (4) The torques F_1d and F_2d of F_1 and F_2 respectively are counterclockwise. The torque F_3d is clockwise. Applying condition for rotational equilibrium,



$$F_1d + F_2d - F_3d = 0 \text{ or } F_3 = F_1 + F_2$$

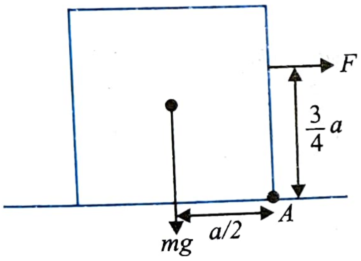
41. (3) Balancing the torque:

for the first case: $16l_1 = ml_2$

for the second case: $ml_1 = 4l_2$

Divide them to get $m = 8$ kg

42. (2) See the figure. For tilting about A the clockwise torque (due to F) should be greater than the anticlockwise torque about A.



$$mg\times\frac{a}{2} = \frac{F3a}{4};\ F = \frac{2mg}{3}$$

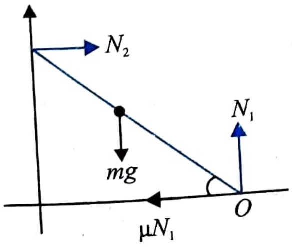
43. (4) $mg = N_1$

$\mu N_1 = N_2$ or $N_2 = \mu mg$

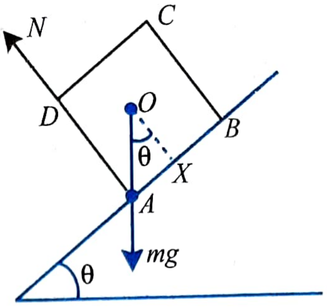
Taking moment about O, we get

$$\mu mgl\sin\theta = mg\frac{l}{2}\cos\theta$$

$$\tan\theta = \frac{1}{2\mu} \text{ or } \theta = \tan^{-1}\left(\frac{1}{2\mu}\right)$$


N₂
N₁
mg
O
μN₁

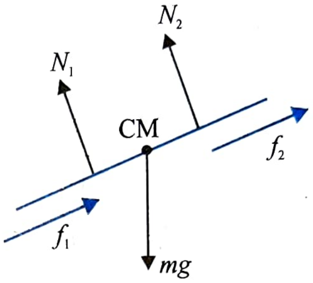
44. (3) The cube will not topple until the line of action of CM passes through the cube.


N
C
O
D
B
θ
X
A
mg
θ

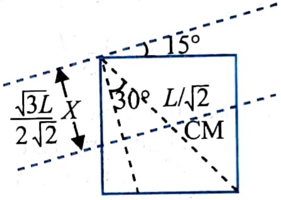
That is, $\tan\theta = \frac{AX}{OX} = \frac{a/2}{a/2}$ or $\theta = 45°$

45. (3) As rod is in equilibrium. The torque of all the forces about the CM is zero.

$$N_1\frac{l}{4} = N_2\frac{l}{4} \Rightarrow N_1 = N_2$$


N₂
N₁
CM
f₂
f₁
mg

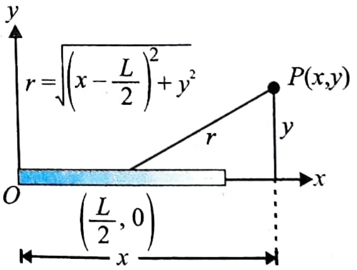
46. (2) From parallel axis theorem,


15°
$\frac{\sqrt{3}L}{2\sqrt{2}}$
X
30°
$L/\sqrt{2}$
CM

$$I_1 = \frac{ML^2}{12},\ I_2 = I_1 + Mx^2 = \frac{ML^2}{12} + M\left(\frac{\sqrt{3}L}{2\sqrt{2}}\right)^2 = \frac{11ML^2}{24}$$

47. (2) Let us take any point P on x–y plane. The MI of the rod about O is

$$I_P = I_{\text{CM}} + Mr^2$$

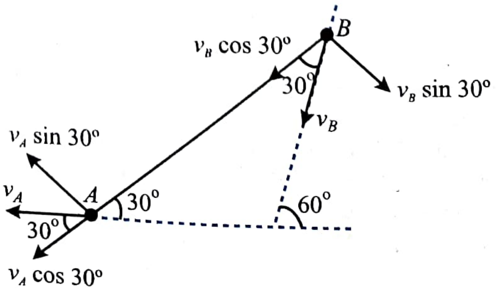

y
$r = \sqrt{\left(x - \frac{L}{2}\right)^2 + y^2}$
P(x,y)
r
y
x
O
$\left(\frac{L}{2}, 0\right)$
x

Hence, $I_P = \frac{ML^2}{12} + M\left[\left(x - \frac{L}{2}\right)^2 + y^2\right]$

MI of the rod about O, $I_0 = \frac{ML^2}{3}$

Applying $I_P = I_0$, we get an equation of a circle.

48. (2)


B
v_B cos 30°
30°
v_B sin 30°
v_B
v_A sin 30°
v_A
A
30°
60°
30°
v_A cos 30°

The velocities of the ends A and B along the length of rod should be the same.

Hence, $v_A \cos 30° = v_B \cos 30° \Rightarrow v_A = v_B = v$

Hence, the angular velocity of the rod is

$$\omega = \frac{(v_{AB})_\perp}{L} = \frac{2v \sin 30°}{L} \Rightarrow \omega = \frac{v}{L}$$

49. (1) $(\mathrm{MI})_{\mathrm{CG}} = M\left(\frac{a^2+b^2}{12}\right)$

According to the theorem of parallel axis,

$$(\mathrm{MI})_{\text{required axis}} = (\mathrm{MI})_{\mathrm{CG}} + M(OA)^2$$

$$= M\left(\frac{a^2+b^2}{12}\right) + M\left(\frac{a^2+b^2}{4}\right) = M\left(\frac{a^2+b^2}{3}\right)$$

50. (2) Total MI $= I_1 + I_2 + I_3 + I_4$

$$= 2I_1 + 2I_2 \qquad [I_1 = I_3;\ I_1 = I_4]$$

$$= 2(I_1 + I_2)$$

Now, $I_2 = I_3 = \frac{Ml^2}{3}$

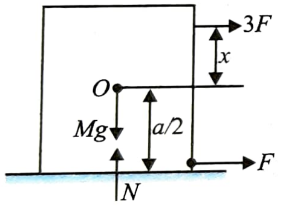
Using parallel axis theorem, we have

$$I = I_{\mathrm{CM}} + Mx^2 \text{ and } x = \sqrt{l^2 + \left(\frac{l}{2}\right)^2}$$

$$I_1 = I_4 = \frac{Ml^2}{12} + M\left[\sqrt{l^2 + \left(\frac{l}{2}\right)^2}\right]^2$$

Putting all values, we get $I = \frac{10\,Ml^2}{3}$

51. (2) The free-body diagram of the block can be drawn as shown. As the body has to move in pure translational motion, the torque about the centre of gravity must be zero.



$$3F \times x = F \times \frac{a}{2} \Rightarrow x = \frac{a}{6}$$

52. (2) $I_1 = \frac{ml^2}{12}$

$$I_2 = m\left[\frac{l}{2\pi}\right]^2 = \frac{ml^2}{4\pi^2}$$

$$\frac{I_1}{I_2} = \frac{ml^2}{12} \times \frac{4\pi^2}{ml^2} = \frac{\pi^2}{3}$$

53. (2) The two forces along the y-direction balance each other.

Hence, the resultant force is $2F$ along the x-direction.

Let the point of application of the force be at $(0, y)$.

(By symmetry, x-coordinate will be zero).

For rotational equilibrium:

$$F(a) + F(a) + F(a+y) - F(a-y) = 0$$

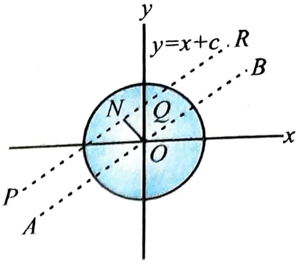
$$\Rightarrow y = -a$$

Alternative method: Torque will only be produced by the two forces acting along y-direction in anticlockwise direction. To balance this torque, we should apply a force $2F$ in order to produce a torque in the clockwise direction, which is only possible if we apply a force at a point below the x-axis.

Then $\tau = F(a) + F(a) - 2F \times y = 0$

$$\Rightarrow y = a$$

54. (2)

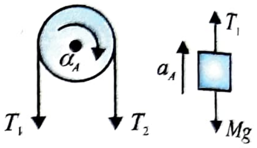


$$I_{PQR} = I_{AOB} + M(ON)^2 = \frac{1}{4}MR^2 + M\left(\frac{c}{\sqrt{2}}\right)^2$$

But $I_z = \frac{1}{2}MR^2$

$$\therefore \quad c = \pm\frac{R}{\sqrt{2}}$$

55. (2) In figure, $(T_2 - T_1)R = \frac{MR^2}{2}\alpha_A$



$$T_1 - Mg = Ma_A$$

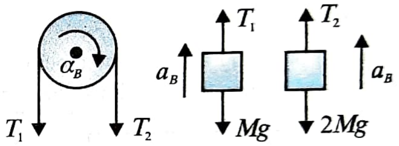
$$T_2 = 2Mg$$

$$a_A = R\alpha_A$$

$$\alpha_A = \frac{2g}{3R}$$

From figure,

$$(T_2 - T_1) \times R = \frac{MR^2}{2}\alpha_B$$



$$T_1 - Mg = Ma_B$$

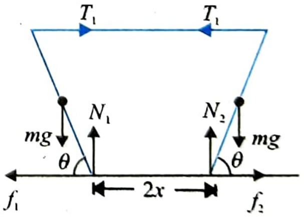
$$2Mg - T_2 = 2Ma_B$$

$$a_B = R\alpha_B$$

$$\alpha_B = \frac{2g}{7R}$$

So, $\alpha_A > \alpha_B$

56. (2) For the equilibrium of the entire structure, $N_1 + N_2 = 2Mg$



$$Mg\left(\frac{l}{2}\cos\theta\right) + N_2 \times 2x - Mg\left(\frac{l}{2}\cos\theta + 2x\right) = 0$$

Given $N_2 = Mg$, $N_1 = Mg$

For individual boards, $T_1 = f_1$, $N_1 = Mg$

and $T_1 \times l \sin\theta = Mg\frac{l}{2}\cos\theta$

$$f_1 = T_1 = \frac{Mg}{2\tan\theta}$$

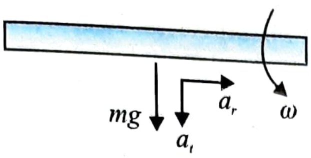
For safe equilibrium, $f_1 < f_L = \mu_s Mg$

$$\frac{Mg}{2\tan\theta} = \mu_s Mg \Rightarrow \tan\theta > \frac{1}{2\mu_s} \Rightarrow \theta = 45°$$

So the minimum value of θ for this type of arrangement is 45°.

57. **(3)** The angular velocity of the rod about the pivot when it passes through the horizontal position is given by

$$mg \times \frac{L}{2}\sin 30^\circ = \frac{mL^2}{3} \times \frac{\omega^2}{2}$$



$$\omega = \sqrt{\frac{3g}{2L}}$$

Radial acceleration of the centre of mass (as centre of mass is moving in a circle of radius $L/2$) is given by

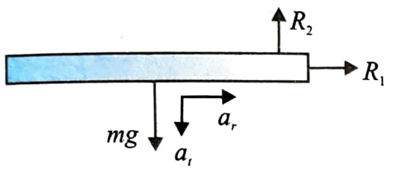
$$a_r = \omega^2\frac{L}{2} = \frac{3g}{4}$$

Torque about pivot, in the horizontal position, is $\tau = mg\frac{L}{2} = I\alpha$

$$\alpha = \frac{mgL/2}{mL^2/3} = \frac{3g}{2L}$$

Tangential acceleration of the centre of mass, $a_t = \frac{L}{2}\alpha = \frac{3g}{4}$

Draw the FBD of the rod at an instant when it passes through the horizontal position. Use Newton's second law of equation.



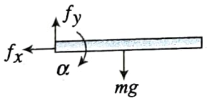
$$R_1 = ma_r = \frac{3mg}{4}$$

$$mg - R_2 = m \times a_t = \frac{3mg}{4}$$

$$R_2 = \frac{mg}{4}$$

So, reaction force by the pivot on the rod, $R = \sqrt{R_1^2 + R_2^2}$ $= \sqrt{10}\,mg/4$ at an angle of $\tan^{-1}(R_2/R_1)$ $[= \tan^{-1}(1/3)]$ with the horizontal.

58. **(2)** $mg\frac{L}{2} = \frac{1}{3}mL^2\alpha$



$$\alpha = \frac{3g}{2L}$$

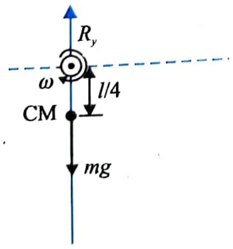
and $\alpha x = g \Rightarrow \frac{3g}{2L}x = g$

$$x = \frac{2L}{3}$$

distance from B: $L - x = \frac{L}{3}$

59. **(3)** KE of the rod is maximum when it is in the vertical position. From conservation of energy, we get

$$\frac{mgl}{4} = \frac{1}{2}\left[\frac{ml^2}{12} + \frac{ml^2}{16}\right]\omega^2$$



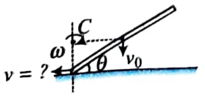
$$\frac{\omega^2 l}{4} = \frac{6g}{7}$$

and $R_y - mg = m(l/4)\,\omega^2$

$$R_y = mg + m\left(\frac{6g}{7}\right)$$

$$= \frac{13mg}{7}$$

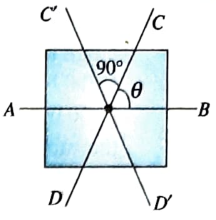
60. **(4)** C is instantaneous centre of rotation



$$\omega\frac{L}{2}\cos\theta = v_0$$

$$\omega\frac{L}{2}\sin\theta = v \Rightarrow v = v_0\tan\theta$$

61. **(1)** As shown in the figure



$$I_{CD} = I_{CD'}$$

So, $2I_{CD} = I \Rightarrow I_{CD} = \frac{I}{2}$

Also, $2I_{AB} = I \Rightarrow I_{AB} = \frac{I}{2}$

So $I_{CD} = I_{AB}$

62. **(2)** Moment of inertia of semicircular portions about x and y axes are same. But moment of inertia of straight portions about x-axis is zero.

$\therefore\quad I_x < I_y$ or $\frac{I_x}{I_y} < 1$

63. **(1)** $m_2g\cdot1 = m_1g\cdot3 \Rightarrow m_2 = 3m_1$

$$4m_1g\cdot3 = m_3g \Rightarrow m_3 = 12m_1$$

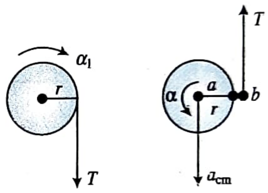
$$16m_1g\cdot3 = m_4g \Rightarrow m_1 = \frac{m_4}{48} = \frac{48}{48} = 1\text{ kg}$$

64. **(2)** $T_r = \frac{mr^2}{2}\alpha_1$ ...(i)

$T_r = \frac{mr^2}{2}\alpha$ ...(ii)

$\alpha_1 = \alpha$ ...(iii)

From (i) and (ii)

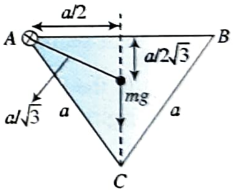


Acceleration of point b = acceleration of point a

$r\alpha_1 = a_{cm} - r\alpha$ ...(iv)

Hence, $a_{cm} = 2r\alpha$

65. **(3)** Torque about: $mg\frac{a}{2} = I\alpha$



$$\Rightarrow\quad \alpha = \frac{mga}{2I}$$

$$\Rightarrow\quad \text{Acceleration} = \frac{a}{\sqrt{3}}\alpha = \frac{mga^2}{2\sqrt{3}I}$$

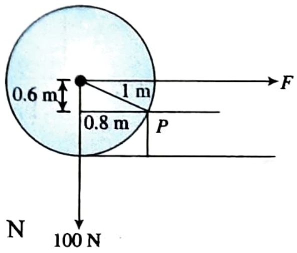
66. **(2)** Net torque, $\tau = 30g \times 2 - 20g \times 2 = 20g$

$$I = 30 \times 2^2 + 20 \times 2^2 = 120 + 80 = 200\text{ kg m}^2$$

$$\alpha = \frac{\tau}{I} = \frac{20g}{200} = \frac{g}{10} = \frac{10}{10} = 1.0\text{ rad s}^{-2}$$

67. **(3)** $F' \times \frac{R}{5} = FR$ or $F' = 5F$

68. **(4)** Taking moments of forces about point P, we get



$$F \times 0.6 = 100 \times 0.8$$

or $F = \frac{100 \times 0.8}{0.6}$ N

$$= \frac{800}{6}\text{ N} = \frac{400}{3}\text{ N} = 133.3\text{ N}$$

69. **(3)** Equation of motion is

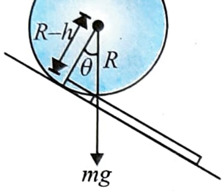
$$T = Mg - Ma = Mg - \frac{2}{3}Mg = \frac{1}{3}Mg$$

70. **(1)** The horizontal component of the force exerted on the bar by the hinge must balance the applied force $\vec{F}$, and so has magnitude 120.0 N and is to the left. Taking torques about point A, (120.0 N) (4.00 m) + F_v(3.00 m) so the vertical component is –160 N, with the minus sign indicating a downward component, exerting a torque in a direction opposite that of the horizontal component. The force exerted by the bar on the hinge is equal in magnitude and opposite in direction to the force exerted by the hinge on the bar.

$$\text{Net force} = \sqrt{[(160)^2 + (120)^2]} = 200 \text{ N}$$

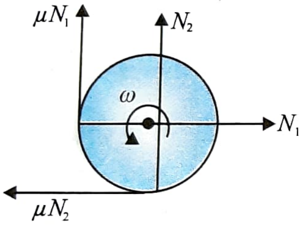
71. **(4)** The sphere is on the verge of toppling when a line of action of weight passes through the edge.

$$\cos\theta = \frac{(R-h)}{R} \quad \Rightarrow \quad h = R - R\cos\theta$$



72. **(2)** In the figure, for translational equilibrium in vertical direction,

$$\mu N_1 + N_2 = mg \qquad \text{...(i)}$$



In horizontal direction,

$$N_1 = \mu N_2 \qquad \text{...(ii)}$$

Solving, Eqs. (i) and (ii), $N_2 = \dfrac{mg}{1+\mu^2}$

The required friction is $\mu N_2 = f_a = \dfrac{3}{10} mg$

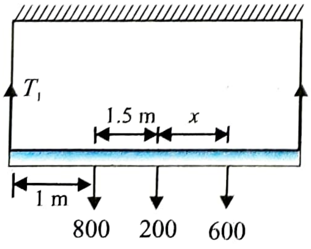
In the figure, the sphere has the tendency to move towards the right;

$$N_1 = 0; N_2 = mg$$

$$f_b = \mu N_2 = \frac{mg}{3}$$

which gives $\dfrac{f_a}{f_b} = \dfrac{9}{10}$

73. **(3)** For vertical equilibrium, $T_1 + T_2 = 1600$ N ...(i)



Take moment about O,

$$T_1 \times 2.5 + 600x = T_2 \times 2.5 + 800 \times 1.5 \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii)

$$T_1 = 1040 - 120x$$

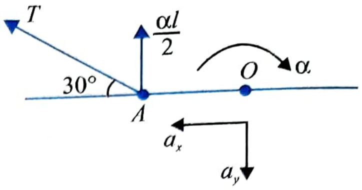
and $T_2 = 560 + 120x$

For safe working, $T_1 < 1040$ N and $T_2 < 1040$ N

$x > 0$ and $x < 2$, so the range is $0 < x < 2$.

74. **(4)** Acceleration of point A along the string is zero. Therefore,

$$a_x \cos 30° - a_y \cos 60° + \frac{\alpha l}{2} \cos 60° = 0$$



75. **(1)** $T = mg$

$$f = Mg\sin\theta + T\sin\theta$$

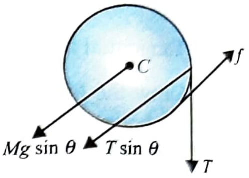
$$\Rightarrow f = Mg\sin\theta + mg\sin\theta$$

Balancing torque about C:

$$Tr = fr$$

$$\Rightarrow mg = Mg\sin\theta + mg\sin\theta$$

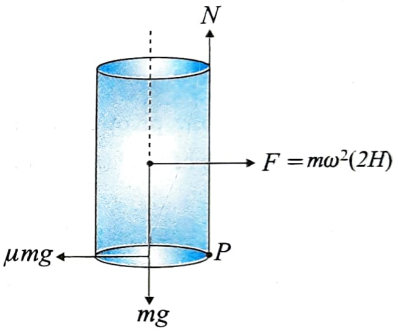
$$\Rightarrow m = \frac{M\sin\theta}{1-\sin\theta}$$



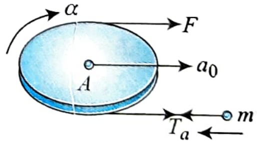
76. **(4)** $N = mg$ ...(i)

Net moment of all the forces about point P is zero.

$$F\frac{H}{2} = mg\frac{H}{8}, \; \omega = \sqrt{\frac{g}{8H}}$$



77. **(1)** For no slipping condition of thread acceleration of point A on the circumference of the disc = acceleration of the small body



$$a = R\alpha - a_0 \qquad \text{...(i)}$$

Other equations are

$$T = ma \qquad \text{....(ii)}$$

$$F + T = ma_0 \qquad \text{....(iii)}$$

And $(F - T)R = \dfrac{1}{2}mR^2 \cdot \alpha$

$$\Rightarrow F - T = \frac{1}{2}mR\alpha \qquad \text{...(iv)}$$

Using (i) in (iv), $F - T = \dfrac{1}{2}m(a + a_0)$

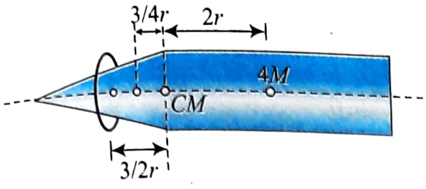
Using (ii) in this, $F = \dfrac{2}{3}ma + \dfrac{1}{2}ma$ ...(v)

From (ii) and (iii), $F = ma_0 - ma$ ....(vi)

Solving (v) and (vi), $a = \dfrac{F}{4m}$

78. (2) In critical case the COM of the system will lie at the centre of the circular base of the conical part (Just above A ; at O)

Volume of cone $= \frac{1}{3}\pi r^2(3r) = \pi r^3$



Volume of cylinder $= \pi r^2(4r) = 4\pi r^3$

$\therefore$ Mass of cylinder is $4M$

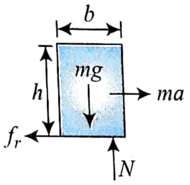
COM of the cone is at a distance of $\frac{h}{4} = \frac{3R}{4}$ from O.

Let mass of the ring $= m$

$$m \times 1.5r + M \times 0.75r = 4M \times 2r \;\; \therefore \; m = \frac{29}{6}M$$

79. (4) From FBD of the block, taking torque about right edge from where normal reaction is passing.

$$ma\frac{h}{2} \le mg\frac{b}{2} \Rightarrow a \le \frac{b}{h}g$$



Final velocity of truck is zero. So that

$$0 = v^2 - 2\left(\frac{b}{h}g\right)l \Rightarrow l = \frac{hv^2}{2bg}$$

Multiple Correct Answers Type

1. (2), (3)

We know that $K = \sqrt{\frac{I}{M}}$, $I \propto M$ hence radius of gyration does not depends upon mass of the body. Further I depends upon distribution of mass of body and also on the axis of rotation.

Hence K also depends upon these factors.

2. (3), (4)

All points in the body, in plane perpendicular to the axis of rotation, revolve in concentric circles. All points lying on the circle of same radius have same speed (and also same magnitude of acceleration) but different directions of velocity (also different directions of acceleration).

Hence there cannot be two points in the given plane with same velocity or with same acceleration. As mentioned above, points lying on circle of same radius have same speed.

Angular speed of body at any instant w.r.t. any point on the body is same by definition.

3. **(1), (2), (3)**

According to the theorem of perpendicular axis:

Moment of inertia of the plate about an axis passing through centre O and perpendicular to the plate is $I_0 = I_1 + I_2 = I_3 + I_4$.

(Because diagonals 1 and 2 as well as 3 and 4 are mutually perpendicular)

Now, according to symmetry,

$$I_1 = I_2 = I \text{ (say)}$$

and $I_3 = I_4 = I'$ (say)

For a square plate, $I_0 = \frac{M(L^2 + L^2)}{12} = \frac{ML^2}{6}$

Hence, $I = I' = \frac{ML^2}{12}$

i.e., $I_1 = I_2 = I_3 = I_4 = \frac{ML^2}{12}$

Hence, $I_0 = I_1 + I_2 = I_3 + I_4 = I_1 + I_3$

i.e., options (1), (2) and (3) are correct.

4. **(1), (2), (3)**

$$210 - T = 21\,a \qquad \text{...(i)}$$

Again, $\tau = I\alpha$

$$TR = \frac{Ia}{R}$$

$$T = \frac{Ia}{R^2} = \frac{\frac{1}{2} \times 2I \times R^2 \times a}{R^2} = \frac{1}{2} \times 21a$$

$$21a = 2T$$

From Eq. (i), $10 - T = 2T$

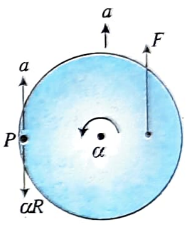
or $3T = 210$

or $T = 70$ N

Again from Eq. (i), $210 - 70 = 21a$

$$a = \frac{140}{21}\,\text{ms}^{-2} = \frac{20}{3}\,\text{ms}^{-2}$$

5. **(1), (2), (3)**

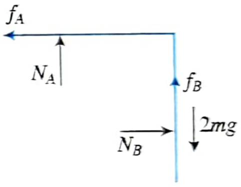


$$F = ma$$

$$F\frac{R}{2} = \frac{1}{2}mR^2\alpha$$

$$a_P = a - \alpha R = 0$$

6. **(1), (3)**



$$N_A + f_B = 2\,mg$$

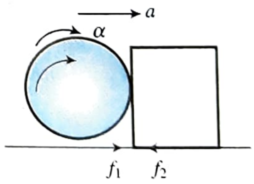
$$N_B = f_A$$

$$mgR - f_A R - f_B R = 0$$

$mg = f_A + f_B$ and given $f_A = f_B$.

7. **(3), (4)**

$$a = \frac{f_1 - f_2}{m_1 + m_2} = \frac{0.4 \times 6g - 0.5 \times 3g}{6 + 3}$$



$\Rightarrow \; a = 1\text{ m s}^{-2}$

$$\alpha = \frac{\tau_{\text{applied}} - \tau_{\text{friction}}}{I} = \frac{6 - f_1 R}{\frac{1}{2}6(R)^2}$$

$$= \frac{6 - 0.4 \times 6g(0.2)}{\frac{1}{2} \times 6 \times (0.2)^2} = 10\text{ rad s}^{-2}$$

8. **(1), (2), (3)**

Parallel axis theorem, check the distance carefully. $I_D = I_B$ (symmetric)

$$I_B = I_A + M\left(\frac{a}{2}\right)^2 = \frac{5}{12}Ma^2$$

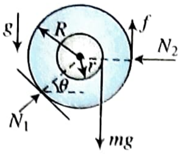
$$I_C = I_A + M\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{2}{3}Ma^2$$

9. (1), (4)

$$mgr = fR \quad \text{...(i)}$$

$$N_1 \sin\theta + f = mg \quad \text{...(ii)}$$

$$N_1 \cos\theta = N_2 \quad \text{...(iii)}$$



From (i), $f = \dfrac{mgr}{R}$

From (ii) and (iii),

$$N_2 = \frac{mg - f}{\tan\theta} = \frac{mg}{\tan\theta}\left[1 - \frac{r}{R}\right]$$

Net force at B: $F_B = \sqrt{f^2 + N_2^2}$

For minimum value of μ: $f \le \mu N_2$

$$\Rightarrow \frac{mgr}{R} \le \frac{\mu mg}{\tan\theta}\left[1 - \frac{r}{R}\right] \Rightarrow \mu \ge \frac{\tan\theta}{R/r - 1}$$

10. (1), (3)

In case of pure rotation ,the axis of rotation should be at rest.The axis of rotation may exist anywhere about which all particles of body will be in a circular motion with all centers lying on the axis of rotation.

11. (2), (4)

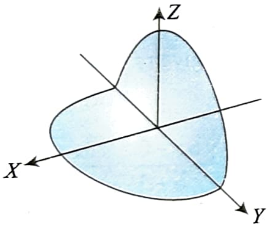
As the sphere is rotating uniformly the tangential acceleration of any particle on sphere will be zero and all particles will only have normal acceleration.

The particles on the diameter do not have any linear acceleration as they have zero radius of rotation.

The particles on the surface have the different radius of rotation hence different linear speeds.

12. (2), (3)

From figure we have



$$I_Y = \frac{1}{2}mR^2 \text{ and } I_X = I_Z = \frac{3}{4}mR^2$$

13. (1), (2), (4)

$$I_{xx} = m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 = ma^2$$

$$I_{yy} = m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 = ma^2$$

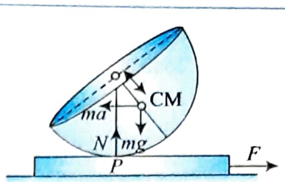
$$I_{AA'} = m\left(\frac{a}{\sqrt{2}}\right)^2 + m\left(\frac{a}{\sqrt{2}}\right)^2 + 0 + 0 = ma^2$$

$$I_{zz} = \left(m\left(\frac{a}{\sqrt{2}}\right)^2\right) \times 4 = 2ma^2$$

14. (1), (3)

From the plank frame net torque about 'P'

$$ma\left(R - \frac{R}{2}\cos\theta\right) = mg \cdot \frac{R}{2}\sin\theta \quad \text{...(i)}$$



$$a = \frac{g\sin\theta}{2 - \cos\theta} \text{ when } \theta = 37°$$

$$a = 5\text{m/s}^2$$

and $f = ma$ ...(ii)

$$\mu N = ma$$

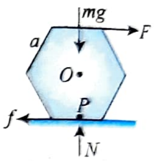
$$\Rightarrow \mu.mg = ma$$

$$\Rightarrow \mu = \frac{a}{g} = \frac{1}{2} \text{ and } F - \mu(2mg) = 2ma \quad \text{...(iii)}$$

So, $F = 100$ N

15. (1), (3)

Torque about 'P':



$$F\sqrt{3}a = mg\frac{a}{2}$$

$$\Rightarrow F = \frac{mg}{2\sqrt{3}}$$

Torque about 'O':

$$\frac{mg}{2\sqrt{3}}\left(\frac{\sqrt{3}}{2}a\right) + \mu mg\left(\frac{\sqrt{3}}{2}a\right) = mg\frac{a}{2}$$

$$\Rightarrow \mu = \frac{1}{2\sqrt{3}}$$

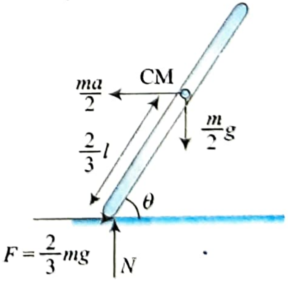
When $\mu = 2\mu_{\min} = \dfrac{1}{\sqrt{3}}$ and $F = \dfrac{mg}{\sqrt{3}}$, taking torque about P

$$\frac{mg}{\sqrt{3}} \cdot \sqrt{3}a - mg\frac{a}{2} = \frac{17}{12}ma^2\alpha$$

$$\Rightarrow \alpha = \frac{6}{17}\frac{g}{a}$$

16. (1), (2), (3)

Mass of the rod $= \displaystyle\int_0^l \frac{m}{l^2}rdr = \frac{m}{2}$



$$\text{Position of centre of mass} = \frac{\int_0^l \left(\frac{m}{l^2}r\,dr\right)r}{\int_0^l \frac{m}{l^2}rdr} = \frac{2}{3}l$$

$$N = \frac{mg}{2} \quad \text{...(i)}$$

$$F = \frac{2mg}{3} = \frac{ma}{2} \Rightarrow a = \frac{4}{3}g \quad \text{...(ii)}$$

Torque about centre of mass, $F\sin\theta = N\cos\theta$

$$\tan\theta = \frac{g}{a} = \frac{3}{4}$$

$$\theta = 37^\circ$$

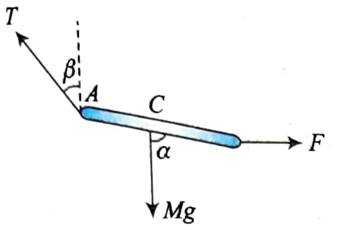
17. (1), (2), (3)

Horizontal and vertical equilibrium of the stick requires -

$$F = T\sin\beta \Rightarrow Mg = T\sin\beta \quad \text{...(i)}$$

and $Mg = T\cos\beta$ ...(ii)

Solving (i) and (ii), $\tan\beta = 1 \Rightarrow \beta = 45^\circ$



$$T = \sqrt{2}\,Mg$$

Rotational equilibrium requires torque about C to be zero ($\tau_c = 0$)

$$T\cos\beta\frac{l}{2}\sin\alpha - T\sin\beta\frac{l}{2}\cos\alpha = F\frac{l}{2}\cos\alpha$$

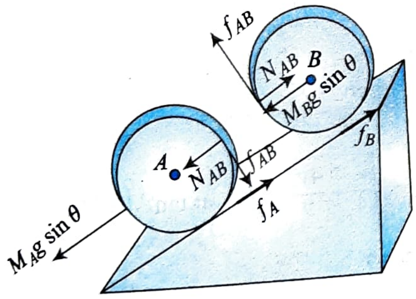
$$T\sin\alpha - T\cos\alpha = \sqrt{2}F\cos\alpha \qquad \left[\because \cos\beta = \sin\beta = \frac{1}{\sqrt{2}}\right]$$

$$\sqrt{2}Mg(\sin\alpha - \cos\alpha) = \sqrt{2}Mg\cdot\cos\alpha$$

$$\therefore \quad \tan\alpha = 2 \Rightarrow \alpha = \tan^{-1}(2)$$

18. (1), (2), (3)

The diagram shows the relevant forces on the two cylinders.



f_{AB} = friction between the two cylinders;

f_A = friction between A and the incline;

f_B = friction between B and the incline

For rotational equilibrium of the cylinders

$$f_A = f_{AB}\ [\text{for } A];\ f_B = f_{AB}\ [\text{for } B] \quad \therefore\ f_A = f_B = f_{AB}$$

For translational equilibrium of $(A + B)$

$$f_A + f_B = (M_A + M_B)g\sin\theta$$

$$\therefore \quad f_A = f_B = f_{AB} = \left(\frac{M_A + M_B}{2}\right)g\sin\theta$$

For A to be in equilibrium

$$N_{AB} + M_A g\sin\theta = \left(\frac{M_A + M_B}{2}\right)g\sin\theta$$

$$N_{AB} = \left(\frac{M_B - M_A}{2}\right)g\sin\theta$$

For the two cylinders to be in contact

$$N_{AB} > 0 \Rightarrow M_B > M_A$$

19. (1), (2), (4)

From perpendicular axis theorem,

$$I_z = I_1 + I_2 \quad \text{...(i)}$$

Moment of inertia of disc about a diameter axis, $I_d = \dfrac{mR^2}{4} = I_3$

Moment of inertia of the disc about an axis passing through the tangent, $I_t = \dfrac{mR^2}{4} + mR^2 = \dfrac{5mR^2}{4}$

$$\text{As } I_1 = I_2 = I_t = \frac{5mR^2}{4} \quad \text{...(ii)}$$

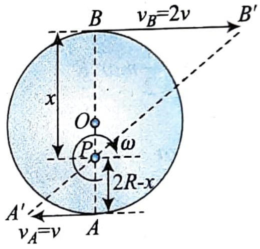
$$\text{From (i) and (ii), } I_z = 2\times\frac{5mR^2}{4} = \frac{5mR^2}{2}$$

20. (1), (3)

Since, there is no relative sliding between the cylinder and the planks 1 and 2, the points A and B of the disc will move with velocities equal to the velocities of the respective surfaces.

$$\vec{v}_A = -v\hat{i} \text{ and } \vec{v}_B = -2v\hat{i}$$

Joining A and B and A' and B' we find the point P as IAR. Then we have the similar triangles PAA' and PBB'. Using the properties of similar triangles we have



$$\omega = \frac{BB'}{AA'} = \frac{BP}{AP}$$

where $AA' = v_A = v$, $BB' = v_B = 2v$, $AP = 2R - x$ and $BP = x$

$$\text{Then, we have } \frac{2v}{v} = \frac{x}{2R - x}$$

$$\text{This gives, } x = \frac{4R}{3}$$

Hence, IAR is located at a distance of $4R/3$ below the top of the disc.

$$\text{As we know } \omega = \frac{v_{AB}}{AB}$$

$$v_{AB} = |\vec{v}_A - \vec{v}_B| = |-v\hat{i} - 2v\hat{i}| = 3v \text{ and } AB = 2R$$

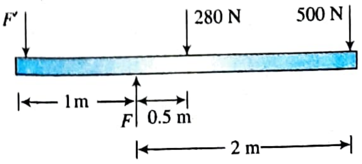
$$\text{This gives } \omega = \frac{3v}{2R} \text{ clockwise}$$

Linked Comprehension Type

For Problem 1–2

1. (3), 2. (4)

Take torques about the left end of the board inward



The force F at the support point is found from

$$F(1.00\text{ m}) = +(280\text{ N})(1.50\text{ m}) + (500\text{ N})(3.00\text{ m})$$

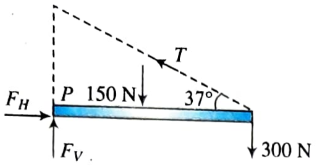
$\Rightarrow F = 1920$ N

The net force must be zero, so the force at the left end is (1920 N) – (500 N) – (280 N) = 1140 N, downward.

For Problems 3–4

3. (4), 4. (2)

Taking torques about the pivot (P)



$$(4\text{ m})(3/5)T = (4\text{ m})(300\text{ N}) + (2\text{ m})(150\text{ N})$$

so $T = 625$ N

The horizontal force must balance the horizontal component of the force exerted by the rope, or $T(4/5) = 500$ N.

The vertical force is 300 N + 150 N – $T(3/5)$ = 75 N, upwards.

For Problem 5–6

5. (3) $Mg - T = ma$ ...(i)

$$TR = \frac{MR^2}{2}\cdot\frac{a}{R} \quad \text{...(ii)}$$

From energy conservation

$$mgh = \frac{1}{2}\frac{MR^2\omega^2}{2} + \frac{1}{2}m\omega^2R^2$$

$$\sqrt{\frac{4mgh}{(M+2m)R^2}} = \omega$$

6. (4) $mgh = \frac{1}{2}\frac{MR^2}{2}\cdot\frac{v^2}{R^2} + \frac{1}{2}mv^2$

$$4mgh = Mv^2 + 2mv^2 \Rightarrow v = \sqrt{\frac{4mgh}{M+2m}}$$

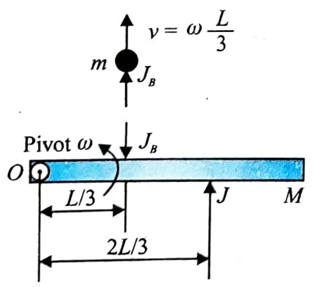
which is independent of R

For Problems 7–9

7. (2), 8. (4), 9. (1)

Let the system starts with angular velocity ω. Angular velocity of the ball will also be ω as it remains struck to the rod.

Velocity of the ball $v = \omega\frac{L}{3}$



For the rod, angular impulse = change in angular momentum:

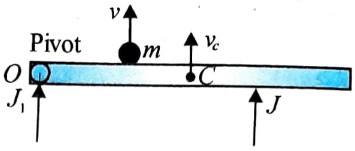
$$J\frac{2L}{3} - J_b\frac{L}{3} = \frac{ML^2}{3}\omega \quad \text{...(i)}$$

For the ball, impulse = change in linear momentum

$$J_b = mv = m\omega\left(\frac{L}{3}\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), $\omega = \frac{6J}{(m+3M)L}$

and $J_b = \frac{2mJ}{(m+3M)}$



Let impulse on the pivot be J_1.

For the rod and ball system,

total impulse = change in linear momentum

$$J + J_1 = Mv_c + mv$$

Solve to get: $J_1 = \frac{mJ}{m+3M}$

For Problems 10–12

10. (2) Let $OA = x$, $OB = y$

$$x^2 + y^2 = l^2 \quad \text{and} \quad x = l\cos\theta$$

Differentiate $x^2 + y^2 = l^2$ with respect to time.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$v_B = \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = -v_A\cot\theta = \frac{4}{5}v_0\downarrow$$

11. (1) Differentiate $x = l\cos\theta$ with respect to time

$$\frac{dx}{dt} = -l\sin\theta\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = -\frac{v_A}{l\sin\theta}$$

(negative sign indicates that θ is decreasing)

$$\omega = \left|\frac{d\theta}{dt}\right| = \frac{v_A}{l\sin 37°} = \frac{5v_0}{3l}$$

12. (4) $x_{CM} = \frac{x}{2}$

$$\Rightarrow v_{CM_x} = \frac{1}{2}\frac{dx}{dt} = \frac{v_0}{2}(\rightarrow)$$

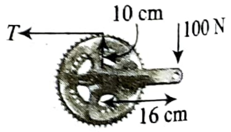
$$\Rightarrow y_{CM} = \frac{y}{2}$$

$$\Rightarrow v_{CM_y} = \frac{1}{2}\frac{dy}{dt} = \frac{v_B}{2} = \frac{2}{3}v_0\downarrow$$

$$v_{CM} = \sqrt{\frac{v_0^2}{4} + \frac{4v_0^2}{9}} = \frac{5}{6}v_0 \text{ at } \tan^{-1}\left(\frac{4}{3}\right) \text{ below horizontal}$$

For Problems 13–17

13. (3) As angular velocity of the disc is constant i.e., $\sum\tau = 0$



$$100\text{ N} \times 16\text{ cm} = T \times 10\text{ cm}$$

$T = 160$ N

14. (1) As angular acceleration of the rear wheel is zero therefore net torque on the wheel is zero.

15. (3) Power delivered $= \vec{F}\cdot\vec{v}$

where $\vec{v}$ is velocity of the point of application of the force.

$$v = 16\text{ cm} \times 2\pi.2\ (= R\omega)$$

$$= 0.64\,\pi\text{ m s}^{-1}$$

$$P = 100 \times 0.64\pi = 64\pi\text{ W}$$

16. (2) $RN = rn \Rightarrow n = \frac{10\text{ cm} \times 2}{4\text{ cm}} = 5$ cy/s

So rear wheel rotates 5 cycles/s.

Hence $V = \frac{35}{100} \times 2\pi \times 5 = 3.5\pi\,\text{m s}^{-1}$

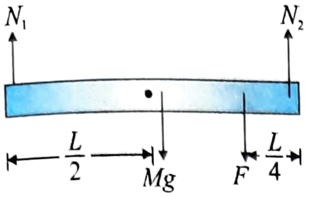
17. (4) As $\sum\tau = 0$

$$160\text{ N}\times 4\text{ cm} = f\times 35\text{ cm}$$

$$f = \frac{160\times 4\text{cm}}{35\text{cm}} = 18.3\text{ N}$$

For Problems 18–20

18. **(3)**



As the rod is in equilibrium,

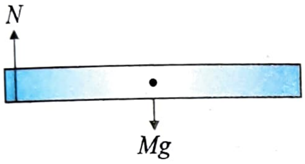
$\therefore\ \sum F_x = 0;\ \sum F_y = 0;\ \sum \tau = 0$

Taking torque about hinge 2, we get

$$N_1\times L = Mg\times\frac{L}{2} + F\times\frac{L}{4}$$

$$\therefore\ N_1 = \frac{Mg}{2} + \frac{F}{4} = 11\text{ N}$$

19. **(3)**



Now, rod is not in equilibrium,

$\therefore\ F = Ma_{\text{CM}}$

$\Rightarrow\ Mg - N = Ma$ ...(i)

and $\tau = I\alpha$

$\Rightarrow\ Mg\left(\frac{L}{2}\right) = \frac{ML^2}{3}\frac{a}{(L/2)}$ $\left(\because I = \frac{ML^2}{3}, a = \alpha\left(\frac{L}{2}\right)\right)$

$\Rightarrow\ a = \frac{3}{4}g$ ...(ii)

From Eqs. (i) and (ii),

$$N = \frac{Mg}{4} = 5\text{ N}$$

20. **(1)** By conservation of energy,

$$Mg\left(\frac{L}{2}\right) = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

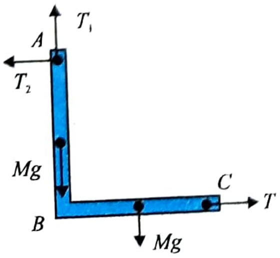
$$\therefore\ \omega = \sqrt{\frac{3g}{L}}$$

Acceleration $= \omega^2 L = 3g = 30\text{ m/s}^2$

For Problems 21–23

21. **(4)** Taking torque about A: $Mg\frac{L}{2} = TL$

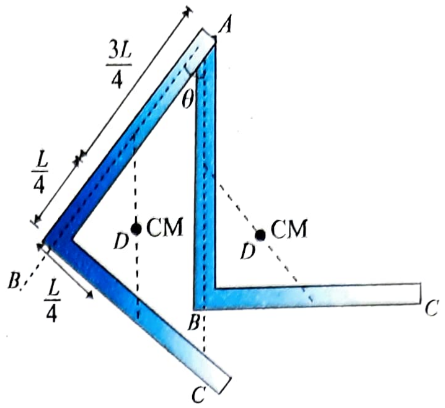
$$\Rightarrow\ T = \frac{Mg}{2}$$



22. **(4)** $T_1 = 2Mg,\ T_2 = T = \frac{Mg}{2}$

Net reaction $= \sqrt{T_1^2 + T_2^2} = \sqrt{(2Mg)^2 + (Mg/2)^2} = \frac{\sqrt{17}Mg}{2}$

23. **(1)** D is the CM of the rods



In triangle ABE:

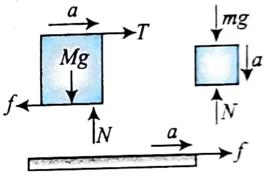
$$\tan\theta = \frac{L/4}{3L/4}$$

$$\Rightarrow\ \tan\theta = \frac{1}{3} \Rightarrow \theta = \tan^{-1}(1/3)$$

Here D is the centre of mass. In equilibrium, D will be vertically below A. So in equilibrium, AB will make angle θ, as calculated above, with the vertical.

For Problems 24–25

24. **(4)** Drawing free body diagrams of all the three (block, plank and mass)



Since we are taking the limiting case.

Therefore $f =$ maximum friction between the block and the plank.

or $f = \mu N = \mu Mg = \mu(4)g$

$f = 4\mu g$ ...(i)

Now writing equations of motion for all three

$mg - T = ma$ $(m = 1\text{ kg})$

or $g - T = a$ ...(ii)

$T - f = Ma$ $(M = 4\text{ kg})$

or $T - 4\mu g = 4a$ ...(iii)

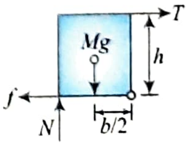
$f = Ma$ or $f = 4a$ or $4\mu g = 4a$

or $a = \mu g$ ...(iv)

Solving Eqs. (ii), (iii) and (iv) for μ, we get $\mu = \frac{1}{9}$

Therefore, minimum value of μ should be $\frac{1}{9}$ or $\mu \geq \frac{1}{9}$

25. **(2)** At the time of toppling, normal reaction N will pass through the front edge as shown below.



So, torque of T about $P \leq$ torque of Mg about P

or $Th \leq (Mg)\left(\frac{b}{2}\right)$ or $T \leq \left(\frac{Mg}{2}\right)\left(\frac{b}{h}\right)$

From Eqs. (ii), (iii) and (iv) we can find that, $T = \frac{8}{9}g$

Therefore, $\frac{8}{9}g \leq \left(\frac{Mg}{2}\right)\left(\frac{b}{h}\right)$

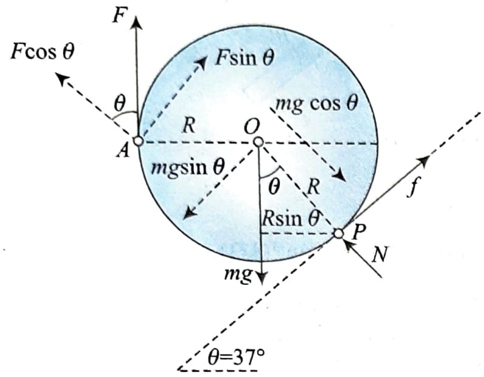
or $\frac{b}{h} \geq \frac{16}{9M}$ $(M = 4\text{ kg})$

or $\left(\frac{b}{h}\right) \geq \frac{4}{9}$

Therefore, minimum value of b/h so that the block does not topple is 4/9.

For Problems 26–28

26. (3) For rotational equilibrium of the cylinder.



Torque about P: $F(R + R\sin\theta) = mgR\sin\theta$

$$F\left(1 + \frac{3}{5}\right) = mg \times \frac{3}{5} \Rightarrow F = \frac{3mg}{8}$$

27. (3) For translational equilibrium

$$f + F\sin\theta = mg\sin\theta$$

$$\Rightarrow \quad f = (mg - F)\sin\theta \qquad \text{...(i)}$$

$$N + F\cos\theta = mg\cos\theta$$

$$\Rightarrow \quad N = (mg - F)\cos\theta \qquad \text{...(ii)}$$

For no sliding $f \leq \mu_s N$

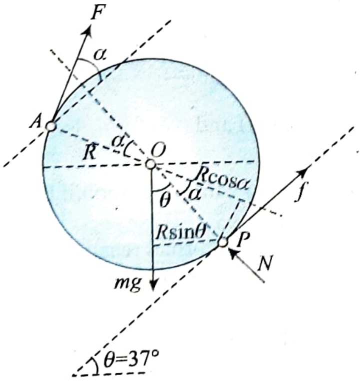
$$(mg - F)\sin\theta \leq \mu_s(mg - F)\cos\theta$$

or $\mu_s \geq \tan\theta$

As $\theta = 37°$. It means $\mu_s \geq \frac{3}{4}$

The minimum value of coefficient of static friction is $\frac{3}{4}$.

28. (1) For rotational equilibrium, balancing torque about P



$$F(R + R\cos\alpha) = mgR\sin\theta$$

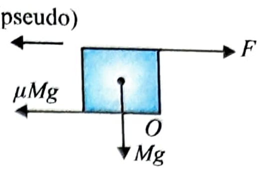
$$\Rightarrow \quad F = \frac{mg\sin\theta}{(1 + \cos\alpha)} = \frac{3mg}{5(1 + \cos\alpha)} \qquad \text{...(i)}$$

For F to be minimum $(1 + \cos\alpha)$ should be maximum, for this $\alpha = 0$

and $F_{\min} = \frac{3}{5}\frac{mg}{(1 + \cos\theta)} = \frac{3}{10}mg$

Matrix Match Type

1. i. → a.; ii.→ d.; iii.→ a.; iv. → c.



The force required to topple the block about O,

$$Fa = \frac{Ma}{g}g + \frac{Ma}{2} \times 1$$

$$F = M + \frac{M}{2} = 55\text{ N}$$

and $f_{\max} = \mu mg = 0.1 \times 10 \times 10 = 10\text{ N}$

Force between 0–10 N will do neither sliding or slipping.

Force between 10–55 N will cause slipping.

Force above 55 N will topple the block.

2. i. → c.; ii. → a.; iii. → a.; iv. → b.

Total mass: M

Mass of cutout portion: $m_1 = \frac{M}{4}$

$$I_1 = \frac{1}{2}MR^2 - \frac{1}{2}m_1\left(\frac{R}{2}\right)^2 = \frac{15}{32}MR^2$$

$$I_2 = I_1 + \frac{3M}{4}\left(\frac{R}{2}\right)^2 = \frac{21}{32}MR^2$$

Also $I_3 + I_4 = I_2$

$$I_2 = I_1 + \frac{3M}{4}\left(\frac{R}{2}\right)^2$$

$$I_3 = \frac{I_1}{2} + \frac{3M}{4}\left(\frac{R}{2}\right)^2$$

$$\Rightarrow I_2 - I_3 = \frac{I_1}{2}$$

3. i.→ b.; ii. → a.; iii. → c.; iv. → a., b., d.

i. Pure translational motion

ii. Pure rotational motion about centre of mass

iii. No motion as $\sum\vec{F} = 0$ and $\sum\vec{\tau} = 0$

iv. Combined rotation and translational motion

4. i. → b.; ii.→ d.; iii.→ d.; iv.→ a.

For sphere, all the three axis are diameter, so $I_x = I_y = I_z$

Perpendicular axis theorem is valid for two dimensional bodies. It is not valid for spheres.

5. (2)

(i) $K = \sqrt{\frac{I}{m}} = \sqrt{\frac{m(2a)^2}{3m}} = \frac{2}{\sqrt{3}}a$

(ii) $K = \sqrt{\frac{m(2a)^2}{12m}} = \frac{a}{\sqrt{3}}$

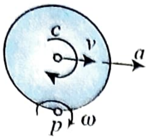
(iii) $K = \sqrt{\frac{ma^2}{3m}} = \frac{a}{\sqrt{3}}$

(iv) $K = \sqrt{\frac{ma^2}{12m}} = \frac{a}{\sqrt{12}}$

6. (1) (i) The centre of rolling body is rotating about O. Hence

$$\omega_{CO} = \frac{v}{(R-r)}.$$

(ii) As the body is rolling ploint P is I.C.R



Hence $\omega_{CP} = \frac{v}{r}$

(iii) As point P is ICR , have

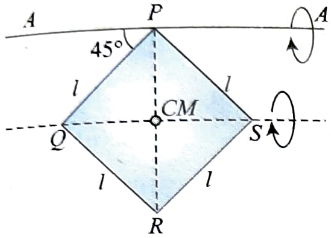
$$a_{C.P} = \alpha_{CP} \times r \Rightarrow \alpha_{CP} = \frac{\alpha_{cp}}{r} = \frac{a}{r}$$

(iv) The centre of the rolling body is rotating about O

Hence $\alpha_{CO} = \frac{a}{(R-r)}$

7. (3)

(i)



Moment of inertia of the frame about an axis passing through QS

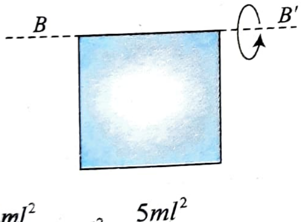
$$I_{QS} = 4 \times \frac{1}{3} ml^2 \sin^2 45° = \frac{2}{3} ml^2$$

Using parallel axis theorem

$$I_{AA'} = I_{QS} + 4m\left(\frac{l}{\sqrt{2}}\right)^2 = \frac{2}{3} ml^2 + 2ml^2$$

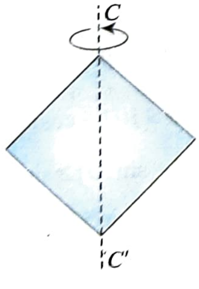
$$\Rightarrow I_{AA'} = \frac{8}{3} ml^2$$

(ii)



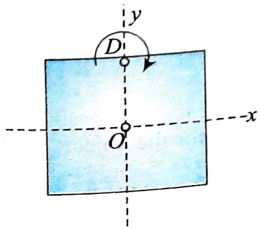
$$I_{BB'} = \frac{2ml^2}{3} + ml^2 = \frac{5ml^2}{3}$$

(iii)



$$I_{CC'} = 4 \times \frac{1}{3} ml^2 \sin^2 45° = \frac{2}{3} ml^2$$

(iv)



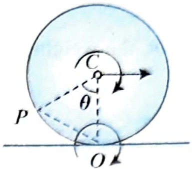
$$I_x = I_y = 2\left[\frac{1}{12} ml^2 + m\left(\frac{l}{2}\right)^2\right] = \frac{2ml^2}{3}$$

$$I_z = I_x + I_y \Rightarrow I_y = \frac{4ml^2}{3}$$

Moment of inertia about an axis passing through D

$$I_D = I_2 + 4m\left(\frac{l}{2}\right)^2 = \frac{4}{3} ml^2 + ml^2 = \frac{7ml^2}{3}$$

8. (2) As the disc is rolling point O should be ICR.



$$v_P = \omega(PO) = \omega 2R \sin\frac{\theta}{2} = 2v \sin\frac{\theta}{2}$$

As $v = \omega R$

(i) If $\theta = 60° \Rightarrow v_P = 2v \sin\frac{60°}{2} = v$

(ii) If $\theta = 90° \Rightarrow v_P = 2v \sin\frac{90°}{2} = \sqrt{2}\, v$

(iii) If $\theta = 120° \Rightarrow v_P = 2v \sin\frac{120°}{2} = \sqrt{3}\, v$

(iv) If $\theta = 180° \Rightarrow v_P = 2v \sin\frac{180°}{2} = 2v$

9. (2) For (i): Acceleration of 1 w.r.t. centre of mass $= r\alpha\hat{i} - \omega^2 r\hat{j}$

$\Rightarrow \vec{a}_1 = r\alpha\hat{i} - \omega^2 r\hat{j} + R\alpha\hat{i} = (R+r)\alpha\hat{i} - \omega^2 r\hat{j}$

For (ii): $\vec{a}_2 = -r\alpha\hat{j} - \omega^2 r\hat{i} + R\alpha\hat{i} = (R\alpha - \omega^2 r)\hat{i} - r\alpha\hat{j}$

For (iii): $\vec{a}_3 = -r\alpha\hat{i} + \omega^2 r\hat{j} + R\alpha\hat{i} = (R\alpha - r\alpha)\hat{i} + \omega^2 r\hat{j}$

For (iv): $\vec{a}_4 = r\alpha\hat{j} + \omega^2 r\hat{i} + R\alpha\hat{i} = (R\alpha + \omega^2 r)\hat{i} + r\alpha\hat{j}$

10. (4)

(i) $I_O = I_1 + I_2$

$$= \frac{m_1 l_1^2}{3} + \frac{m_2 l_2^2}{3} = \frac{1}{3}\left(\frac{2m}{3}\right)(2l)^2 + \frac{1}{3}\left(\frac{m}{3}\right)l^2 = ml^2$$

(ii) $I_O = ml^2 + m\left(\frac{l^2}{12}\right) + m\left(l^2 + \frac{l^2}{4}\right) = \frac{7}{3} ml^2$

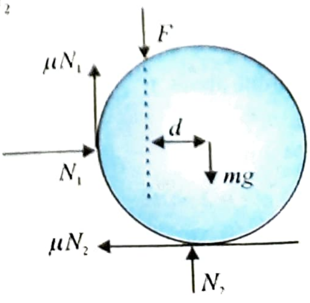
(iii) $I_O = \frac{ml^2}{12} + m\left[\left(\frac{l}{4}\right)^2 + l^2\right] = \frac{55}{48} ml^2$

(iv) $I_O = I_1 + I_2 = \frac{ml^2}{3} + \frac{ml^2}{12} + ml^2 = \frac{17ml^2}{12}$

Numerical Value Type

1. (6) For equilibrium of the cylinder in the horizontal direction,

$N_1 = \mu N_2$ ...(i)



In the vertical direction,

$$N_2 + \mu N_1 = F + Mg \quad \text{...(ii)}$$

Solving these two equations with $F = 40$ N, $M = 2$ kg and $\mu = 1/3$, we get

$$N_1 = 18 \text{ N}$$

and $N_2 = 54$ N

Now, $Fd = \mu(N_1 + N_1)r$

$$\therefore \quad d = \frac{M(N_1 + N_2)r}{F} = \frac{(1/3)(18+54)(0.1)}{40} = 0.06 \text{ m} = 6 \text{ cm}$$

2. **(5)** When the string is cut, the weight of the rod constitutes torque about the hinge, so

$$\tau_A = mg\frac{l}{2} \quad \text{...(i)}$$

According to Newton's second law,

$$\tau_A = I\alpha \quad \text{...(ii)}$$

where α is the angular acceleration of the rod about the end A.

From Eqs. (i) and (ii), we get

$$I\alpha = mg\frac{l}{2} \text{ or } \alpha = \frac{mg\frac{l}{2}}{I}$$

Here $I = ml^2/3$, therefore

$$\therefore \quad \alpha = \frac{mgl/2}{ml^2/3} = \frac{3}{2}\frac{g}{l}$$

Acceleration of the CM of the rod is

$$a_{\text{CM}} = \alpha r = \frac{3}{2}\frac{g}{l} \times \frac{l}{2} = \frac{3g}{4}$$

Again by Newton's second law,

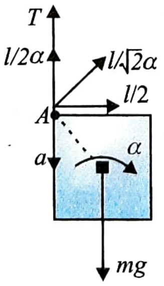
$$mg - R_A = ma_{\text{CM}}$$

$$\text{or} \quad mg - R_A = m \times \frac{3g}{4}$$

$$\therefore \quad R_A = \frac{Mg}{4} = \frac{2 \times 10}{4} = 5 \text{ N}$$

3. **(6)** $T\frac{l}{2} = \frac{ml^2}{6} \times \alpha;\ mg - T = ma$

$$\alpha = \frac{3T}{ml} = \frac{3}{ml}(mg - ma)$$



Constraint relation: Acceleration of A in the vertical direction should be zero

$$\frac{l}{2}\alpha = a$$

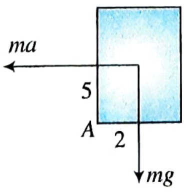
$$\Rightarrow \quad \alpha = \frac{2a}{l} = \frac{3(g-a)}{l}$$

$$5a = 3g \Rightarrow a = 3 \times \frac{g}{5} = 6 \text{ m/s}^2$$

4. **(4)** Maximum acceleration of cart so that cylinder does not slip:

$a_m = \mu g = 0.5 \times 10 = 5$ ms^{-2}

For tipping over: Let acceleration of cart is a.



Considering torque about A:

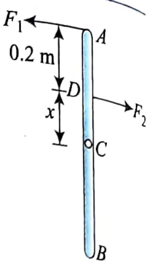
$$Ma \times 5 \geq mg \times 2$$

$$\Rightarrow \quad a \geq 2g/5 = 4 \text{ ms}^{-2}$$

5. **(2.00)** We have $F_2 - F_1 = 2 \Rightarrow F_2 = 10$ N Balancing torque,

$$F_1\frac{l}{2} = F_2\left(\frac{l}{2} - 0.2\right) \Rightarrow l = 2 \text{ m}$$

6. **(1.0)** Let x be the distance of centre point C of rod from D. Then, $F_2 - F_1 = ma$



or $\quad F_1 = 3$ N

Further, $\tau_C = 0$

$$\therefore \quad F_2 x = F_1(0.2 + x)$$

$$5x = F_1(0.2 + x)$$

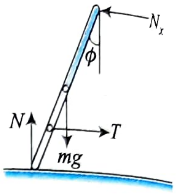
$$\therefore \quad 5x = 3(0.2 + x) \text{ or } x = 0.3 \text{m}$$

$$\therefore \quad \text{Length of rod} = 2(x + 0.2) = 1.0 \text{m}$$

7. **(3.00)** $a_1 = R\alpha - a_2 \Rightarrow 2 = 2\alpha - 4 \Rightarrow \alpha = 3$ rad/s^2

8. **(30.00)** Drawing FBD

Taking torque about A



$$N_x l\cos\phi - mg\frac{l}{2}\sin\phi - T\frac{l}{3}\cos\phi = 0$$

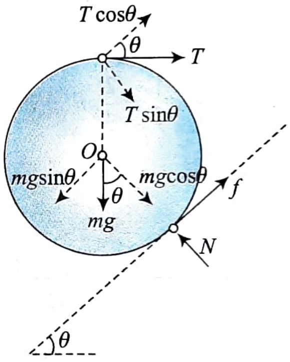
$$N_x - \frac{T}{3} = \frac{mg}{2}\tan\phi \quad \text{...(i)}$$

$$N_x = T \quad \text{...(ii)}$$

Then $T = N_x = \frac{3mg}{4}$

On substituting $m = 4$ kg we get $T = 30$ N

9. **(0.50)** Drawing FBD of the cylinder



$$T = f \quad \text{...(i)}$$

$$T\cos\theta + f = mg\sin\theta \quad \text{...(ii)}$$

$$T\sin\theta + mg\cos\theta = N \quad \text{...(iii)}$$

$$f \leq \mu N \quad \text{...(iv)}$$

$$\frac{mg\sin\theta}{1+\cos\theta} \leq \mu\left(\frac{mg\cos\theta}{1-\mu\sin\theta}\right)$$

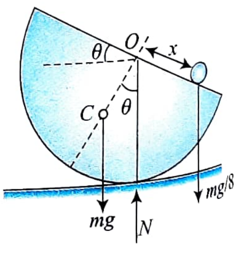
$$(1 - \mu\sin\theta)\sin\theta \leq \mu(1+\cos\theta)\cos\theta$$

$$\mu[\cos\theta + \cos^2\theta + \sin^2\theta] \geq \sin\theta$$

$$\mu_{\min} \geq \frac{\sin\theta}{1+\cos\theta} = \frac{\sin 53^\circ}{1+\cos 53^\circ} = 0.50$$

10. **(6.00)** $\Sigma\tau_0 = 0 \Rightarrow mg\frac{3R}{8}\sin\theta = \frac{mgx}{8}\cos\theta$

$$\therefore \quad x = 3R\tan\theta$$



$$f = \frac{mg}{8}\sin\theta$$

and $N = \frac{mg}{8}\cos\theta$ (for the object)

$$\therefore \quad \tan\theta = \frac{f}{N}$$

$$\therefore\ \tan\theta_m = \frac{f_m}{N} = \mu$$

$$\therefore\ x_m = 3R\tan\theta_m = 3R\mu = (3)(10)(0.2) = 6\text{ m}$$

11. (5.00) Torque about A

$$F\frac{\sqrt{3}}{2}d = mg\frac{d}{2} + ma\frac{\sqrt{3}}{2}d\frac{1}{3}$$

$$\therefore\ F = \frac{mg}{\sqrt{3}} + \frac{ma}{3}$$

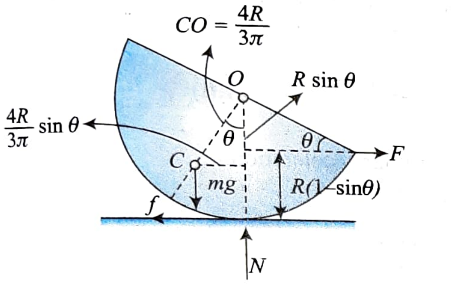
$$\therefore\ F_H = \frac{mg}{\sqrt{3}} - \frac{2ma}{3}$$

$$F_v = mg$$

So, $F_A = 5$ N

12. (30) As the disc moves with uniform velocity $F = \mu mg$ and $N = mg$

The force F and friction force will form a couple.



Balancing torque about centre of mass

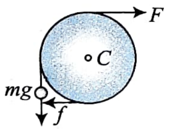
$$\mu mgR(1-\sin\theta) = mg\frac{4R}{3\pi}\sin\theta$$

$$\Rightarrow\ \sin\theta = \frac{1}{2}\ \Rightarrow\ \theta = 30°$$

13. (25.00) For cylinder $F = f$ (friction)

and $FR + fR = mgR$

$$\Rightarrow 2F = mg \Rightarrow F = \frac{mg}{2} = \frac{5\times 10}{2} = 25\text{ N}$$

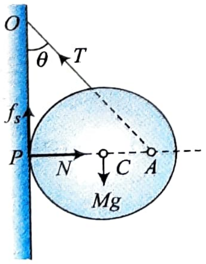


14. (0.50) Net torque about $A = 0$

$$f_s\frac{3R}{2} = Mg\cdot\frac{R}{2}$$

$$\therefore\ f_s = \frac{Mg}{3} \quad \text{...(i)}$$

$$\left[\therefore\ PA = \frac{3R}{2}\tan 45° = \frac{3R}{2}\right]$$



Horizontal equilibrium: $N = \frac{T}{\sqrt{2}}$ ...(ii)

Vertical equilibrium: $\frac{T}{\sqrt{2}} + f_s = Mg$

$$T = \frac{2\sqrt{2}}{3}Mg \quad \text{...(iii)}$$

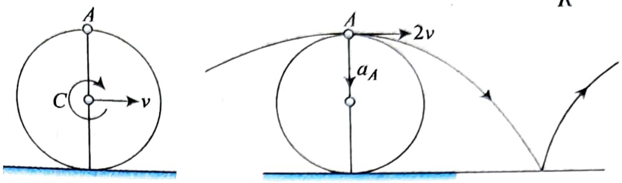
Put in (ii) $N = \frac{2}{3}Mg$

$$f_s \le \mu_s N \Rightarrow \frac{Mg}{3} \le \mu_s\cdot\frac{2}{3}Mg \Rightarrow \frac{1}{2} \le \mu_s$$

Hence, $\mu_{\min} = 0.50$

15. **(4.00)** Radius of curvature of the path is the radius of the circle that matches up with the path locally at a point.

Let's first find the acceleration of point A with respect to the COM of the wheel. An observer at COM sees that point A is spinning about it. Acceleration of A is $\frac{v^2}{2R}(\downarrow)$ with respect to COM. Because the COM does not have any acceleration hence acceleration of point A with respect to the ground is $a_A = \frac{v^2}{R}(\downarrow)$

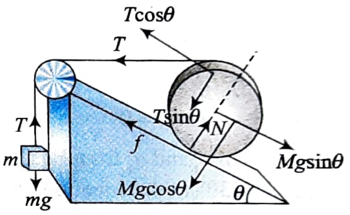


For a person on ground, speed of point A is $v_A = v + \omega R = 2v$.

If the radius of curvature of the path of A is r then $a_A = \frac{v_A^2}{r}$ i.e.,

$$\frac{v^2}{R} = \frac{(2v)^2}{r} \Rightarrow r = 4R = 4\text{ m}$$

16. **(30.00)** Let the mass of the block be m



For equilibrium of the block, $T = mg$ ...(i)

For rotational equilibrium of the cylinder, net torque about its centre must be zero.

$\therefore\ TR = fR.$ [R = radius of cylinder]

$\Rightarrow\ mg = f$ ...(ii)

For translational equilibrium of the cylinder

$$N = Mg\cos\theta + T\sin\theta$$

$$\Rightarrow\ N = Mg\cos\theta + mg\sin\theta \quad \text{...(iii)}$$

and $f + T\cos\theta = Mg\sin\theta$ ...(iv)

$$\Rightarrow\ f(1+\cos\theta) = Mg\sin\theta$$

$$\Rightarrow\ f = \frac{Mg\sin\theta}{(1+\cos\theta)} \Rightarrow mg = \frac{Mg\sin\theta}{1+\cos\theta}$$

$$m = \frac{M\sin\theta}{1+\cos\theta} \quad \text{...(v)}$$

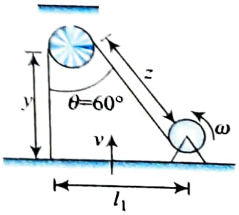
On substituting the values of M and θ we get $m = 30$ kg

17. **(1.00)** Total length of the string at any time t

$$L = y + z = y + \sqrt{y^2 + l_1^2}$$

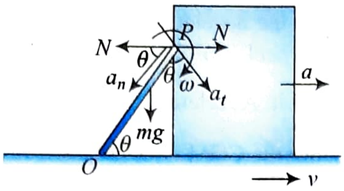
$$\frac{dL}{dt} = \frac{dy}{dt} + \frac{y}{\sqrt{y^2+l_1^2}}\frac{dy}{dt}$$

$$\omega R = v(1+\cos\theta),\ v = \frac{\omega R}{1+\cos\theta}$$



On substituting the values of $\omega = 15$ rad/s, $\theta = 60°$ and $R = 0.10$ m, we get $v = 1$ m/s

18. (4)



Decrease in potential energy of rod = increase in rotational kinetic energy of rod + translational kinetic energy of block.

But $v = v_P \sin\theta = (l\omega)\sin\theta$

$$\therefore \quad mg(l - l\sin\theta) = \frac{1}{2}(ml^2)\omega^2 + \frac{1}{2}M(\omega l\sin\theta)^2$$

From above equation we get,

$$a_n = l\omega^2 = \frac{2mg(1-\sin\theta)}{m + M\sin^2\theta} \qquad \text{...(i)}$$

At the time of losing contact

$$a_t = l\alpha = l\left(\frac{\tau}{I}\right) = l\left[\frac{mgl\cos\theta}{ml^2}\right] = g\cos\theta \qquad \text{...(ii)}$$

At the time of separation,

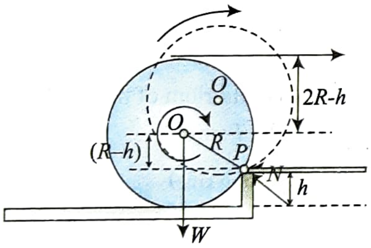
$$\Sigma F_x = 0 \text{ and } a = 0$$

For block: $a_t\sin\theta - a_n\cos\theta = 0$ ...(iii)

Now putting values of a_t and a_n from Eqs. (i) and (ii) in Eq. (iii) and then putting $\theta = 30°$ in the equation we get,

$$\frac{M}{m} = 4$$

19. (50.00) From figure



$$d = \sqrt{R^2 - (R-h)^2} = \sqrt{2Rh - h^2}$$

Taking torque about $P, \tau_F > \tau_W$

For minimum F, $mgd = F(2R - h)$

$$\text{or} \quad F = \frac{mg\sqrt{2Rh - h^2}}{(2R-h)} = \frac{mgh\sqrt{\frac{2R}{h} - 1}}{h\left(\frac{2R}{h} - h\right)} = \frac{mg}{\sqrt{\frac{2R}{h} - 1}}$$

$$= \frac{15 \times 10}{\sqrt{10 - 1}} = 50 \text{ N}$$

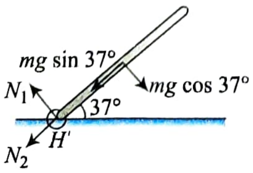
20. (40.00) Torque about hinge $\tau_H = I\alpha$

$$Mg\cos 37° \frac{l}{2} = \frac{ml^2}{3}\alpha$$

$$\Rightarrow \alpha = 6g/5l$$

$$a_t = \alpha\frac{l}{2} = \frac{3g}{5}$$

$$\Rightarrow Mg\cos 37° - N_1 = ma_t$$



$$\therefore \quad N_1 = \frac{mg}{5}$$

Angular velocity of rod is zero. So

$$N_2 = mg\sin 37° = 3mg/5$$

$$N = \sqrt{N_1^2 + N_2^2} = \sqrt{\left(\frac{mg}{5}\right)^2 + \left(\frac{3mg}{5}\right)^2} = \frac{mg\sqrt{10}}{5}$$

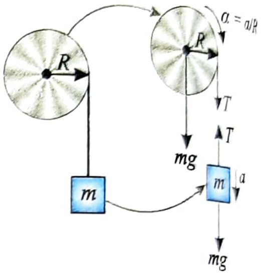
$$\Rightarrow \quad N = \frac{2\sqrt{10} \times 10 \times \sqrt{10}}{5} = 40 \text{ N}$$

Archives

JEE Main

Single Correct Answer Type

1. (3) The free body diagram of pulley and mass



Equation of motion of block: $mg - T = ma$

$$\therefore \quad a = \frac{mg - T}{m} \qquad \text{...(i)}$$

∴ Angular acceleration of pulley

$$\alpha = \frac{\tau}{I} \qquad \text{...(ii)}$$

Here, $\tau = T \times R$

and $I = \frac{1}{2}mR^2$ (For circular disc)

$$\therefore \quad T = \frac{mR\alpha}{2}$$ (Using (ii

$$\text{Therefore,} \quad a = \frac{mg - \frac{mR\alpha}{2}}{m}$$ (Using (

$$ma = mg - \frac{ma}{2} \qquad \left(\because \quad \alpha = \frac{a}{R}\right)$$

$$\therefore \quad a = \frac{2g}{3}$$

2. (2) $\tau = (20t - 5t^2)2 = 40t - 10t^2$

$$\alpha = \frac{\tau}{I} = \frac{40t - 10t^2}{10} = 4t - t^2$$

$$\omega = \int_0^t \alpha\, dt = 2t^2 - \frac{t^3}{3}$$

When direction is reversed, ω is zero. So

$$2t^2 - \frac{t^3}{3} = 0 \Rightarrow t^3 = 6t^2 \Rightarrow t = 6 \text{ s}$$

$$\theta = \int \omega\, dt = \int_0^6 \left(2t^2 - \frac{t^3}{3}\right)dt = \left[\frac{2t^3}{3} - \frac{t^4}{12}\right]_0^6 = 36 \text{ rad}$$

Number of revolution $= \frac{36}{2\pi}$ = Less than 6

3. (4) Applying Newton's second law on hanging block

$$mg - T = ma \qquad \text{...(i)}$$

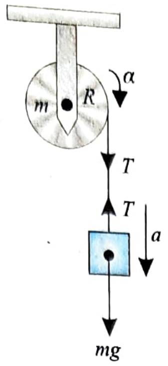
Torque on cylinder due to tension in string about center of pulley

$$T \times R = I\alpha$$

$$T \times R = mR^2\alpha$$

($\because$ $I = mR^2$ for hollow cylinder)

$$\Rightarrow T = mR\alpha \qquad \text{...(ii)}$$



As string is not slipping over pulley.

$$a = R\alpha \qquad \text{...(iii)}$$

From Eqs. (ii) and (iii)

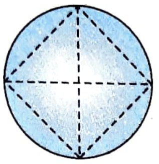
$$\Rightarrow T = ma \qquad \text{...(iv)}$$

From Eqs. (i) and (iv)

$$mg = 2ma$$

$$\Rightarrow a = \frac{g}{2}$$

4. (3) $d = 2R = a\sqrt{3}$



$$\Rightarrow a = \frac{2}{\sqrt{3}}R$$

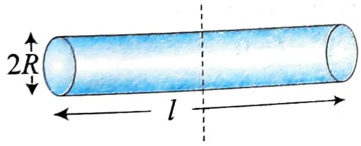
$$\frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3} = \frac{\sqrt{3}}{2}\pi$$

$$\Rightarrow M' = \frac{2M}{\sqrt{3}\pi}$$

$$I = \frac{M'a^2}{6} = \frac{2M}{\sqrt{3}\pi} \times \frac{4}{3}R^2 \times \frac{1}{6}$$

$$I = \frac{4MR^2}{9\sqrt{3}\pi}$$

5. (3)



$$I = \frac{ml^2}{12} + \frac{mR^2}{4} \qquad \text{...(1)}$$

or $$I = \frac{m}{4}\left(\frac{l^2}{3} + R^2\right)$$

Also $$m = \pi R^2 l\rho \qquad \text{...(2)}$$

$$\Rightarrow R^2 = \frac{m}{\pi l\rho} \text{ (put in equation)}$$

$$I = \frac{m}{4}\left(\frac{l^2}{3} + \frac{m}{\pi l\rho}\right)$$

For maxima and minima

$$\frac{dI}{dl} = \frac{m}{4}\left(\frac{2l}{3} - \frac{m}{\pi l^2\rho}\right) = 0$$

$$\frac{2l}{3} = \frac{m}{\pi l^2\rho} \Rightarrow \frac{2l}{3} = \frac{\pi R^2 l\rho}{\pi l^2\rho}$$

or $$\frac{2l}{3} = \frac{R^2}{l}$$

$$\Rightarrow \frac{l^2}{R^2} = \frac{3}{2}$$

or $$\frac{l}{R} = \sqrt{\frac{3}{2}}$$

6. (3) Torque at angle θ

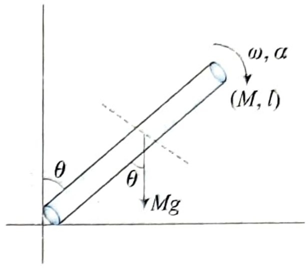
$$\tau = Mg\sin\theta.\frac{l}{2}$$

Taking torque about pivot

$$\tau = I\alpha$$

$$mg\sin\theta\frac{l}{2} = \frac{ml^2}{3}\alpha$$

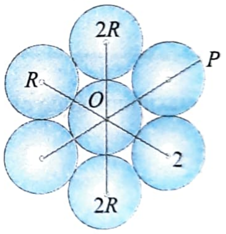
$$\alpha = \frac{3g}{2l}\sin\theta$$



7. (1) The moment of inertia of the arrangement about the axis normal to the plane and passing through the point O is

$$I_O = \frac{MR^2}{2} + 6\left(\frac{MR^2}{2} + M\times(2R)^2\right)$$

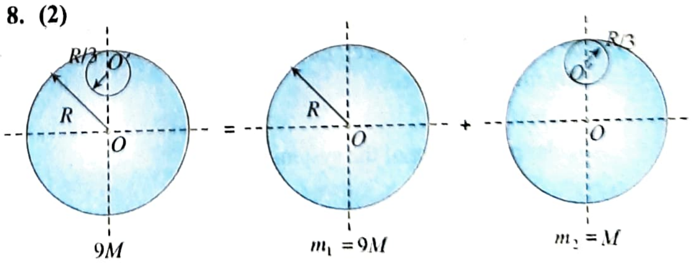
$$= \frac{7MR^2}{2} + 24MR^2 = \frac{55MR^2}{2}$$



$$I_P = I_O + 7M(3R)^2$$

$$= \frac{55MR^2}{2} + 7M(3R)^2 = \frac{181}{2}MR^2$$

8. (2)



Surface mass density $= \frac{9M}{\pi R^2}$

Mass of removed disc,

$$m_2 = \sigma\times\pi\left(\frac{R}{3}\right)^2 = \frac{9M}{\pi R^2}\times\frac{\pi R^2}{9} = M$$

Moment of inertia about an axis passing through O of original disc of mass m;

$$I_1 = \frac{(9M)R^2}{2} = \frac{9MR^2}{2}$$

Moment of inertial of removed disc of mass m_2 about an axis passing through O,

$$I_2 = \frac{M}{2}\left(\frac{R}{3}\right)^2 = \frac{MR^2}{18}$$

Now using parallel axis theorem to get moment of inertia of structure about O

$$I_0 = I_1 - \left[I_2 + m_2\left(\frac{2R}{3}\right)^2\right] = \frac{9MR^2}{2} - \left[\frac{MR^2}{18} + \frac{4}{9}MR^2\right]$$

$$= \frac{72\,MR^2}{18} = 4\,MR^2$$

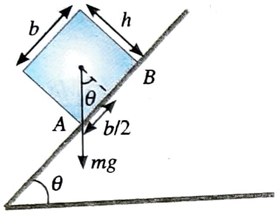
JEE Advanced

Single Correct Answer Type

1. (2) The maximum angle when the block will not slide should be equal to the angle of repose.

$$\phi = \tan^{-1}\mu = \tan^{-1}\sqrt{3} = 60°$$

If the block does not topple, the line of action of weight should pass within the base of the block.



And for toppling, maximum angle of inclination.

$$\theta = \tan^{-1}\frac{a/2}{h/2}$$

$$\theta = \tan^{-1}\frac{(10/2)}{(15/2)} = \tan^{-1}\frac{2}{3}$$

$$\theta = 34° < 60°$$

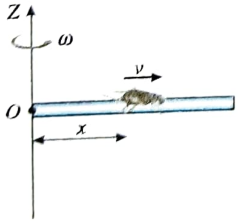
For 60°, mg is already outside the base.

∴ As the angle θ is increased first it will topple before it starts sliding.

2. (2) Let M and l be the mass and length of the rod respectively and m be the mass of the insect. Let insect be at a distance x from O at any instant of time t.

∴ $x = vt$ ...(i)

Angular momentum of the system about O,



$$L = \left[\frac{Ml^2}{3} + mx^2\right]\omega = \left[\frac{Ml^2}{3} + m(vt)^2\right]\omega \quad \text{(using (i))}$$

$$= \left[\frac{Ml^2}{3} + mv^2t^2\right]\omega$$

As $|\vec{\tau}| = \frac{dL}{dt} = \frac{d}{dt}\left[\frac{Ml^2}{3} + mv^2t^2\right]\omega$

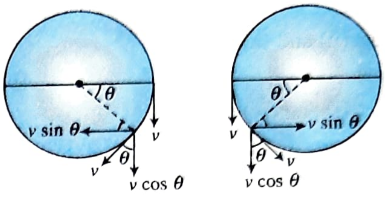
As ω and v remain constant.

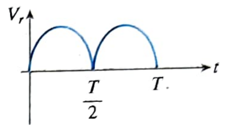
$\therefore \quad |\vec{\tau}| = 2m\omega v^2 t$

$|\vec{\tau}| \propto t$

Hence, the graph $|\tau|$ and t is a straight line passing through the (0, 0). Option (2) represents correct plot.

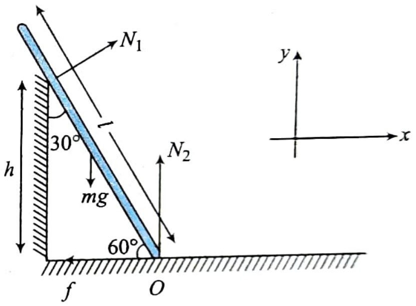
3. (1)





$$v_r = |2v\sin\theta| = |2v\sin\omega t|$$

4. (4)



Force equation in x-direction,

$N_1\cos 30° - f = 0$ ...(i)

Force equation in y-direction,

$N_1\sin 30° + N_2 - mg = 0$ ...(ii)

Torque equation about O,

$$mg\frac{l}{2}\cos 60° - N_1\frac{h}{\cos 30°} = 0 \quad \text{...(iii)}$$

Also, given $N_1 = N_2$ ...(iv)

[Note taking reaction from floor as normal reaction only]

Solving (i), (ii), (iii) and (iv) we have

$$\frac{h}{l} = \frac{3\sqrt{3}}{16} \text{ and } f = \frac{16\sqrt{3}}{3}$$

Multiple Correct Answers Type

1. (2), (3)

$V_A = 0,\ V_B = \omega r,\ V_C = 2\omega r$

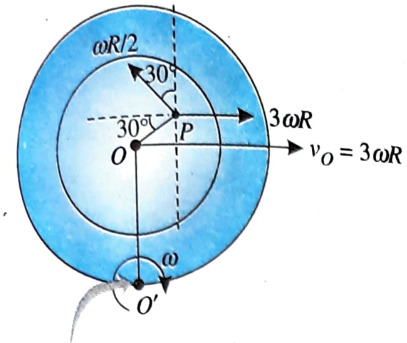
$\vec{V}_C - \vec{V}_A$ is towards the right and $\vec{V}_B - \vec{V}_C$ is towards left,

Hence, option (1) is incorrect.

But in magnitude, option (3) is correct.

$$\vec{V}_C - \vec{V}_A = 2\omega r = 2\vec{V}_B$$

2. (1), (2) As ring is rolling the point of contact of ring with ground (O') will be instantaneous center of rotation. Hence for pure rolling $v_0 = \omega(3R) = 3R\omega$ and its direction is along positive x.



Instantaneous center of rotation

The velocity at the point P

$$v_P = 3R\omega\hat{i} - \frac{R\omega}{2}\sin 30°\hat{i} + \frac{R\omega}{2}\cos 30°\hat{k}$$

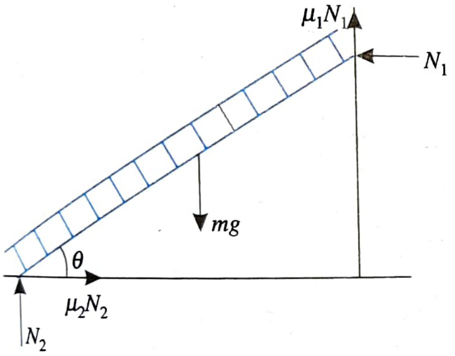
$$v_P = \frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$$

3. (3), (4)

Consider a translational equilibrium

$$N_1 = \mu_2 N_2 \qquad \text{...(i)}$$

$$N_2 + \mu_1 N_1 = Mg \qquad \text{...(ii)}$$



Solving $N, N_2 = \dfrac{mg}{1+\mu_1\mu_2}$

$$N_1 = \frac{\mu_2 mg}{1+\mu_1\mu_2}$$

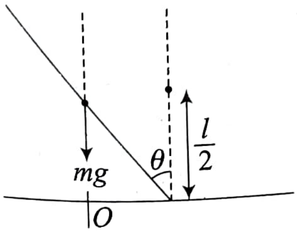
Applying torque equation about corner (left) point on the floor

$$mg\frac{l}{2}\cos\theta = N_1 l\sin\theta + \mu_1 N_1 l\cos\theta$$

Solving $\tan\theta = \dfrac{1-\mu_1\mu_2}{2\mu_2}$

4. (1), (2), (3) Torque about O, at any instant is $mg\cdot\dfrac{l}{2}\sin\theta$

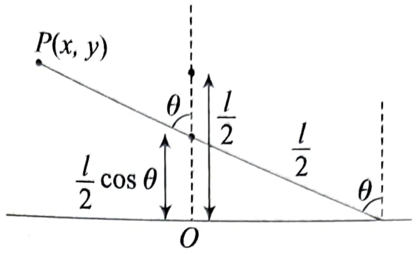
$\therefore$ Option (3) is correct.



As no external force acts along x-axis, therefore centre of mass will fall vertically downward.

$\therefore$ Option (2) is correct.

When the bar makes an angle θ; the height of its COM (mid-point) is $\dfrac{l}{2}\cos\theta$



Displacement of centre of mass along y-axis

$$= \frac{l}{2} - \frac{l}{2}\cos\theta = \frac{l}{2}(1-\cos\theta)$$

$\therefore$ Option (1) is correct.

$$x = -\frac{l}{2}\sin\theta,\ y = l\cos\theta$$

$$\Rightarrow \left(-\frac{2x}{l}\right)^2 + \frac{y^2}{l^2} = 1 \text{ or } \frac{x}{\left(\frac{l}{2}\right)}^2 + \frac{y^2}{l^2} = 1$$

$\therefore$ Trajectory is not parabola. Path of A is an ellipse.

5. (1), (3) Here we are given

$$\vec{F} = (\alpha t)\hat{i} + \beta\hat{j} \quad [\text{At } t = 0, v = 0, \vec{r} = \vec{0}]$$

Substituting, we get $\alpha = 1, \beta = 1 \Rightarrow \vec{F} = t\hat{i} + \hat{j}$

or $\quad m\dfrac{d\vec{v}}{dt} = t\hat{i} + \hat{j} \quad \Rightarrow \quad m\,d\vec{v} = (t\hat{i} + \hat{j})dt$

On integrating; $m\displaystyle\int_0^v d\vec{v} = \int_0^t t\,dt\,\hat{i} + t\,dt\,\hat{j}$

$$\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j} \qquad [\text{as } m = 1 \text{ kg}]$$

$$\frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j} \qquad [\vec{r} = \vec{0} \text{ at } t = 0]$$

$$d\vec{r} = \frac{t^2}{2}dt\hat{i} + t\,dt\,\hat{j}$$

On integrating, we get $\Rightarrow \vec{r} = \dfrac{t^3}{6}\hat{i} + \dfrac{t^2}{2}\hat{j}$

At $t = 0$ s

$$\vec{\tau} = (\vec{r}\times\vec{F}) = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right)\times(\hat{i}+\hat{j})$$

$$= \frac{1}{6}(\hat{i}\times\hat{j}) + \frac{1}{2}(\hat{j}\times\hat{i}) = \frac{1}{6}\hat{k} - \frac{1}{2}\hat{k} \quad \text{or} \quad \vec{\tau} = -\frac{1}{3}\hat{k}$$

$$\Rightarrow |\vec{\tau}| = \frac{1}{3}\text{Nm}$$

Hence option (1) is correct.

$$\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$$

At $t = 0$ s

$$\vec{v} = \left(\frac{1}{2}\hat{i} + \hat{j}\right) = \frac{1}{2}(\hat{i} + 2\hat{j})\text{m/s}$$

Hence option (3) is correct.

At $t = 1\text{s}$, $\vec{s} = \vec{r}_1 - \vec{r}_0$

$$\vec{s} = -\left[\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right] - [\vec{0}] \quad \text{or} \quad \vec{s} = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}$$

$$|\vec{s}| = \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{10}}{6}\text{m}$$

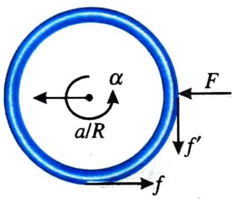
Linked Comprehension Type

1. **(4)** Angular speed about the instantaneous axis passing through centre of mass is ω for both the cases.
2. **(1)** Instantaneous axis passing through cm is vertical in both the cases.

Numerical Value Type

1. **(4)** There are two possibilities for applied force by string.
   (i) If net force applied by the rod is considered to be 2 N.
   (ii) If only normal reaction applied by the rod is considered to be 2 N.

Let us consider 1st possibility that if net force applied by the rod is considered to be 2 N.



Hence, net force applied by the rod

$$\sqrt{f'^2 + F^2} = 2 \qquad \text{...(i)}$$

Applying torque equation about point of contact

$$FR - f'R = 2mR^2\frac{a}{R}$$

$$F - f' = 2ma = 1.2 \qquad \text{...(ii)}$$

From (i) and (ii), $(1.2 + f')^2 + f'^2 = 2^2$

$$2f'^2 + 2.4f' + 1.44 = 4$$

$$f'^2 + 1.2f' + 0.72 - 2 = 0$$

$$f'^2 + 1.2f' - 1.28 = 0$$

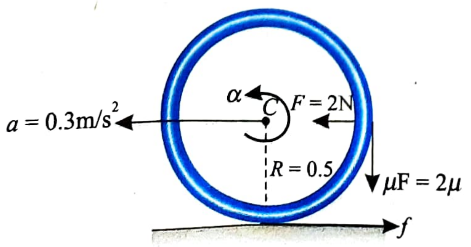
$$f' = \frac{-1.2 \pm \sqrt{1.44 + 4 \times (1.28)}}{2}$$

$$= 0.6 \pm \sqrt{0.36 + 1.28} = -0.6 \pm \sqrt{1.64} = 0.68$$

From Eq. (ii), $F = 1.88$

$$\mu = \frac{0.68}{1.88} = \frac{P}{10} \quad \Rightarrow \quad P = 3.16 \approx 4$$

Now let us consider the possibility that if only normal reaction applied by the rod is considered to be 2 N.



From FBD of ring, writing equation of motion of ring

$$2 - f = 2(0.3) \quad \Rightarrow \quad f = 2 - 0.6 = 1.4\text{ N} \qquad \text{...(i)}$$

As disc is rolling, $A = R\alpha \Rightarrow 0.3 = \alpha(0.5)$

$$\alpha = \frac{3}{5}\text{rad/s} \qquad \text{...(ii)}$$

Now torque equation, $\tau_c = I_c\alpha$

$$fR - 2\mu R = mR^2\alpha$$

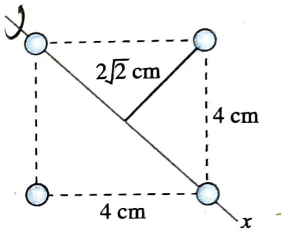
$$f - 2\mu = mR\alpha$$

$$1.4 - 2\mu = \frac{2}{2}\left(\frac{3}{2}\right)$$

$$0.8 = 2\mu \Rightarrow \mu = 0.4 = \frac{P}{10} \quad \because \quad P = 4$$

2. **(9)** The moment of inertia of the system about the diagonal of the square

$$I = \left(\frac{2}{5}MR^2\right)2 + \left(\frac{2}{5}MR^2 + Mx^2\right)2$$



$$= \left(\frac{2}{5}MR^2\right)2 + \left(\frac{2}{5}MR^2\right)2 = (Mx^2)2$$

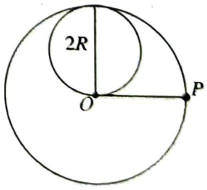
$$= 4\left(\frac{2}{5}MR^2\right) + 2Mx^2 = \frac{8}{5}MR^2 + 2mx^2$$

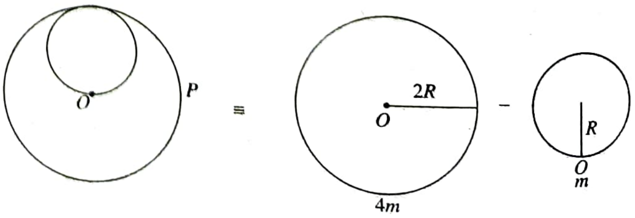
$$= \left[\frac{8}{5} \times 0.5 \times \left(\frac{\sqrt{5}}{2}\right)^2 + 2 \times (0.5) \times (4 \times 2)\right]10^{-4}$$

$$= \left[\frac{5}{5} + 8\right] \times 10^{-4} = 9 \times 10^{-4} = N \times 10^{-4}$$

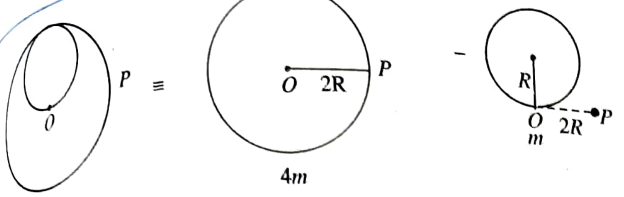
So, $N = 9$

3. **(3)**





$$I_0 = \frac{(4m)(2R)^2}{2} - \frac{3}{2}mR^2 = mR^2\left[8 - \frac{3}{2}\right] = \frac{13}{2}mR^2$$

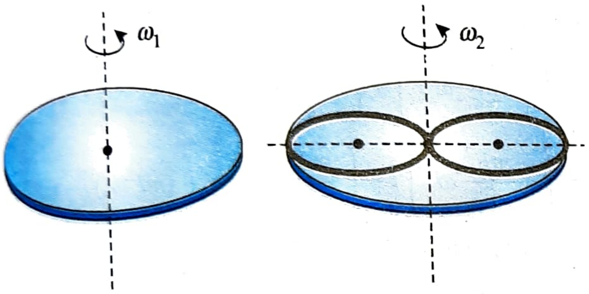


$$I_P = \frac{3}{2}(4m)(2R)^2 - \left[\frac{mR^2}{2} + m\{(2R)^2 + R^2\}\right]$$

$$= 24mR^2 - \frac{11}{2}mR^2 = \frac{37}{2}mR^2$$

$$\frac{I_P}{I_O} = \frac{\frac{37}{2}}{\frac{13}{2}} = \frac{37}{13} \approx 3$$

(8)



(a) Only disc is rotating (b) Rotation of disc with ring

Moment of inertia of the disc about its axis

$$I_1 = \frac{1}{2}MR^2 = \frac{1}{2} \times 50\text{ kg} \times (0.4\text{ m})^2 = 4\text{ kg m}^2$$

∴ Initial angular momentum of the disc is $L_1 = I_1\omega_1$

When two uniform circular rings are placed symmetrically on the disc such that they are touching each other along the axis of the disc and are horizontal as shown in Fig. (b). Then the moment of inertia of the system about the given axis

$$I_2 = \frac{1}{2}MR^2 + 2mr^2 + 2mr^2 = \frac{1}{2}MR^2 + 4mr^2$$

$$= \frac{1}{2} \times 50\text{ kg} \times (0.4\text{ m})^2 + 4 \times 6.25\text{ kg} \times (0.2\text{ m})^2$$

$$= 4\text{ kg m}^2 + 1\text{ kg m}^2 = 5\text{ kg m}^2$$

Let ω_2 be the final angular speed of the system.

Final angular momentum of the system is $L_2 = I_2\omega_2$

According to law of conservation of angular momentum, we get

$$L_1 = L_2 \quad \text{or} \quad I_1\omega_1 = I_2\omega_2$$

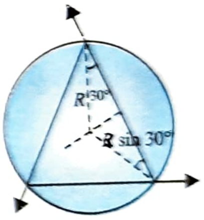
$$\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{4\text{ kg m}^2}{5\text{ kg m}^2} \times 10\text{ rad s}^{-1} = 8\text{ rad s}^{-1}$$

5. (2) $\tau_{\text{net}} = 3\tau_1 = 3Fr\sin 30° = 3(0.5)(0.5)\frac{1}{2} = \frac{3}{8}$ N-m

$$I = \frac{1.5(0.5)^2}{2} = \frac{3}{16}$$

$$\alpha = \frac{\tau}{I} = 2\text{ rad/s}^2$$

$$\omega = \alpha t = 2\text{ rad/s}$$



Chapter 4

Concept Application Exercises

Exercise 4.1

1. $\vec{L} = \vec{r} \times \vec{p}$

The magnitude of $\vec{L}$ is given by $L = rp \sin \theta(\hat{k})$
In this example, velocity of the particle at any time
$v = gt;\ r \sin \theta = x_0$ and $p = mv = mgt$

$$\therefore \quad \vec{L} = mgx_0 t(\hat{k}) \qquad \text{...(i)}$$

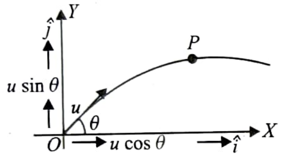
The right hand rule shows that $\vec{L}$ is parallel to torque, i.e., directed perpendicular to paper inwards.

2. Initial velocity is $\vec{u} = u(\cos \theta\hat{i} + \sin \theta\hat{j})$

$\therefore$ Velocity after time t will be $v = \vec{u} + \vec{a}t$

$$\therefore \quad \vec{v} = u(\cos \theta\hat{i} + \sin \theta\hat{j}) - gt\,\hat{j}$$
$$= u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$$

If $\vec{r}$ be the radius vector of the particle P at time t (as shown in figure), then from $\vec{s} = \vec{u}t + \vec{a}t^2/2$



$$\therefore \quad \vec{r} = ut(\cos\theta\hat{i} + \sin\theta\hat{j}) - \frac{gt^2}{2}\hat{j}$$
$$= ut\cos\theta\,\hat{i} + \left(ut\sin\theta - \frac{gt^2}{2}\right)\hat{j}$$

$\therefore$ Angular momentum $\vec{L}$ of the particle at time t will be

$$\vec{L} = m(\vec{r} \times \vec{v})$$

i.e., $$\vec{L} = m\left[ut\cos\theta\hat{i} + \left(ut\sin\theta - \frac{gt^2}{2}\right)\hat{j}\right]$$
$$\times [ut\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}]$$

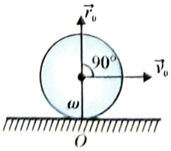
$$= \frac{-mut^2 g\cos\theta\,\hat{k}}{2}$$

3. $\vec{L} = \vec{L}_{CM} + m(\vec{r}_0 \times \vec{v}_0) \qquad \text{...(i)}$

Here, $\vec{L}_{CM} = I\omega = \left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right) = \frac{1}{2}mvR$ and $m(\vec{r}_0 \times \vec{v}_0)$
(perpendicular to the paper inwards)
Since, both the terms of right hand side of Eq. (i) are in the same direction,

$$\therefore \quad |\vec{L}| = \frac{1}{2}mvR + mvR$$

or $$|\vec{L}| = \frac{3}{2}mvR$$



4. (i) $\vec{L}_0 = \vec{L}_{cm} + \vec{r} \times m\vec{v}_{cm}$

where, $\vec{L}_{cm} = \frac{ml^2}{12}\omega(\hat{j})$

(as rotation about com with same ω)

$$\vec{r} \times m\vec{v}_{cm} = \left(a\hat{j} + \frac{l}{2}\hat{i}\right) \times \left(m\frac{l}{2}\omega(-\hat{k})\right) = \frac{mla\omega}{2}(-\hat{i}) + \frac{ml^2}{4}\omega\hat{j}$$

$$\vec{L}_0 = \frac{ml^2}{3}\omega\hat{j} + \frac{mla\omega}{2}(-\hat{i})$$

(ii) $\vec{L}_0 = \vec{r} \times m\vec{v}$ (as it is a particle)

$$\vec{L}_0 = (-l\cos\theta\,\hat{j} - l\sin\theta\,\hat{i}) \times (ml\sin\theta\,\omega\hat{k})$$
$$\vec{L}_0 = 2m\omega l^2\cos\theta\sin\theta(-\hat{i}) + ml^2\sin^2\theta\omega\hat{j}$$

5. We know, that for any body $\vec{L}_0 = \vec{L}_{cm} + \vec{r} \times m\vec{v}_{cm}$

So, we will calculate angular momentum of rod and disc about O and vectorially add them.

For rod: $$\vec{L}_0 = \frac{ml^2\omega_1}{12}(\hat{j}) + \left(\frac{l}{2}\hat{i} + R\hat{j}\right) \times m\left(-\omega_1\frac{l}{2}\hat{k}\right)$$
$$= \frac{ml^2\omega_1}{12}(\hat{j}) + \frac{ml^2\omega_1}{4}(\hat{j}) - \frac{m\omega_1 lR}{2}(\hat{i})$$
$$= \frac{ml^2}{3}\omega_1(\hat{j}) - \frac{ml}{2}R\omega_1(\hat{i}) \qquad \text{...(i)}$$

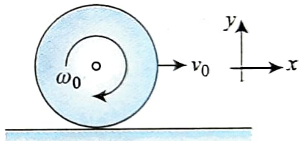
For disc:

$$\vec{L}_0 = \left(\frac{-mR^2}{2}\omega_2\hat{i}\right) + \left(\frac{mR^2}{4}\omega_1\hat{j}\right) + (l\hat{i} + R\hat{j}) \times m(-\omega_1 l\hat{k})$$
$$= \left(\frac{-mR^2}{2}\omega_2\right)\hat{i} + \left(\frac{mR^2}{4}\omega_1\right)\hat{j} + (ml^2\omega_1)\hat{j} - mRl\omega_1(\hat{i}) \qquad \text{...(ii)}$$

So total angular momentum by adding Eqs. (i) and (ii),

$$\vec{L} = \frac{4ml^2}{3}\omega_1\hat{j} - \left(\frac{mR^2}{2}\omega_2 - \frac{3}{2}m\omega_1 lR\right)\hat{i} + \frac{mR^2}{4}\omega_1\hat{j}$$

6. Since $\vec{L}_P = \vec{L}_{cm} + \vec{r} \times \vec{p}_{cm} = I_{cm}\omega_0(-\hat{k}) + Mv_0R(-\hat{k})$



Since sphere is in pure rolling motion hence $\omega = v_0/R$

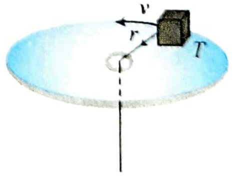
$$\Rightarrow \quad \vec{L}_P = \left[\frac{2}{5}MR^2\left(\frac{v_0}{R}\right) + Mv_0R\right](-\hat{k}) = \frac{7}{5}Mv_0R(-\hat{k})$$

Exercise 4.2

1. From conservation of angular momentum for the isolated system of two disks:

$$(I_1 + I_2)\omega_f = I_1\omega_i \quad \text{or} \quad \omega_f = \frac{I_1}{I_1 + I_2}\omega_i$$

2. The tension of the rope is the only net force on the block and it does not exert any torque about the axis of rotation. Hence, the angular momentum of the block about the axis should remain conserved.



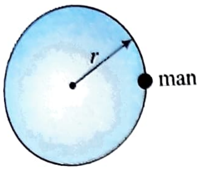
$\Rightarrow mvr = \text{constant}$
Let $r_1 = 0.5$ m and $v_1 = 4$ m/s
Let r_2, v_2, T_2 be the radius, velocity and tension, respectively, when the string breaks.
$\Rightarrow T_2 = 600$ N

$$mv_1r_1 = mv_2r_2 \text{ and } T_2 = mv_2^2/r_2$$

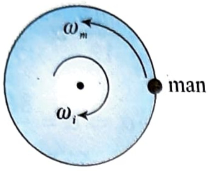
$$\Rightarrow \quad mv_1r_1 = m\sqrt{\frac{r_2T_2}{m}}r_2$$

$$r_2 = \left(\frac{m v_1^2 r_1^2}{T_2}\right)^{\frac{1}{3}} = \left(\frac{4\times 16\times 0.25}{600}\right)^{\frac{1}{3}} = \left(\frac{16}{600}\right)^{\frac{1}{3}} \approx 0.3\text{ m}$$

3. As for the man and table system, initial angular momentum is zero and it will remain zero as no external torque about rotation axis. When the man starts moving in the anticlockwise direction to make angular momentum zero, the table will start rotating in the clockwise direction.



Initially



When the man starts moving

(a) By conservation of angular momentum on the man-table system, $L_i = L_f$

$$\Rightarrow 0 + 0 = I_m\omega_m + I_t\omega_t$$

$$\Rightarrow \omega_t = -\frac{I_m\omega_m}{I_t}\text{, where } \omega_m = \frac{v}{r} = \frac{1}{2}\text{ rad/s}$$

$$= -\frac{100(2)^2\times\left(\frac{1}{2}\right)}{4000}$$

(negative sign represents clockwise direction)

$$\Rightarrow \omega_t = -\frac{1}{20}\text{ rad/s}$$

Thus, the table rotates clockwise (opposite to the man) with angular velocity 0.05 rad/s.

(b) If the man completes one revolution relative to the table. Angular displacement of the man w.r.t. table is $\theta_{mt} = 2\pi$.

$$\Rightarrow\ 2\pi = \theta_m - \theta_t \text{ or } 2\pi = \omega_m t - \omega_t t$$

(where t is the time taken)

$$t = \frac{2\pi}{\omega_m - \omega_t} = \frac{2\pi}{0.5-(-0.05)} = \frac{2\pi}{0.55}\text{ s}$$

Angular displacement of table is

$$\theta_t = \omega_t t = 0.05\times\left(\frac{2\pi}{0.55}\right) = \left(\frac{2\pi}{11}\right)\text{rad}$$

The table rotates through $(2\pi/11)$ rad clockwise.

(c) If the man completes one revolution relative to the earth, then $\theta_m = 2\pi$

$$\text{Time } t = \frac{2\pi}{\omega_m} = \frac{2\pi}{0.5}\text{ s}$$

During this time, angular displacement of the table

$$\theta_t = \omega_t(t) = 0.05\times\left(\frac{2\pi}{0.5}\right) = \frac{\pi}{5}\text{ rad} = 36°$$

(in clockwise direction)

4. There will be no torque on the system if we ignore frictional torque at the axle. Therefore, the angular momentum of the system is conserved.

$$\vec{L}_{\text{initial}} = \vec{L}_{\text{final}}$$

$$I_{\text{initial}}\,\omega_0 = I_{\text{final}}\,\omega$$

$$4ma^2\omega_0 = 4m(2a)^2\omega$$

from which we have, $\omega = \dfrac{\omega_0}{4}$

5. When the girl lands on the plank, she exerts a force on it, similarly the plank exerts an equal and opposite force on the girl. For the system consisting of girl and plank these forces are internal; therefore angular momentum is conserved.

$$\vec{L}_i = \frac{1}{2}ml\sqrt{2gh}$$

$$\vec{L}_f = \frac{1}{6}ml^2\omega + \frac{1}{4}ml^2\omega$$

$$\vec{L}_f = \vec{L}_f$$

$$\frac{1}{2}ml\sqrt{2gh} = \frac{5}{12}ml^2\omega$$

$$\Rightarrow\quad \omega = \frac{6}{5}\sqrt{\frac{2gh}{l}}$$

6. Initially the system was at rest, thus the initial angular momentum was zero. As child jumps off from the platform, it gains an angular momentum in the opposite direction. This implies that the platform must also gain the same amount of angular momentum in opposite direction.

When the child jumps off, the platform gains an angular velocity ω. Then the net velocity of child with respect to earth is $u - R\omega$. As no external torque is present, the net angular momentum of the system must be finally zero, thus

$$m(u - R\omega) = I\omega$$

$$\omega = \frac{muR}{mR^2 + I}$$

7. Again in this case no external torque is acting on the system so we can equate the angular momentum before catch and after catch. If after catching the ball, platform starts rotating with an angular speed ω, we have

Angular momentum of the ball about centre of platform before catch = Angular momentum of the platform plus child plus ball after catch

$$m_1uR = (I + mR^2 + m_1R^2)\omega$$

$$\text{or}\qquad \omega = \frac{m_1uR}{I + mR^2 + m_1R^2}$$

Here we can conserve angular momentum with respect to the centre pivot of the platform. This is because when the child catches the ball, it has a linear momentum and due to it the platform tends to gain a linear momentum, which develops a normal reaction on the pivot in opposite direction and it will not allow the platform to move translationally. If the platform was resting on a smooth plane without pivot at the centre, after catching the ball, the platform would also move translationally with its rotational motion and in that case linear momentum of system will also remain conserved.

8. As no external torque is present, we can conserve angular momentum before and after starting the walk by the child. As the child walks in opposite direction to the rotation of platform, its angular momentum will be in opposite direction which will tend to decrease the total angular momentum, hence the angular speed of the platform must increase to maintain the angular momentum conservation we have

$$(I + mR^2)\omega_1 = I\omega_2 - m(u - R\omega_2)R$$

The term on left side of the above expression is the angular momentum of platform plus child system when the child was at rest and the first term on right side of the expression is the angular momentum of the platform with increased angular speed (ω_2), when the child starts walking. The second term on right side is the final angular momentum of the child. This term is negative due to his walking in opposite direction and we have taken the velocity of the child as $(u - R\omega_2)$ because u is the relative speed of the child on platform which is rotating with angular speed ω_2 in opposite direction.

Again as we have used in the above problem, be careful, that angular momentum conservation should always be used with respect to earth or some inertial reference frame.

$$\omega_2 = \frac{(I + mR^2)\,\omega_1 + muR}{I + mR^2}$$

9. For circular motion of a body tied to a string on a horizontal plane

$$\left(\frac{mv^2}{r}\right) = T$$

Here as tension is provided by the hanging mass M, i.e., $T = g$, therefore

so, $\left(\frac{mv^2}{r}\right) = Mg$

According to the given problem,

$$\frac{(mv_1^2/r_1)}{(mv_2^2/r_2)} = \frac{M_1 g}{M_2 g} = \frac{8}{1}$$

or, $$\frac{v_1^2 r_2}{v_2^2 r_1} = \frac{8}{1} \qquad \text{...(i)}$$

Now as force T is central, so angular momentum is also conserved, i.e.,

$$mv_1 r_1 = mv_2 r_2 \qquad \text{...(ii)}$$

So substituting the value of v_1/v_2 from Eq. (ii) in Eq. (i),

$$\left[\frac{r_2}{r_1}\right]^2 \times \frac{r_2}{r_1} = \frac{8}{1}$$

i.e., $$\frac{r_2}{r_1} = 2 \qquad \text{...(iii)}$$

so that $$\frac{\Delta r}{r_1} = \frac{r_2 - r_1}{r_1} = \frac{r_2}{r_1} - 1 = 2 - 1 = 1$$

Furthermore as in circular motion, $v = r\omega$

so, $$\frac{\omega_2}{\omega_1} = \frac{v_2}{v_1} \times \frac{r_2}{r_1} = \left[\frac{r_1}{r_2}\right]^2 \qquad \left[\text{as from Eq. (ii), } \frac{v_2}{v_1} = \frac{r_1}{r_2}\right]$$

$$\Rightarrow \frac{\omega_2}{\omega_1} = \left[\frac{1}{2}\right]^2 = \frac{1}{4} \qquad \left[\text{as from Eq. (iii), } \frac{r_2}{r_1} = 2\right]$$

so, $$\frac{\Delta\omega}{\omega} = \frac{\omega_2 - \omega_1}{\omega_1} = \frac{\omega_2}{\omega_1} - 1 = \frac{1}{4} - 1 = -\frac{3}{4}$$

Exercise 4.3

1. (a) Equation of motion

$$F_1 + F_2 = ma$$

or $$2F + F = ma \Rightarrow a = \frac{3F}{m} \qquad \text{...(i)}$$

Torque equation about centre of mass,

$$2F \cdot \frac{l}{2} - F \cdot x = \left(\frac{ml^2}{12}\right)\alpha$$

$$\Rightarrow \alpha = \frac{12F(l - x)}{ml^2} \qquad \text{...(ii)}$$

(b) If point P does not acceleration

$$\vec{a}_P = \vec{a}_{P,C} + \vec{a}_C$$

As $a_P = 0 = -\alpha x + a \Rightarrow \alpha x = a$

$$\frac{12F(l - x) \cdot x}{ml^2} = \frac{3F}{m}$$

which given $x = \frac{l}{2}$

2. Force equation, $F - 2F = ma \Rightarrow a = -\frac{F}{m}$

Hence $\vec{a} = -\frac{F}{m}\hat{i}$

Torque equation about centre of mass

$$F \cdot \frac{l}{2} + \left(\frac{2}{3}l - \frac{l}{2}\right) = I_C\alpha$$

$$F \cdot \frac{l}{2} + F\frac{l}{6} = \frac{ml^2}{12} \cdot \alpha$$

$$\frac{3}{2}Fl = \frac{ml^2}{12} \cdot \alpha \Rightarrow \alpha = \frac{8F}{ml}$$

Hence $\vec{\alpha} = -\frac{8F}{ml}\hat{k}$

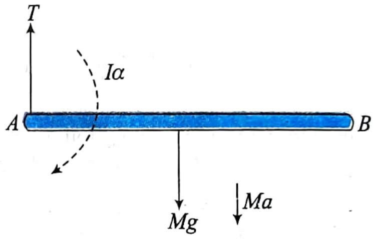
3. When right string snaps, only two forces act on the rod,

(i) Weight Mg of the rod, and

(ii) Tension T in left string

Since, both these two forces are vertical and non-collinear therefore they produce a moment and rod starts to rotate and has no horizontal acceleration. But string is inextensible, therefore left end of rod remains stationary. Hence, the rod rotates about its left end.

Let angular acceleration of rod be a clockwise. Then downward acceleration of its centre is $a = \frac{l}{2}\alpha$



Considering free body diagrams of the rod

For vertical forces, $Mg - T = Ma$

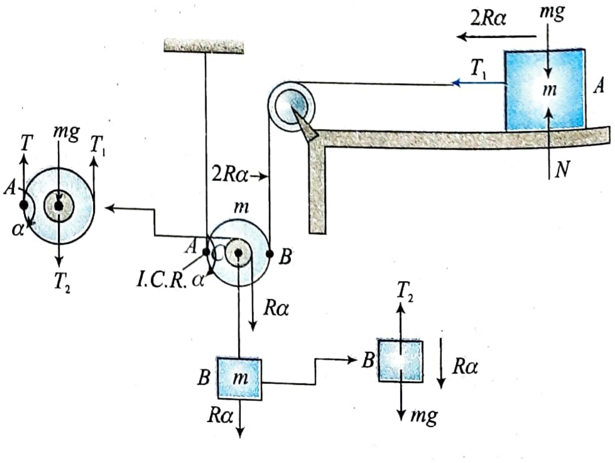
$$Mg - T = M\frac{l\alpha}{2} \qquad \text{...(i)}$$

Point A is ICR. Applying torque equation about A. $Mg\frac{l}{2} = I_A\alpha$

$$Mg\frac{l}{2} = \frac{Ml^2}{3}\alpha \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), $T = \frac{1}{4}Mg$

4. The instantaneous axis of rotation will pass through the point A as shown in figure.



Let angular acceleration of disc be α (clockwise), then downward acceleration of its centre and block B will be equal to $a = R\alpha$ where R is the radius of the disc and acceleration of upper block A will be equal to the acceleration of point B, $b = 2R\alpha$ (leftward).

FBD of different parts will be as shown in figure.

For horizontal forces on A, $T_1 = m(2R\alpha)$ ...(i)

For vertical forces on B, $mg - T_2 = m(R\alpha)$

or $\quad T_2 = mg - mR\alpha$ ...(ii)

Now taking moment of forces acting on the pulley about A,

$$(mg + T_2)\cdot R - T_1\cdot(2R) = I\alpha \quad \text{...(iii)}$$

where, $I = \frac{3}{2}mR^2$

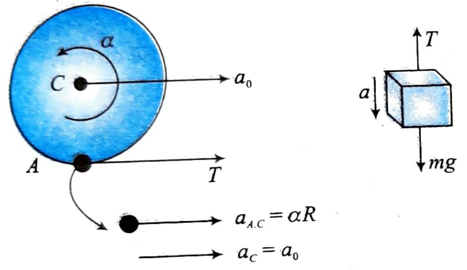
Substituting, $T_1 = 2mR\alpha$ and $T_2 = (mg - mR\alpha)$, we get

$$R\alpha = \frac{4g}{13}$$

Hence, acceleration of block B is equal to $\frac{4g}{13}$

Acceleration of block A is equal to, $b = 2R\alpha$ or $b = \frac{8g}{13}$

5. Let a = acceleration of mass m, a_0 = acceleration of COM of the disc, α = angular acceleration of the disc about its COM



$$\vec{a}_A = \vec{a}_{A,C} + \vec{a}_C \Rightarrow a = \alpha R + a_0$$

Acceleration of point A is: $a_A = a_0 + R\alpha$

But a_A must be equal to a

$\therefore \quad a = a_0 + R.\alpha$ ...(i)

For motion of 'm': $mg - T = ma$ ...(ii)

For translation of disc: $T = Ma_0$ ...(iii)

For rotation of disc: $TR = \frac{1}{2}MR^2\alpha$

or $\quad T = \frac{1}{2}MR\alpha$ ...(iv)

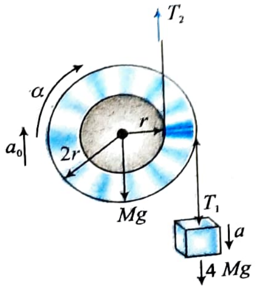
From above equations we get

$$a = \frac{3mg}{3m + M} \quad \text{and} \quad a_0 = \frac{mg}{3m + M}$$

6. Let α = angular acceleration of the pulley

a_0 = upward acceleration of the centre of the pulley

a = Acceleration of the block in downward direction.



The constraint relations are $a_0 = r\alpha$

and $\quad a = R\alpha - a_0 = 2r\alpha - r\alpha = r\alpha$

For motion of the block:

$$4Mg - T_1 = 4M(r\alpha) \quad \text{...(i)}$$

For translation of pulley:

$$T_2 - T_1 - Mg = Ma_0 \quad \text{...(ii)}$$

For rotation of the pulley:

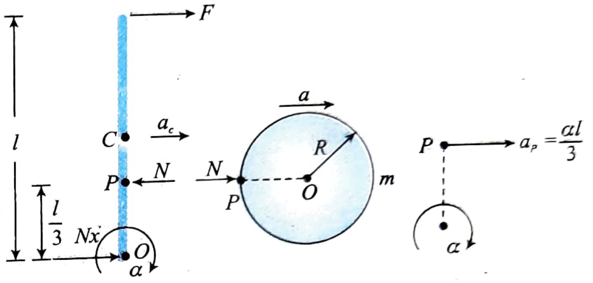
$$T_1(2r) - T_2(r) = I\alpha \quad \text{...(iii)}$$

Adding Eqs. (i), (ii) and (iii)

$$3Mg = \left(4 + 1 + \frac{1}{2}\right)Ma_0 \quad [\because r\alpha = a_0]$$

$$\therefore \quad a_0 = \frac{6}{11}g$$

7. The following figure demonstrates the given condition.



Torque equation for rod about O,

$$I_0\alpha = Fl - N\frac{l}{3}$$

$$\therefore \quad \frac{ml^2}{3}\cdot\alpha = Fl - \frac{Nl}{3} \quad \text{or} \quad m\frac{l\alpha}{3} = F - \frac{N}{3} \quad \text{...(i)}$$

The motion of the sphere is translational only

acceleration of the sphere (a) = acceleration of point P

$$\frac{l\alpha}{3} = a \quad \text{...(ii)}$$

From (i) and (ii) $ma = F - \frac{N}{3}$ ...(iii)

For sphere, $N = ma$ ...(iv)

Solving (iii) and (iv) we get, $a = \frac{3F}{4m}$ and $N = \frac{3F}{4}$

Acceleration of the COM of the rod is

$$a_{\text{cm}} = \alpha\frac{l}{2} = \frac{3a}{2} = \frac{9F}{8m}$$

Let horizontal component of hinge reaction is N_x

Force equation for rod

$$F - \frac{3F}{4} + N_x = ma_{\text{cm}} \Rightarrow F - \frac{3F}{4} + N_x = m\left(\frac{9F}{8m}\right)$$

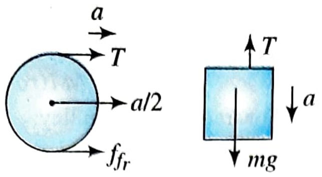
$$N_x = \frac{9F}{8} - \frac{F}{4} = \frac{7F}{8}$$

Exercise 4.4

1. Zero. Since lowest point of the sphere is stationary with respect to the plank, therefore friction force acting on the plank is zero. Thus, no force is required to keep the plank stationary.
2. The cylinder will roll without slipping so long as the friction force is sufficiently large and it never reaches the limiting value. The forces acting on the cylinder are: (a) its weight Mg, (b) normal reaction N, (c) tension T to the right, (d) f_{fr} to the right.

Let a be the acceleration of the falling mass m.

Then $a_{CM} = \dfrac{a}{2}$



For the load,

$$mg - T = ma$$

$$\Rightarrow \quad T = mg - ma$$

For the cylinder

$$T + f_{fr} = \frac{Ma}{2}$$

and $\quad \tau = TR - f_{fr}R = \left(\tfrac{1}{2}MR^2\right)\alpha$

$$T - f_{fr} = \tfrac{1}{2}MR\alpha = \tfrac{1}{2}MR\left(\frac{a_{CM}}{R}\right) = \tfrac{1}{2}\,Ma_{CM} = \tfrac{1}{4}Ma$$

Solving $f_{fr} = \frac{1}{8}Ma$

$$T = \tfrac{3}{8}Ma \text{ and } a = \frac{mg}{m + \frac{3}{8}M}$$

There will be no slip if

$$f_{fr} \le \mu mg \Rightarrow \mu \ge \frac{Ma}{8mg}$$

or $\quad \mu \ge \dfrac{M}{8m + 3M}$

Rolling + slipping: Now $a_{CM} \ne \alpha R$, rather acceleration of the highest point of the cylinder = $a_{CM} + \alpha R$ = acceleration of m.

and $\quad \tau = TR - f_{\text{lim}}R = \left(\tfrac{1}{2}MR^2\right)\alpha$

$$T - \mu Mg = \tfrac{1}{2}MR\alpha$$

For load: $mg - T = ma$

Solving for α,

$$a = \frac{3mg - \mu Mg}{M + 3m}$$

3. Since the cylinder does not slip, the displacement, velocity and acceleration of the hanging mass are, respectively, equal to the displacement, velocity and acceleration of the upper point B of the cylinder as the string leaves the upper point tangentially.

Now v (velocity of the upper point) = $v_m + \omega r = 2v_{CM}$ $(\therefore \omega r = v_m)$

From conservation of energy, when m lowers through h

$$mgh = \tfrac{1}{2}mv^2 + \left(\tfrac{1}{2}mv_{CM}^2 + \tfrac{1}{2}I\omega^2\right)$$

$(\therefore$ loss in PE = gain in KE)

$$= \tfrac{1}{2}mv^2 + \frac{1}{2}\frac{Mv^2}{4} + \frac{1}{2}\left(\tfrac{1}{2}Mr^2\right)\frac{v_{CM}^2}{r^2} \qquad \left(\because I = \tfrac{1}{2}Mr^2\right)$$

$$= \tfrac{1}{2}mv^2 + \tfrac{1}{8}Mv^2 + \tfrac{1}{4}M\frac{v^2}{4}$$

$$= \tfrac{1}{2}mv^2 + \tfrac{3}{16}Mv^2$$

$$16mgh = 8mv^2 + 3Mv^2$$

$$\Rightarrow \quad v^2 = \frac{16mgh}{8m + 3M}$$

From $\quad v^2 = 2ah$

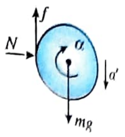
$$\Rightarrow \quad a = \frac{8mg}{8m + 3M} = \frac{g}{1 + 3M/8m} = 4 \text{ m/s}^2$$

Considering downward motion of m,

$$mg - T = ma$$

or $\quad T = mg - ma = 2(10 - 4) = 12$ N

4. Let sphere rolls down with acceleration a', writing force and torque equations for sphere



$$N = \frac{mg}{2} \qquad \text{...(i)}$$

$$mg - f = ma' \qquad \text{...(ii)}$$

$$fR = \left(\frac{2}{5}mR^2\right)\cdot\alpha$$

$$f = \frac{2}{5}mR\alpha \qquad \text{...(iii)}$$

$$\alpha R = a' \qquad \text{...(iv)}$$

$$\Rightarrow \quad f = \frac{2}{5}ma' \text{ or } a' = \frac{5}{2}\frac{f}{m} \qquad \text{...(v)}$$

From (ii) and (v),

$$mg - f = \frac{5}{2}f \Rightarrow mg = \frac{7}{2}f \text{ or } f = \frac{2}{7}mg$$

But, $\quad f \le \mu N$

$$\frac{2}{7}mg \le \frac{\mu mg}{2} \text{ or } \mu \ge \frac{4}{7}$$

5. Since there is no slipping, the velocity of the point of contact of the string with the pulley is the vector sum of the velocity of the point of contact due to rotation plus the velocity of the centre of mass of the spool. That is

$$v_B = v_{CM} - \omega r, \ \omega \text{ is in clockwise direction}$$

But $\quad v_{CM} = \omega R$

$$v_B = v_{CM} - v_{CM}\frac{r}{R} = v_{CM}\left(\frac{R - r}{R}\right)$$

This is also the velocity (v) of the rolling mass.

$$v_B = v_{CM}\left(\frac{R - r}{R}\right) = v\left(\frac{R - r}{R}\right)$$

Considering the conservation of energy when m lowers by h, we get

$$mgh = \tfrac{1}{2}mv^2 + \left(\tfrac{1}{2}Mv_{CM}^2 + \tfrac{1}{2}I\omega^2\right)$$

$$= \tfrac{1}{2}mv^2 + \left(\tfrac{1}{2}Mv_{CM}^2 + \tfrac{1}{2} \times \tfrac{1}{2}MR^2\,\frac{v_{CM}^2}{R^2}\right)$$

$$= \tfrac{1}{2}mv^2 + \tfrac{3}{4}Mv_{CM}^2$$

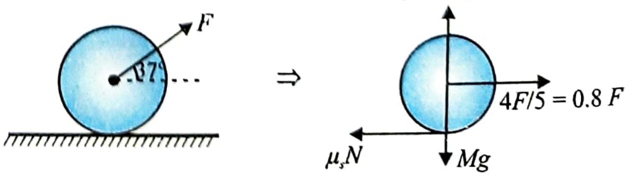
$$= \tfrac{1}{2}\,mv^2 + \tfrac{3}{4}Mv^2\left(\frac{R}{R - r}\right)$$

$$v^2 = \frac{4mgh}{2m + 3M[R/(R - r)]^2}$$

comparing with $v^2 = 2ah$

$$a = \frac{v^2}{2h} = \frac{2mg}{2m + 3M[R/(R - r)]^2}$$

6. FBD is shown in figure



$$N = Mg - \frac{3F}{5} = Mg - 0.6F$$

For pure rolling, $a = R\alpha$

or $\quad \dfrac{0.8F - \mu_s N}{M} = \dfrac{(R)(\mu_s N)(R)}{MR^2/2} = \dfrac{2\mu_s N}{M}$

$\therefore \quad 0.8\,F = 3\,(0.4)\,(Mg - 0.6\,F)$

or $\quad F = 0.79\,Mg$

Therefore, maximum value of F is 0.79 Mg.

7. The force acting on the cylinder are (a) mg, downwards, (b) F, applied force downwards, (c) N, reaction of each plank upwards, (d) f_r frictional force (not limiting) between each plank and the cylinder to the right.

Considering vertical motion

$$F + mg = 2N \quad \text{...(i)}$$

Considering horizontal motion

$$2f_r = ma \quad \text{...(ii)}$$

where a is the horizontal acceleration of the centre of mass of the cylinder to the right. Considering rotational dynamics of the cylinder about its axis

$$\tau = FR - 2f_rR = \left(\tfrac{1}{2}mR^2\right) \times \alpha$$

$$(F - 2f_r)R = \frac{1}{2}mR^2 \times \frac{a}{R} \qquad (\because \alpha = \alpha R)$$

$$F - 2f_r = \frac{1}{2}ma$$

$$F = \frac{1}{2}ma + 2f_r = \frac{3}{2}ma \quad \text{...(iii)}$$

$[(\therefore 2f_r = ma$ by Eq. (ii)$]$

$$F = 3f_r \quad \text{...(iv)}$$

Now $f_r \le f_{\text{lim}}$ or $f_r \le \mu N \quad (\therefore f_{\text{lim}} = \mu N)$

$$\frac{1}{3}F \le \mu\frac{F + mg}{2}$$

$$2F \le 3\mu F + 3\mu mg$$

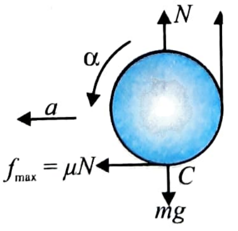
$$F \le \frac{3\mu mg}{2 - 3\mu}$$

$$F_{\text{max}} = \frac{3\mu mg}{2 - 3\mu}$$

When the force is maximum, acceleration is automatically maximum.

$$a_{\text{max}} = \frac{2F_{\text{max}}}{3m} = \frac{2\mu g}{2 - 3\mu}$$

8. As the force F is applied, the cylinder tends to rotate along anticlockwise direction with the point of contact 'C' tending to move to the right. Hence, friction acts along the left (as shown in figure). If α is the angular acceleration of the cylinder, then from $\tau = I\alpha$, we have



$$r(F - f) = (mr^2/2)\alpha \quad \text{...(i)}$$

If a is the linear acceleration of the cylinder's CM, then from $F = ma$, we have

$$f = ma \quad \text{...(ii)}$$

For pure rolling,

$$a = \alpha r \quad \text{...(iii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{(F - f)}{f} = \frac{r\alpha}{2a} = \frac{1}{2}$$

$\Rightarrow \quad F = (3f/2) \Rightarrow F_{\text{max}} = (3/2)f_{\text{max}}$

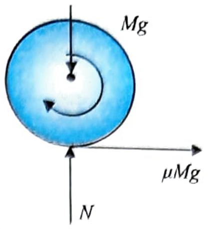
$[f_{\text{max}} = \mu N = \mu(mg - F)]$

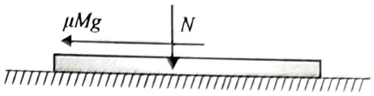
$$\Rightarrow \quad F_{\text{max}} = \frac{3\mu}{2}[mg - F_{\text{max}}] = \frac{3\mu mg}{2 + 3\mu}$$

From Eq. (ii), $a = \dfrac{f}{m}$

$$\Rightarrow \quad a_{\text{max}} = \frac{f_{\text{max}}}{m} = \frac{2}{3m}F_{\text{max}} = \frac{2}{3m}\left(\frac{3\mu mg}{2 + 3\mu}\right) = \frac{2\mu g}{2 + 3\mu}$$

9. Step 1: Free-body diagram,





Step 2: Equation of motion,

Plank: $\mu Mg = MA \Rightarrow A = \mu g$ (backward)

(A will be positive)

Sphere: $\mu Mg = Ma \Rightarrow a = \mu g$ (forward)

(a will be positive)

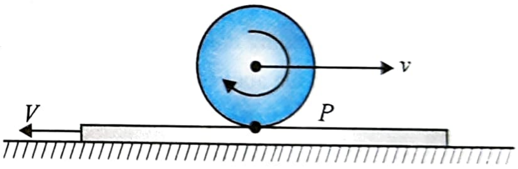
Step 3: Torque equation,

$$\mu Mg\,R = \left(\tfrac{2}{5}MR^2\right)\alpha$$

$$\alpha = \frac{5}{2}\frac{\mu g}{R} \quad \text{(anticlockwise)}$$

(it will be negative)

Step 4: Constraint relation



$$V_P = V = \omega R - v$$

$$(0 + \mu g\,t) = \left(\omega_0 - \frac{5}{2}\frac{\mu g}{R}t\right)R - (0 + \mu g\,t)$$

$$\left(2\mu g\,t + \frac{5}{2}\mu g\,t\right) = \omega_0 R \Rightarrow \frac{9\mu g t}{2} = \omega_0 R$$

$$\Rightarrow \quad t = \frac{2\omega_0 R}{9\mu g}$$

Velocity of the plank when rolling starts,

$$V = 0 + \mu g\,t$$

$$V_{\text{plank}} = \mu g \times \frac{2\omega_0 R}{9\mu g} = \frac{2}{9}\omega_0 R$$

Velocity of the sphere when rolling starts,

$$V_{\text{sphere}} = 0 + \mu g\,t = \frac{2}{9}\omega_0 R$$

Exercise 4.5

1. (a) Kinetic energy of rotation, $K_{\text{rotational}} = \dfrac{1}{2}I_C\,\omega^2$

$$= \frac{1}{2}\left(\frac{ml^2}{12}\right)\left(\sqrt{\frac{g}{l}}\right)^2 = \frac{1}{24}mgl$$

(b) Kinetic energy of translation, $K_{\text{translational}} = \dfrac{1}{2}mv_c^2$

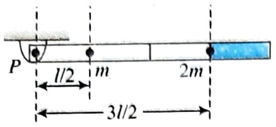
$$= \frac{1}{2}m\left[\left(\omega\frac{l}{2}\right)^2\right] = \frac{1}{2}m\left[\sqrt{\frac{g}{l}} \times \frac{l}{2}\right]^2 = \frac{1}{8}mgl$$

(c) Total kinetic energy,

$$K_{\text{total}} = K_{\text{translational}} + K_{\text{rotational}}$$

$$= \frac{1}{6}mgl = \frac{1}{2}I_P\omega^2 = \frac{1}{2}\left[\frac{ml^2}{3}\right]\left[\sqrt{\frac{g}{l}}\right]^2 = \frac{1}{6}mgl$$

2. Moment of inertia of composite rod about P



$$I_P = \frac{ml^2}{3} + \frac{(2m)l^2}{12} + 2m\left(\frac{3}{2}l\right)^2 = 5ml^2$$

Total kinetic energy, $K_{\text{total}} = \frac{1}{2}I_P\omega^2$

$$= \frac{1}{2}(5ml^2)\omega^2 = \frac{5}{2}ml^2\omega^2 = 250 \text{ J}$$

3. $\Delta K + \Delta U = 0$

$$\left[\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 - 0\right] + \left[-mg\frac{l}{2}\right] = 0 \Rightarrow \omega = \sqrt{\frac{3g}{l}}$$

Velocity of point P, $V_P = \omega l = \sqrt{3gl}$

4. Using principle of conservation of mechanical energy, we have

$$\frac{mgl}{2} + mgl = \frac{1}{2}I\omega^2$$

where ω is the angular speed of the rod when it becomes vertical, and $I = (ml^2/3) + ml^2$, assuming mass of each rod is m.

Substituting value of I in the above equation, we get

$$\omega = \sqrt{\frac{9g}{4l}}$$

5. Using conservation of energy principle, we have fall in PE of the block = Gain in KE of the block + Rotational KE of the pulley + Rotational KE of the shell.

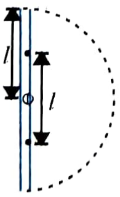
or $$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}\left(\frac{2MR^2}{3}\right)\omega'^2$$

where $\omega = \frac{v}{r}$ and $\omega' = \frac{v}{R}$

After substituting these values in the above equation and solving, we get

$$v^2 = \frac{mgh}{\left(\frac{m}{2} + \frac{I}{2r^2} + \frac{M}{3}\right)}$$

6. As the rod reaches its lowest position, the centre of mass is lowered by a distance l. Its gravitational potential energy is decreased by mg. As no energy is lost against friction, this should be equal to the increase in the kinetic energy. As the rotation occurs about the horizontal axis through the clamped end, the moment of inertia is $I = ml^2/3$. Thus,



$$\frac{1}{2}I\omega^2 = mg\,\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 = mg$$

or $$\omega = \sqrt{\frac{6g}{l}}$$

The linear speed of the free end is $v = l\omega = \sqrt{6gl}$

7. $\Delta K + \Delta U = 0$

$$\left(\frac{1}{2}I_0\omega^2 - 0\right) + \left[-mg\frac{l}{4}\right] = 0 \quad \text{...(i)}$$

$$I_0 = \frac{ml^2}{12} + m\left(\frac{l}{4}\right)^2 = \frac{7}{48}ml^2 \quad \text{...(ii)}$$

From (i) and (ii), $\omega = 2\sqrt{\frac{6g}{7l}}$

8. The conservation of energy yields

$$\Delta KE = \Delta PE$$

$$\frac{1}{2}I\omega^2 = mg(2r) + mgr$$

$$\Rightarrow \frac{1}{2}m(2r)^2\omega^2 + \frac{1}{2}\left\{\left(\frac{mr^2}{2}\right) + mr^2\right\}\omega^2 = 3mgr$$

$$\Rightarrow \frac{11}{4}mr^2\omega^2 \Rightarrow 3mgr \Rightarrow \omega = \sqrt{\frac{12g}{11r}}$$

9. $\text{KE} = \frac{1}{2}\left(\frac{2}{5}\right)mv^2 \Rightarrow \text{KE} = \frac{2}{10}mv^2$

If ω is to be doubled, v is also doubled.

Therefore, $\Delta\text{KE} = \text{KE}_2 - \text{KE}_1 = \frac{2}{10}m\left[v_2^2 - v_1^2\right]$

$$\Rightarrow \Delta\text{KE} = \frac{2}{10}m\left[r^2\omega_2^2 - r^2\omega_1^2\right]$$

$$= \frac{2}{10} \times 1\left[\left(\frac{1}{10}\right)^2\right]\left[4^2 - 2^2\right] = 2.4 \times 10^{-2} \text{ J}$$

10. Rolling can be considered as pure rotation about point of contact

$$K_{\text{rolling}} = \frac{1}{2}I_P\omega^2$$

$$= \frac{1}{2}\left(\frac{mR^2}{2} + mR^2\right)\omega^2 = \frac{3}{4}mR^2\omega^2 \quad \text{...(i)}$$

The rod translates with the velocity v hence velocity of centre of disc will also be v

$$v = \omega R \quad \text{...(ii)}$$

Form (i) and (ii), $K_{\text{rolling}} = \frac{3}{4}mv^2$

Kinetic energy of the rod, $K_{\text{rod}} = \frac{1}{2}\left(\frac{m}{2}\right)v^2 = \frac{mv^2}{4}$ ...(iii)

$$K_{\text{total}} = K_{\text{rolling}} + K_{\text{rod}} = mv^2$$

11. In the process, angular momentum remains constant

$$I_1\omega_1 = I_2\omega_2$$

where $I_1 = 2m\,(0.15)^2$, $\omega_1 = 4$ rad/s

$$I_2 = 2m\,(0.05)^2$$

Substituting these values in above equation, we get

$$\omega_2 = \frac{I_1\omega_1}{I_2} = 36 \text{ rad/s}$$

Work done on the collar, $W = \frac{1}{2}I_2\,\omega_2^2 - \frac{1}{2}I_1\omega_1^2$

or $W = \frac{1}{2}(0.05)^2 \times 36^2 - \frac{1}{2}2\,m\,(0.015)^2 \times 4^2 = 1.44$ J

where $m = \frac{500}{1000} = 0.05$ kg

Exercise 4.6

1. (d) Since net force along the incline is zero, so cylinder will remain in position till it stops rotating. After that it will start moving downwards.

2. (a) Since for $a = g \tan \theta$, no force acts along the incline, so it will continue its pure rolling.

3. (a) If $v_0 > R\omega_0$, the sliding tendency of lowest point of the sphere will be in upward direction. Hence friction force will act downwards

If $v_0 < R\omega_0$, the sliding tendency of lowest point of the sphere will be in downward direction. Hence friction force will act upwards

If $v_0 = R\omega$, in this case the sphere is rolling up hence friction force will act upwards

4. (d) Since energy is conserved for all the three bodies, so, final kinetic energy of all the three bodies will be same.

5. (a) $v = \sqrt{\dfrac{2gh}{1+c}}$ where $c = \dfrac{k^2}{R^2}$

c for ring = 1

c for disc $= \dfrac{1}{2} = 0.5$

c for sphere $\dfrac{2}{5} = 0.4$

Hence, v (sphere) > v (disc) > v (ring)

6. (b) We know $a = \dfrac{g\sin\theta}{1+c}$

Since a (sphere) > a (disc) > a (ring)

So, t (sphere) < t (disc) < t (ring)

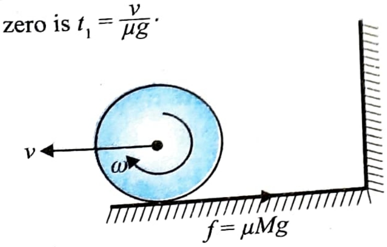
Higher the acceleration smaller will be the time taken.

Exercise 4.7

1. (a) True: Collision is elastic and the frictional impulse is not generated.
   (b) True: The net impulse passes through the point of contact with the wall. Therefore, angular impulse about the point of contact with the wall is zero and hence the angular momentum is conserved.
   (c) False: Angular momentum is not conserved because the net impulse exerts torque about a stationary point on the floor.
   (d) False: The point of contact with the horizontal surface moves away from the wall.
   (e) False: Friction force decreases both the speeds.
   (f) True: When the sphere starts rolling, it is moving away from the wall because the angular velocity becomes zero before the linear velocity. The linear acceleration of the CM of the sphere is $a = -\mu g$.

The angular acceleration about the CM is $\alpha = \dfrac{-5}{2}\dfrac{\mu g}{R}$.

The negative sign shows that both the accelerations have the retarding effect. The time after which the linear velocity will become zero is $t_1 = \dfrac{v}{\mu g}$.



and the time after which the angular velocity will become zero is

$$t_2 = \frac{2}{5}\frac{\omega R}{\mu g} = \frac{2}{5}\frac{v}{\mu gh} \qquad (\therefore v = \omega R)$$

Note that $t_2 < t_1$.

2. (a) In the process of collision, the linear momentum remains constant

$$2mv - m(2v) + 0 = (2m + m + 8m)\, v_{CM}$$

or $v_{CM} = 0$

(b) Angular momentum of the system also remains constant. Therefore,

$$2mva + m(2v)(2a) + 0 = I\omega \qquad \text{...(i)}$$

where I is the moment of inertia of the system after collision.

I = MI of the rod + MI of the mass $2m$ + MI of the mass m

or $I = \dfrac{(8m)(6a)^2}{12} + 2ma^2 + m(2a)^2$

Substituting this value in Eq. (i), we get $\omega = \dfrac{v}{5a}$

(c) Total energy after collision

$$E = \frac{1}{2}I\omega^2 = \frac{1}{2}(30m)a^2 \times \left(\frac{v}{5a}\right)^2$$

or $E = \dfrac{3mv^2}{5}$

3. Conservation of linear momentum

$$mv_0 = M\frac{v_0}{8} - m\frac{v_0}{4}$$

$$8\,m = M - 2m \Rightarrow 10\,m = M$$

$\Rightarrow$ $M/m = 10$

Conservation of angular momentum about the point of collision

$$0 = \frac{M(8l)^2}{12}\omega - M\frac{v_0}{8l}$$

$$\Rightarrow \quad \omega = \frac{3v_0}{128l}$$

Using coefficient of restitution equation

$$\left[\left(\frac{v_0}{8} + \omega l\right) - \left(-\frac{v_0}{4}\right)\right] = e(v_0 - 0) \Rightarrow e = \frac{51}{128}$$

Velocity at end A

$$v_A = \frac{v_0}{8} + \omega.4l = \frac{28}{128}v_0$$

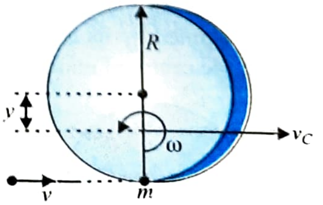
$$\Rightarrow \quad v_A = \frac{7}{32}v_0$$

Velocity at end B

$$v_B = \frac{v_0}{8} + \omega.4l = \frac{4}{128}v_0$$

$$\Rightarrow \quad v_B = \frac{1}{32}v_0$$

4. Let the velocity of the CM of the system after the strike be v_c. By conservation of linear momentum,



$$mv = 2mv_c \Rightarrow v_c = \frac{v}{2}$$

Position of CM, $y = \dfrac{m.0 + m.R}{(m+m)} + m = \dfrac{R}{2}$

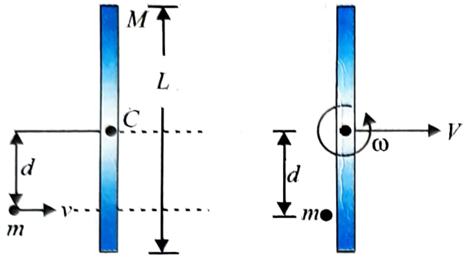
Using conservation of angular momentum about CM of the system (hoop + bullet),

$$mv\frac{R}{2} = I_c\omega = (I_{\text{bullet}} + I_{\text{hoop}})_c\,\omega$$

$$= \left[m\left(\frac{R}{2}\right)^2 + \left\{mR^2 + m\left(\frac{R}{2}\right)^2\right\}\omega\right]$$

After solving, we get $\omega = \frac{v}{R}$

5.



By conservation of linear momentum and angular momentum, we have

$$mv + 0 = m.0 + MV \Rightarrow = V = \frac{mv}{M}$$

and $mvd = I\omega \Rightarrow \omega = \frac{mvd}{I} = \frac{mvd}{\left(\frac{ML^2}{12}\right)}$

Since collision is elastic, therefore

$$\frac{1}{2}mv^2 = \frac{1}{2}M\left(\frac{mv}{M}\right)^2 + \frac{1}{2}\left(\frac{ML^2}{12}\right)\left(\frac{mvd}{\frac{mL^2}{12}}\right)^2$$

$$\Rightarrow\ m = \frac{ML^2}{L^2 + 12d^2}$$

6. Let Δt be the duration of impact, then

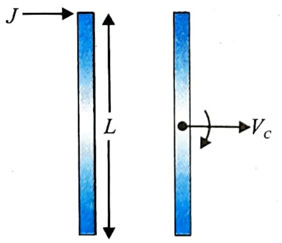
$$J = \Delta\vec{P}$$

or $J = (Mv_c - 0)$ ...(i)

Also angular impulse is equal to change in angular momentum

or $\frac{Jl}{2} = I\Delta\omega$

$= I(\omega - 0) = \frac{ml^2}{12}\omega$ ...(ii)



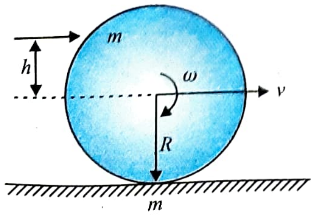
Dividing Eq. (i) by Eq. (ii), we get $\omega = \frac{6v_0}{l}$

Let t be the time in which the rod rotates at an angle 2π, then

$$\omega t = 2\pi\ \Rightarrow\ t = \frac{\pi L}{3v_0}$$

Hence, distance travelled by centre of mass $\Rightarrow x_c = \frac{l\pi}{3}$

7. Let v_z be the velocity attained by its centre of mass and ω be the angular velocity about the centre of mass by the impact of the cue.



Suppose J is the impulse provided by the cue

$J = m(v - 0)$ ...(i)

and $J.h = I(\omega - 0)$ ...(ii)

From Eqs. (i) and (ii), $Mv \times h = I\omega$

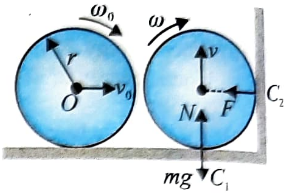
For pure rolling, $\omega = \frac{v}{R}$

$\therefore\ mv \times h = \frac{2}{3}mR^2 \times \frac{v}{R}$

$\Rightarrow\ h = \frac{2R}{3}$

8. Figure shows the two situations of a rolling wheel before and after its striking the wall.

If C_2 is the point on the wall where the wheel strikes, then the impulsive force F passes through it. Moreover, the weight of the wheel mg and the normal reaction offered by the horizontal surface on the wheel to balance each other. So, the angular momentum of the wheel remains conserved about point C_2.



$\therefore$ Initial angular momentum = final angular momentum

$\therefore\ I\omega_0 = I\omega + mvr$

[where ω and v are the angular and linear speeds of the wheel, just after its striking]

Since slippage is prevented, so $v = \omega r$

$\therefore\ I\omega_0 = I\omega + m\omega r^2$

$$\Rightarrow\ \frac{mr^2\omega_0}{2} = \frac{mr^2\omega}{2} + m\omega r^2$$

$$\Rightarrow\ \omega = \frac{\omega_0}{3}$$

Exercises

Single Correct Answer Type

1. **(2)** $E_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}(I_1\omega_1)\omega_1 = \frac{1}{2}k\omega_1$

Now $I_1\omega_1 = I_2\omega_2 = k$ (say)

$E_2 = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}(I_2\omega_2)\omega_2 = \frac{1}{2}k\omega_2$

$I_1 > I_2$ and $\omega_2 > \omega_1$

so, $E_2 > E_1$

2. **(4)** $\vec{L} = I_{CM}\vec{\omega} + m\vec{r} \times \vec{v}_{CM}$

$= \frac{MR^2}{2}\omega(-\hat{k}) + M\frac{3}{2}R(-\hat{j}) \times v_0\hat{i}$

$= MRv_0\hat{k}$

3. **(1)** F will provide anticlockwise torque about the centre due to which the bottommost point will tend to move towards right, so friction will act towards left. So it will move towards left.

4. **(2)** As there is no friction, net force is F which is towards right. Torque due to this force will be in anticlockwise direction.

5. **(2)** Friction force will act towards right.

6. **(1)** For a body to roll without slipping on an incline of angle θ:

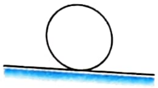
$$\mu \geq \frac{\tan\theta}{1 + \frac{mR^2}{I}}$$

I is minimum for the sphere, hence μ required is least for the sphere. Here it is given that μ is just sufficient to roll the sphere purely. It means the ring and the cylinder will not roll purely, so kinetic friction will act on them. Ultimately, friction acting on each of the three will be $\mu mg\cos\theta$ and each of them will have the same acceleration. Hence, all of them reach at the same time.

7. **(3)** In first case, the impulse on the platform will act radially inward and it will not produce any angular impulse.

In second case, when man steps into platform, moment of inertia of system increases, so rotational speed decreases.

8. **(4)** Since $v_{\text{point of contact}} = 0$, no friction acts between ball and plank.



No impulse acts on plank or ball.

9. **(2)** Since all the forces pass through point of contact, so angular momentum remains conserved.

10. **(4)** $$\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2} = \frac{40}{60} = \frac{2}{3} \text{ or } \frac{\frac{Iv^2}{R^2}}{Mv^2} = \frac{2}{3} \text{ or } I = \frac{2}{3}MR^2$$

Clearly, the body is a hollow sphere.

11. **(4)** $$\frac{MgL}{2}\cos\theta = \frac{ML^2}{3}\alpha$$

$$\alpha = \frac{3}{2}\frac{g\cos\theta}{L}$$

$$a = \alpha L = \frac{3}{2}g\cos\theta$$

12. **(4)** $$\frac{mv^2}{2} + \frac{I\omega^2}{2} = mg \times \frac{3v^2}{4g} \Rightarrow I = \frac{mR^2}{2}$$

For pure rolling condition, $v = R\omega$.

13. **(2)** In both the cases, the loss of gravitational potential energy and the resulting gain 'total kinetic energy' is same.

14. **(2)** Equation of motion is

$$Mg - T = Ma \quad \text{...(i)}$$

Taking torque about the axis passing through the centre of the spool and perpendicular to it,

$$TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$$

$$T = \frac{1}{2}Ma \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$Ma = Mg - \frac{1}{2}Ma$$

$$a = \frac{2g}{3}$$

$$\therefore\ T = \frac{Mg}{3}$$

15. **(1)** Work done = increase in KE of the rotation

$$W = \frac{1}{2}I\omega^2 = \frac{1}{2}\times\frac{1}{2}MR^2\omega^2 = \frac{1}{4}MR^2\omega^2$$

16. **(1)** From conservation of angular momentum,

$$\frac{ML^2}{12}\omega_0 = \left[\frac{ML^2}{12} + 2m\left(\frac{L}{2}\right)^2\right]\omega,$$

$$\omega = \left(\frac{M}{M+6m}\right)\omega_0$$

17. **(2)** $$mg\left(\frac{l}{4}\right) = \frac{1}{2}\left[\frac{ml^2}{12} + m\left(\frac{l}{4}\right)^2\right]\omega^2$$

$$\omega = \sqrt{\frac{24g}{7l}} = 2\sqrt{\frac{6g}{7l}}$$

18. **(4)** As the inclined plane is smooth, the sphere can never roll, rather it will just slip down. Hence, the angular momentum remains conserved about any point on a line parallel to the inclined plane and passes through the centre of the ball.

19. **(4)** As $\sum\tau = 0$, angular momentum and linear momentum remain conserved. As the two balls will move radially out, I changes. In order to keep the angular momentum ($L = I\omega$) conserved, angular speed (ω) should change.

20. **(3)** If the track is smooth (case A), only translational kinetic energy changes to gravitational potential energy.

But, if the track is rough (case B), both translational and rotational kinetic energies change to potential energy.

Therefore, potential energy (= mgh) will be more in case B than in case A. Hence, $h_1 > h_2$.

21. **(4)** Since linear acceleration is same for all ($a = Mg\sin\theta - \mu Mg\cos\theta$) as they have same mass 'M' and same 'μ'.

Hence, all will reach the bottom simultaneously.

22. **(2)** For all the bodies, torque is the same.

$$\text{Now, KE} = \frac{1}{2}mv^2 + \frac{L^2}{2I}$$

Linear velocity 'v' is the same for all as the same force acts on them.

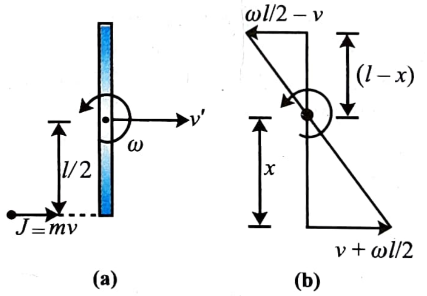
Therefore, the more value of moment of inertia implies the lesser kinetic energy.

Among all, the hollow sphere has the maximum moment of inertia, $I = \left(\frac{2}{3}MR^2\right)$

23. **(4)** $J = mv = mv'$

$\Rightarrow$ Velocity of the CM of rod = v

Applying impulse momentum equation about the CM of rod



(a) (b)

$$J\frac{l}{2} = I_{\text{CM}}\,\omega \Rightarrow \frac{mvl}{2} = \left(\frac{ml^2}{12}\right)\omega \Rightarrow \omega = \frac{6v}{l}$$

About instantaneous axis of rotation the rod is considered to have pure rotation.

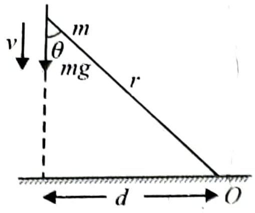
Let instantaneous axis of rotation be located at a distance x from the colliding end [Fig. (b)]

$$\frac{\frac{\omega l}{2} - v}{l - x} = \frac{v + \frac{\omega l}{2}}{x} \quad \text{...(i)}$$

Substituting the value of $\omega = 6v/l$ in Eq. (i), we get $x = \frac{2}{3}l$.

24. **(3)** The magnitude of angular momentum of the particle about $O = mvd$.

Since speed v of the particle increases, its angular momentum about O increases.



Magnitude of inertia of the particle about $O = mr^2$

Hence, MI of the particle about O decreases.

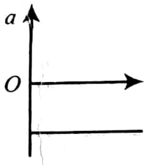
Angular velocity of the particle about $O = \frac{v\sin\theta}{r}$

Therefore, v and $\sin\theta$ increase and r decreases.

Therefore, angular velocity of the particle about O increases.

25. (2) As the normal force exerted by the horizontal surface passes through point B, the external torque on the ball is zero about point B. So the angular momentum of the ball is conserved about point B. ($\because \tau = \frac{dL}{dt}$)

26. (4) As the sphere rolls up its speed is decreasing and while rolling down its speed is increasing. Hence the acceleration of its centre of mass is down the incline and is thus always negative. Therefore the correct graph is shown in figure.



27. (3) $a = \dfrac{g\sin\theta}{1+\dfrac{I}{MR^2}}$

a is minimum, if I is maximum.

$I_{\max} = MR^2$ [for ring]

$$\Rightarrow a_{\min} = \frac{g\sin\theta}{1+\dfrac{MR^2}{MR^2}} = \frac{g\sin\theta}{2}$$

28. (2) For A and B, energy will remains conserved but energy of C will dissipate due to action of frictional force.

29. (3) Both the sphere and the plank will slide down with same acceleration $g\sin\theta$

30. (1) As $\sqrt{\dfrac{2gh}{1+\dfrac{I}{MR^2}}}$

Hence velocity is independent of the inclination of the plane and depends only on height h through which body descends.

But because $t + \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}\left(1+\dfrac{I}{MR^2}\right)}$ depends on the inclination also, hence greater the inclination lesser will be the time of descend. Hence, in present case, the speeds will be same (because h is same) but time in descend will be different (because of different inclinations)

31. (4) Let the angular velocity of disc after child jumps off. be ω'

$\therefore$ From conservation of angular momentum

$(I + mR^2)\omega = mvR + I\omega'$

$$\therefore \ \omega' = \frac{(I+mR^2)\omega - mvR}{I}$$

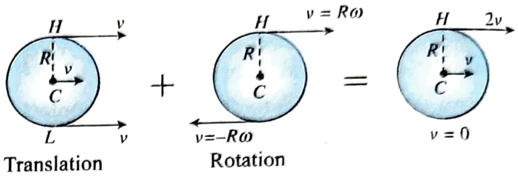
32. (3) Case-1 $mgh = \frac{1}{2}mv^2 + \frac{1}{2}mk^2\omega^2$

Case-2 $mgh = \frac{1}{2}m\times\left(\frac{5v}{4}\right)^2$

$$\frac{1}{2}mk^2\frac{v^2}{R^2} + \frac{1}{2}mv^2 = \frac{1}{2}m\times\frac{25v^2}{16}$$

$$\frac{k^2}{R^2} + 1 = \frac{25}{16} \Rightarrow k = \frac{3R}{4}$$

33. (1) In case of rolling without slipping, all points of a rigid body have same angular speed but different linear speed. The linear speed is maximum ($= 2v$) at the highest point H, while minimum for the lowest point ($=$ zero) L. This all clearly follows from the given figure.



34. (3) Applying conservation of angular momentum,

$(5 + 5)\omega' = 5 \times 10 + 5 \times 20 = 150$

or $\omega' = 15$ rad s^{-1}

Initial kinetic energy $= \frac{1}{2}\times 5\times 10\times 10 + \frac{1}{2}\times 5\times 20\times 20$

$= 250 + 1000 = 1250$ J

Final kinetic energy $= \frac{1}{2}\times 10\times 15\times 15 = 1125$ J

Loss of kinetic energy $= (1250 - 1125)$ J $= 125$ J

35. (4) $F + f = ma$ ...(i)

$FR - fR = \dfrac{mR^2}{2}\dfrac{a}{R}$ ...(ii)

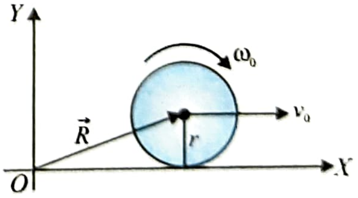
$F - f = \dfrac{ma}{2}$ ...(iii)

$2F = \dfrac{3ma}{2} \Rightarrow a = \dfrac{4F}{3m}$

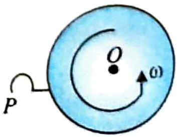
36. (1) $\vec{L_O} = I_{\text{CM}}\vec{\omega} + m(\vec{r}\times\vec{v}) = \frac{1}{2}mR^2\omega\hat{k} + m(4R\hat{i} + 3R\hat{j})\times(\omega R\hat{i})$

$= \frac{1}{2}mR^2\omega\hat{k} - 3mR^2\omega\hat{k} = -\frac{5}{2}mR^2\omega\hat{k}$

37. (4) $\vec{L} = \vec{R}\times m\vec{v}_0 + I\vec{\omega}_0 = mv_0 + I\omega_0$ (which is constant)



38. (2) During the impact, the impact forces pass through point P. Therefore, the torque produced by it about P is equal to zero. Consequently, the angular momentum of the disc about P, just before and after the impact, remains the same



$\Rightarrow L_2 = L_1$ ...(i)

where L_1 = angular momentum of the disc about P just before the impact

$I_0\omega = \frac{1}{2}mr^2\omega$

L_2 = angular momentum of the disc about P just after the impact

$$I_0\omega = \left(\frac{1}{2}mr^2 + mr^2\right)\omega' = \frac{3}{2}mr^2\omega'$$

Just before the impact, the disc rotates about O. But just after the impact, the disc rotates about P.

$$\Rightarrow \frac{1}{2}mr^2\omega = \frac{3}{2}mr^2\omega' \quad \Rightarrow \quad \omega' = \frac{1}{3}\omega$$

39. **(3)** Using conversation of angular momentum about O, we get

$$mvR = (mR^2 + mR^2)\omega;\ \omega = \frac{v}{2R}$$

40. **(4)** Apply conservation of angular momentum about the hinge, we get

$$mvR = \frac{m}{2}R^2\omega + m(R\omega)R$$

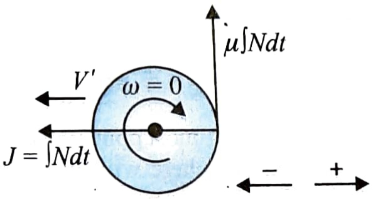
$$R\omega = \frac{2v}{3} = \frac{2\times 5}{3} = \frac{10}{3}\ \text{m/s}$$

41. **(1)** Impulse provided by the edge in the horizontal direction:

$$-\int N dt = -mV' - (mV) \qquad \text{...(i)}$$

Friction impulse in the vertical direction:

$$\mu R\int N dt = \frac{2}{5}mR^2\left(\frac{V}{R}\right) \qquad \text{...(ii)}$$



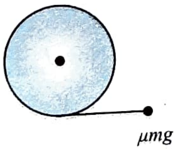
From Eqs. (i) and (ii), we get

$$\int N dt = 2mV \text{ and } V' = V$$

42. **(3)** **Method 1:** The distance of $\vec{L}$ is perpendicular to the line joining the bob at point C. Since this line keeps changing its orientation in space, direction of $\vec{L}$ keeps changing, however, as ω is constant, magnitude of $\vec{L}$ remains constant.

Method 2: The torque about point is perpendicular to the angular momentum vector about point C. Hence it can only change the direction of L, and not its magnitude.

43. **(4)** Taking τ about CM



$$\mu mgR = MR^2\alpha$$

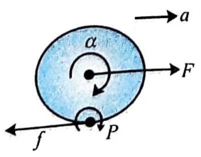
$$\mu g = R\alpha$$

$$\alpha = \frac{\mu g}{R}$$

$$\omega = \omega_0 - \frac{\mu g}{R}f = \frac{\omega_0}{2}$$

$$\frac{\omega_0}{2} = \frac{\mu g}{R}f \Rightarrow f = \frac{\omega_0 R}{2\mu g}$$

44. **(4)** As sphere is rolling we can apply torque equation about point of contact (about P).



$$FR = I_P\alpha$$

$$\Rightarrow FR = \left(\frac{7}{5}MR^2\right)\alpha$$

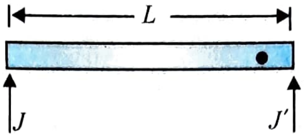
$$\Rightarrow R\alpha = \frac{5F}{7M} = a \text{ (acceleration of CM)}$$

Equation of translatory motion

$$F - f = Ma \Rightarrow f = \frac{2}{7}F \qquad \text{...(i)}$$

For no sliding, $f \le \mu mg \quad \Rightarrow \quad F = \frac{7}{2}\mu mg$

45. **(2)** Let J' be the impulse exerted by the pivot on the rod. Then from impulse momentum theorem, $J + J' = mv_0$, where v_0 is the velocity acquired by the centre of mass.



Now apply, angular impulse = change in angular momentum about the pivot.

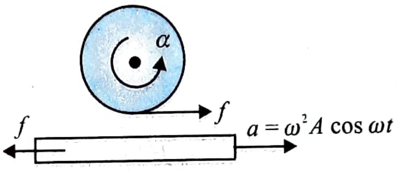
$$J \times L = \frac{ML^2}{3}\omega \text{ or } J = \frac{ML}{3}\omega,$$

where ω is the angular velocity of the rod.

$$v_0 = \frac{\omega L}{2} \Rightarrow Mv_0 = \frac{M\omega L}{2}$$

$$J + J' = \frac{3J}{2} \Rightarrow J' = \frac{J}{2}$$

46. **(2)** The cylinder is having a fixed axis, so it can perform rotational motion only.



$$fR = \frac{MR^2}{2}\alpha \Rightarrow f = \frac{MR}{2}\alpha$$

For no slipping,

$R\alpha$ = acceleration of the platform

$$\frac{2f}{M} = \omega^2 A\cos\omega t$$

For the maximum torque, f would be maximum and

$f_{\max} = M\omega^2A/2$

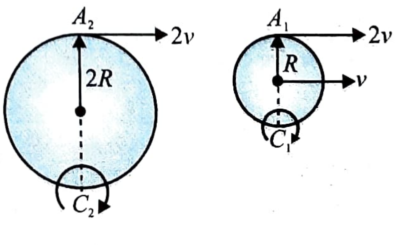
So the maximum torque, $\tau_{\max} = f_{\max} \times R = \dfrac{M\omega^2AR}{2}$

47. **(2)** Since the disc comes to rest, it stops rotating and translating simultaneously $v = 0$ and $\omega = 0$. That means, the angular momentum about the instantaneous point of contact just after the time of stopping is zero. We know that the angular momentum of the disc about P remains constant because frictional force f, N and mg pass through point P and thus produce no torque about this point.

$$\Rightarrow L_{\text{initial}} = L_{\text{final}} \Rightarrow mvr - I_0\omega_0 = 0$$

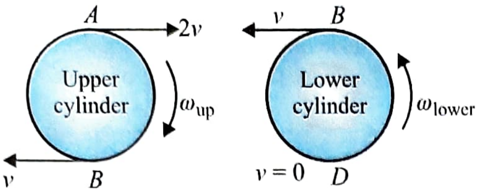
$$\Rightarrow mvr = \frac{1}{2}mr^2\omega_0 \Rightarrow 2v_0 = \omega_0 r$$

48. **(2)** In figure, C_1 and C_2 are ICR (instantaneous centre of rotation) of the two cylinders. The cylinder can be considered as rotating about C_1 and C_2. In the absence of slipping between the plank and the cylinders, points A_1 and A_2 have the same velocity. Angular velocity of the larger cylinder is $\dfrac{2v}{4R} = \dfrac{v}{2R}$



$$v_{CM} = (2R)\left(\frac{v}{2R}\right) = v$$

49. (2)



Angular velocity of the upper cylinder is

$$\omega_{up}=\frac{2v-(-v)}{2R}=\frac{3v}{2R}$$; for the lower cylinder

$$\omega_{lower}=\frac{v-0}{2R}=\frac{v}{2R}$$

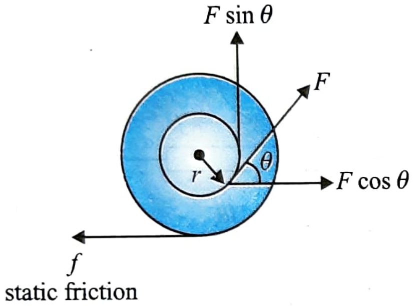
Hence, ratio, $\frac{\omega_{up}}{\omega_{lower}}=3$

50. (1) If the spool is not to translate,

$$F\cos\theta=f \qquad \text{...(i)}$$

If the spool is not to rotate,

$$Fr=fR \qquad \text{...(ii)}$$

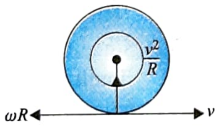


or $\frac{FR}{r}\cos\theta=F$

or $\cos\theta=\frac{r}{R} \Rightarrow \theta=\cos^{-1}\left(\frac{r}{R}\right)$

Also the line of action of F should pass through the bottom-most point.

51. (4) As the disc is in combined rotation and translation, each point has a tangential velocity and a linear velocity in the forward direction.



From the figure,

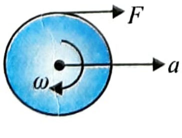
v_{net} (for the lowest point) $= v - R\omega = v - v = 0$

and acceleration $\frac{v^2}{R}+0=\frac{v^2}{R}$

(Since the linear speed is constant)

52. (2) $F = Ma$

$FR = I\alpha$



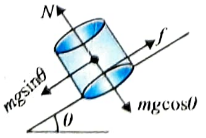
For pure rolling, $a = R\alpha$

Solving the above equations, we get

$I = MR^2$.

So the object may be a ring or a hollow cylinder.

53. (1) $f = Mg\sin\theta$

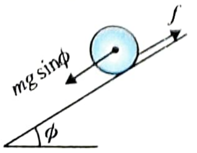


For no sliding, $f < f_L$

$mg\sin\theta < \mu_s mg\cos\theta$

$\tan\theta = \mu_s$

For rolling friction, force



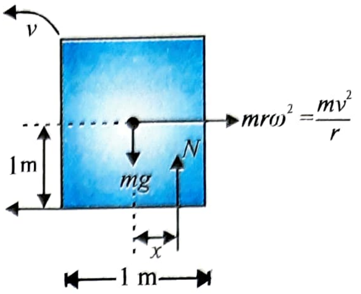
$$f=\frac{mg\sin\phi}{1+\frac{R^2}{k^2}}$$

$$\Rightarrow f=\frac{mg\sin\phi}{3}$$

For pure rolling, $\frac{mg\sin\phi}{3}<\mu_s mg\cos\phi$

$\tan\phi<3\mu_s$, so $\frac{\tan\phi}{\tan\theta}=3$

54. (2) The free-body diagram of the box w.r.t. the truck is as shown in figure.



For vertical equilibrium, $mg = N$

For horizontal equilibrium, $f = (mv^2)/r$

For rotational equilibrium, $f \times 1 = N \times x$

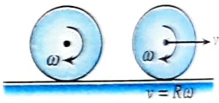
For no tipping to take place, $x < (1/2)$ m

So, $mg\times x=\frac{mv^2}{r} \Rightarrow x=\frac{v^2}{rg}<\frac{1}{2} \Rightarrow v<\sqrt{\frac{rg}{2}}=10$ m/s

55. (4) Since $v_0 > \omega_0 R$, so bottom-most point will have forward velocity initially. So friction on sphere will act in backward direction. Minimum velocity will occur when $v = \omega R$

$$\Rightarrow v_0-\mu g t=\left(\omega_0+\frac{5\mu g}{2R}t\right)R \Rightarrow t=\frac{2(v_0-\omega_0 R)}{7\mu g}$$

56. (3) From conservation of angular momentum about point of contact.



$$I\omega_0 = I\omega + mR^2\omega$$

$$I\omega_0 = I\frac{v}{R}+mRv$$

$$v=\frac{I\omega_0}{\frac{I}{R}+mR} \Rightarrow v=\frac{\omega_0}{\frac{1}{R}+\frac{mR}{I}}$$

Now $I_{\text{solid sphere}} < I_{\text{hollow}}$

$v_{\text{solid}} < v_{\text{hollow}}$

$v_1 < v_2$

57. (2) Friction force μmg acts in forward direction till pure rolling is started. Hence linear acceleration

$$a=\frac{\mu mg}{m}=\mu g$$

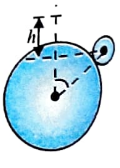
$v = at$

or $t=\frac{v}{a}=\frac{\omega_0}{\mu g\left(\frac{1}{R}+\frac{mR}{I}\right)}$

Again $I_{\text{solid}} < I_{\text{hollow}}$

$\therefore\ t_{\text{solid}} < t_{\text{hollow}}$ or $t_1 < t_2$

58. (2) $\frac{mv^2}{(R+r)}=mg\cos\theta$,



$$mgh=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2 \qquad \text{...(i)}$$

$$mg(R+r)(1-\cos\theta) = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

$$= \frac{7}{10}mv^2$$

$$mv^2 = \frac{10}{7}mg(R+r)(1-\cos\theta) \quad \text{...(ii)}$$

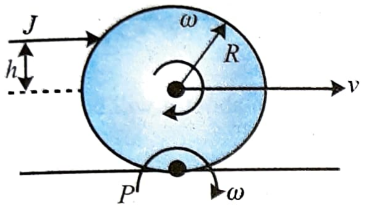
From (i) and (ii)

$$\frac{10}{7} = \frac{17}{7}\cos\theta \text{ or } \cos\theta = \frac{10}{17}$$

$$v = \sqrt{g(R+r)\cos\theta} = \sqrt{\frac{10}{7}g(R+r)}$$

$$\text{and } \omega = \frac{v}{r} = \sqrt{\frac{10g(R+r)}{17r^2}}$$

59. **(2)** Rolling is rotation about point of contact. Applying impulse momentum equation about P.



$$J(R+h) = I_P\omega \quad \text{...(i)}$$

$$\text{and } J = mv \quad \text{...(ii)}$$

As sphere rolls $v = \omega R$, and $I = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2$

After solving, we get $\frac{h}{R} = \frac{2}{5}$

60. **(3)** From the figure in question, position vector of mass

$$\vec{r} = R\hat{i} - \sqrt{l^2 - R^2}\,\hat{k}$$

Linear moment $\vec{p} = Mv\hat{j}$

$$\vec{L} = \vec{r} \times \vec{p}$$

Substituting the values, we get result

$$Mvl\left[\frac{\sqrt{l^2 - R^2}}{l}\hat{i} + \frac{R}{l}\hat{k}\right]$$

61. **(1)** Let the impulse due to the particle be J, $J = mv_{CM}$

$$\Rightarrow v_{CM} = \frac{J}{m}$$

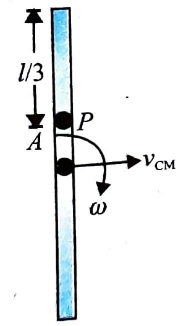
Angular impulse about centre of mass is

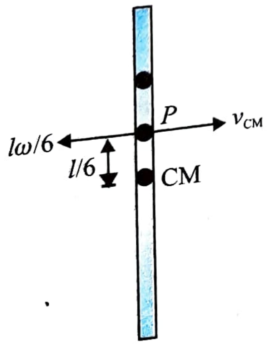
$$Jx = \frac{ml^2}{12}\omega \Rightarrow \omega = \frac{12Jx}{ml^2}$$

For point A to be at rest instantaneously,

$$\vec{v}_{\text{trans}} + \vec{v}_{\text{rot}} = 0$$

$$\text{or } v_{\text{cm}} = \frac{\omega l}{6}$$





$$\text{Thus } \frac{J}{m} = \frac{l}{6} \times \frac{12Jx}{ml^2}$$

which gives, $x = \frac{l}{2}$

62. **(2)** When the ring is at the maximum height, the wedge and the ring have the same horizontal component of velocity. As all the surfaces are smooth, in the absence of friction between the ring and the wedge surface, angular velocity of the ring remains constant.

For conservation of mechanical energy, we get

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv'^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv'^2 + mgh$$

where v' is final common velocity;

$$v' = \frac{v}{2} \text{ (from conservation of momentum) and } h = \frac{v^2}{4g}$$

63. **(4)** As torque = change in angular momentum

$$F\Delta t = mv \text{ (linear momentum)} \quad \text{...(i)}$$

$$\text{and } \left(F\frac{l}{2}\right)\Delta t = \frac{ml^2}{12}\omega \text{ (angular momentum)} \quad \text{...(ii)}$$

Dividing Eqs. (i) and (ii), we get

$$2 = \frac{12v}{\omega l} \Rightarrow \omega = \frac{6v}{l}$$

Using $S = ut$,

Displacement of CM is $\frac{\pi}{2} = \omega t = \left(\frac{6v}{l}\right)t$

and $x = vt$

Dividing, we get

$$\frac{2x}{\pi} = \frac{l}{6} \Rightarrow x = \frac{\pi l}{12}$$

Coordinates of A will be $\left[\frac{\pi l}{12} + \frac{l}{2}, 0\right]$.

64. **(1)** By conservation of angular momentum about pivot

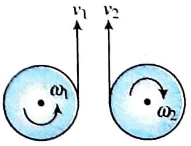
$$L = I\omega$$

$$\frac{mvd}{2} = \left[\frac{Md^2}{12} + m\left(\frac{d}{2}\right)^2\right]\omega = \left(\frac{md^2}{2} + \frac{md^2}{4}\right)\omega$$

$$\frac{mvd}{2} = \frac{3}{4}md^2\omega \Rightarrow \frac{2}{3}\frac{v}{d} = \omega$$

65. **(3)** Angular momentum of system can not remain conserved as some external unbalanced torque is present due to forces at axels. Kinetic energy is not conserved, because slipping is there and work is done against friction.

66. **(2)** Let ω_1 and ω_2 be the final angular velocities when the slipping is ceased. Then



$$v_1 = v_2 \text{ or } \omega_1 R = \omega_2(2R)$$

$$\text{or } \omega_2 = \frac{\omega_1}{2} \quad \text{...(i)}$$

Now applying angular impulse = change in angular momentum we have

$$J \cdot R = I\,\omega_1 \quad \text{...(ii)}$$

$$\text{and } J \cdot 2R = 4I(\omega_0 - \omega_2) \quad \text{...(iii)}$$

Here J = linear impulse due to friction which will be tangential and equal for both the cylinders.

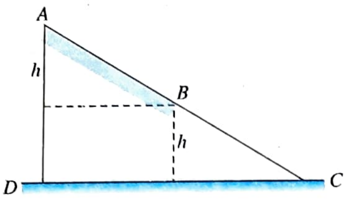
Solving Eqs. (i), (ii) and (iii) we get

$$\omega_1 = \omega_0 \text{ and } \omega_2 = \frac{\omega_0}{2}$$

67. **(2)** For a cylinder $\frac{K_T}{K_R} = 2$ in case of pure rolling.

Up to B:

$$K_T = \frac{2}{3} mgh \text{ and } K_R = \frac{1}{3} mgh$$



After B: rotational kinetic energy is constant; however, translational kinetic energy increases.

At C: $K_T = \frac{2}{3} mgh + mgh = \frac{5}{3} mgh$

While $K_R = \frac{1}{3} mgh$, $\frac{K_T}{K_R} = 5$

68. **(3)** Let v be the linear velocity of centre of mass of the spherical body and ω its angular velocity about centre of mass. Then

$$\omega = \frac{v}{2R}$$

KE of spherical body, $K_1 = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

$$K_1 = \frac{1}{2} mv^2 + \frac{1}{2} (2mR)^2 \left(\frac{v^2}{4R^2} \right) = \frac{3}{4} mv^2 \qquad \text{...(i)}$$

Speed of the block, $v' = (\omega)(3R) = 3R\omega = (3R)\left(\frac{v}{2R} \right) = \frac{3}{2} v$

$$\therefore \text{ KE of block, } K_2 = \frac{1}{2} mv'^2 = \frac{1}{2} m \left(\frac{3}{2} v \right)^2 = \frac{9}{8} mv^2 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), $\frac{K_1}{K_2} = \frac{2}{3}$

69. **(3)** Since the cylinder rolls with a constant velocity, so there is no friction between the upper surface of the plank and the cylinder. The plank is at rest, under the influence of the weight of the cylinder. So, no external force is required to keep the plank in equilibrium.

70. **(1)** Given that I_0 is the moment of inertia of table and gun and m the mass of bullet.

Initial angular momentum of system about centre:

$$L_i = (I_0 + mr^2)\omega_0 \qquad \text{...(i)}$$

Let ω be the angular velocity of table after the bullet is fired.

Final angular momentum

$$L_f = I_0\omega - m(v - r\omega)r \qquad \text{...(ii)}$$

where $(v - r\omega)$ is absolute velocity of bullet to the right.

Equating (i) and (ii): we get $(\omega - \omega_0) = \frac{mvr}{I_0 + mr^2}$

This is also the increase in angular velocity.

71. **(4)** Net external torque is zero. Therefore angular momentum of system will remain conserved, i.e., $L_i = L_f$

Initial angular momentum $L_i = 0$

$\therefore$ Final angular momentum should also be zero.

or angular momentum of man = angular momentum of platform in opposite direction

or $mv_0 r = I\omega$

$$\omega = \frac{mv_0 r}{I} = \frac{(70)(1.0)(2)}{200}$$

$$\omega = 0.7 \text{ rad/s}$$

72. **(2)** Angular velocity of man relative to platform is

$$\omega_r = \omega + \frac{v_0}{r} = 0.7 + \frac{1}{2} = 1.2 \text{ rad/s}$$

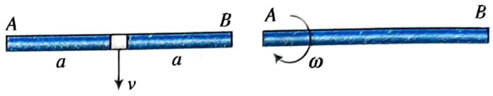
$\therefore$ Time taken to complete one round

$$t = \frac{2\pi}{\omega_r} \text{ or } t = \frac{2\pi}{1.2} \text{ s}$$

Angle rotated with respect ground in this time:

$$\theta = \left(\frac{v_0}{r} \right) \cdot t = \left(\frac{1}{2} \right) \left(\frac{2\pi}{1.2} \right) = \frac{5}{6}\pi$$

73. **(4)**



Angular momentum about A will be conserved i.e., $L_i = L_f$

or $mva = I\omega$

or $mva = \frac{m(2a)^2}{3} \cdot \omega \therefore \omega = \frac{3v}{4a}$

74. **(2)** KE of ball in position $B = mg(R - r)$

Here m = mass of ball.

Since it rolls without slipping the ratio of rotational to translational kinetic energy will be 2/5.

$$\frac{K_R}{K_T} = \frac{2}{5}$$

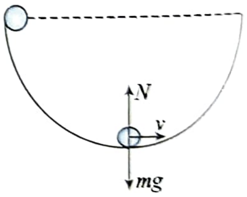
$$K_T = \frac{2}{7} mg\,(R - r);\ \frac{1}{2} mv^2 = \frac{2}{7} mg\,(R - r)$$

$$v = \frac{2}{\sqrt{7}} \sqrt{g(R - r)};\ \omega = \frac{v}{R - r} = 2\sqrt{\frac{g}{7(R - r)}}$$

75. **(3)** From conservation of mechanical energy

$$mg(R - r) = \frac{7}{10} mv^2 \qquad \text{...(i)}$$

$$\text{and } N - mg = \frac{mv^2}{R - r} \qquad \text{...(ii)}$$



Solving Eqs. (i) and (ii), we get

$$N = \frac{17}{7} mg$$

Note: (7/10) mv^2 is the translational + rotational kinetic energy of sphere at bottom.

76. **(1)** Equilibrium of m gives: $T = mg$ (T = tension in string)

Net torque about point of contact of spool should be zero.

Hence, $(2R)(Mg \sin \alpha) = TR$

or $2Mg \sin \alpha = mg$

or $m = 2M \sin \alpha$

77. **(3)** KE of $P = \frac{1}{2} m(2v)^2 = 2mv^2$

$$\text{KE of } S = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{3}{4}mv^2 \Rightarrow \frac{\text{KE of } P}{\text{KE of } S} = \frac{8}{3}$$

78. (3) Double torque, double angular acceleration, double linear acceleration, double distance.

79. (1) The kinetic energy is same in both the cases.

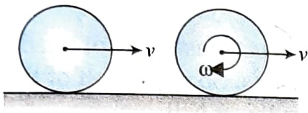
80. (3) Apply conservation of angular momentum:

$$I_i\omega_i = I_f\omega_f$$

$$\Rightarrow \frac{1}{2}mR^2 \times 2.4 = \left(\frac{1}{2}mR^2 + \frac{m(2R)^2}{12}\right)\omega_f$$

$$\Rightarrow \omega_f = 1.44 \text{ rev/s}$$

81. (2) Let v be the velocity of COM of ring just after the impulse is applied and v' its velocity when pure rolling starts. Angular velocity ω of the ring at this instant will be $\omega = v'/r$.



From impulse = change in linear momentum, we have

$$J = mv$$

or $v = J/m$ ...(i)

Between the two positions shown in figure, force of friction on the ring acts backwards. So, angular momentum of the ring about bottommost point will remain conserved

$$\therefore\ L_i = L_f \quad \text{or} \quad mvr = mv'r + I\omega$$

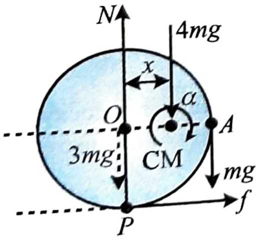
$$mv'r + (mr^2)(v'/r) = 2mv'r$$

$$v' = \frac{v}{2} = \frac{J}{2m} \quad \text{(from equation)}$$

82. (2) The distance of CM from the ring centre O

$$x = \frac{3m(0) + m(r)}{3m + m} = \frac{r}{4}$$

We can apply torque equation about point of contact as the ring is rolling.



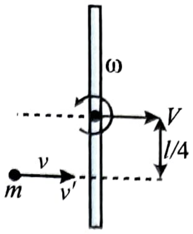
$$\tau_P = I_P\alpha$$

$$4mg\left(\frac{r}{4}\right) = \left[(3mr^2 + mr^2) + m(AP)^2\right]\alpha$$

$$\Rightarrow mgr = \left[4mr^2 + m(\sqrt{2}\,r)^2\right]\alpha$$

$$\Rightarrow mgr = 6mr^2\alpha \quad \Rightarrow \alpha = \frac{g}{6r}$$

83. (1) Applying conservation of linear momentum,



$$mv = mv' + mV \Rightarrow v = v' + V \qquad \text{...(i)}$$

Applying conservation of angular momentum about point of collision.

$$0 = \left(\frac{ml^2}{12}\right)\omega - mV\left(\frac{l}{4}\right) \Rightarrow l\omega = 3V \qquad \text{...(ii)}$$

Applying restitution equation,

$$(u_1 - u_2)_n = (v_2 - v_1)_n \Rightarrow (v - 0) = (V - v') \qquad \text{...(iii)}$$

Solving Eqs. (i), (ii), and (iii) we get $V = v$ and $\omega = \dfrac{3v}{l}$

Time taken to complete three revolutions ($\theta = 6\pi$)

$$t = \frac{\theta}{\omega} = \frac{6\pi}{\omega} = \frac{6\pi l}{3v} = \frac{2\pi l}{v}$$

Hence, distance travelled by the centre of the rod is

$$s = vt = v\left(\frac{2\pi l}{v}\right) = 2\pi l$$

84. (1) Just before collision between two balls

Potential energy lost by ball A = kinetic energy gained by ball A.

$$mg\frac{h}{2} = \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}mv_{\text{cm}}^2$$

$$= \frac{1}{2} \times \frac{2}{5}mR^2 \times \left(\frac{v_{\text{cm}}}{R}\right)^2 + \frac{1}{2}mv_{\text{cm}}^2 = \frac{1}{5}mv_{\text{cm}}^2 + \frac{1}{2}mv_{\text{cm}}^2$$

$$\Rightarrow \frac{5}{7}mgh = mv_{\text{cm}}^2 \Rightarrow \frac{mgh}{7} = \frac{1}{5}mv_{\text{cm}}^2$$

After collision, only translational kinetic energy is transferred to ball B.

So just after collision, rotational kinetic energy of ball A

$$= \frac{1}{5}mv_{\text{cm}}^2 = \frac{mgh}{7}$$

85. (4) Let after collision velocity of rod be v and angular velocity be ω, then

$$mv_0 = mv + MV \Rightarrow mv_0 x = mvx + \frac{Ml^2}{12}\omega$$

$$mv_0 = mv + \frac{M\omega\eta^2 x}{12}$$

From above equations, $\dfrac{M\omega\eta^2 x}{12} = MV \Rightarrow \omega\eta^2 x = 12V$

Velocity of farther end of the rod $= V - \dfrac{l\omega}{2}$

$$= \frac{\omega\eta^2 x}{12} - \frac{\omega\eta x}{2} = \frac{\omega\eta x}{2}\left(\frac{\eta}{6} - 1\right)$$

If this velocity is opposite to v_0, then,

$$\frac{\eta}{6} - 1 < 0 \Rightarrow \eta < 6$$

86. **(1)** $\vec{\tau}_{av} \cdot \Delta t = \Delta \vec{L}$

$|\Delta \vec{L}| = |\vec{L}_f - \vec{L}_i|$ above point of projection.

$= (mu \sin \theta)$ (Range)

$$= \frac{(mu \sin \theta)(u^2 \sin 2\theta)}{g} = \frac{mu^3 \sin \theta \sin 2\theta}{g}$$

$$|\vec{\tau}_{av}| = \frac{\Delta \vec{L}}{\Delta t} = \frac{mu^3 \sin \theta \sin 2\theta}{g} \frac{g}{2u \sin \theta}$$

$$= \frac{mu^2 \sin 2\theta}{2}$$

87. **(1)** Since the disc was rolling, the velocity of its top point at the instant of leaving the track was zero. It means $v_H = \omega r$.

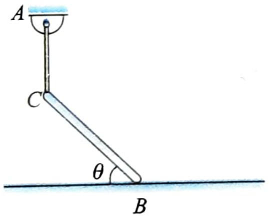
When the disc is in air v_H and ω both do not change. Hence the horizontal component of the velocity of the top point P of the disc at every instant is zero and the vertical component of the velocity of the point P is equal to the vertical component of velocity of the CM of disc.

$$\Rightarrow \sqrt{2gh} = \sqrt{2 \cdot g \cdot 2(R - 0.1)} = 6$$

$$\Rightarrow 4 \times 10(R - 0.1) = 36$$

$$R = 1 \text{ m}$$

88. **(2)** When the string gets vertical, the centre of the rod has horizontal velocity and the angular speed of the rod will be zero. At this point angle θ has become minimum (i.e. $\frac{d\theta}{dt} = 0$). Velocity of C is horizontal. Velocity of B is also horizontal.



∴ Rod has a horizontal velocity and no angular velocity.

Energy conservation gives:

$$\frac{1}{2}Mv^2 = Mg\frac{b}{4} - Mg\left(\frac{b-a}{2}\right)$$

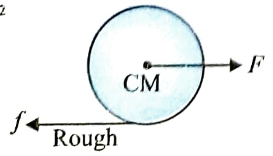
$$\therefore \; v = \sqrt{g\left(a - \frac{b}{2}\right)}$$

Multiple Correct Answers Type

1. **(1), (2), (3), (4)**

Acceleration of centre of mass,

$$a_c = \frac{F}{M}\frac{1}{1 + \frac{K^2}{R^2}} = \frac{5F}{7M} \qquad \left(\because K = \sqrt{\frac{2}{5}}\,R\right)$$



Frictional force, $f = F\left[\frac{1}{1 + \frac{R^2}{K^2}}\right] = \frac{2F}{7}$

Since $f = +$ ve, friction is directed backward.

Options (1), (2), (3) and (4) are incorrect.

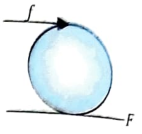
2. **(3), (4)**

The velocity of the disc when rolling begins can be obtained using the conservation of angular momentum principle about the point through which the friction force acts. So, the coefficient of friction has no bearing on the final velocity. The work done by the force of friction will simply be changed to kinetic energy.

3. **(3), (4)**

$mgh = 1/2\, mv^2 \left[1 + (k^2/R^2)\right]$, k = radius of gyration. For all solid spheres, $k^2/R^2 = 2/5$ = independent of m and R.

4. **(2), (4)**



$$F + f = mg \qquad \text{...(i)}$$

$$FR - fR = \frac{2}{5}mR^2\,\frac{a}{R} \qquad \text{...(ii)}$$

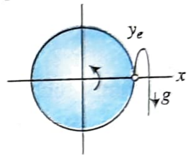
After solving these two equations we get the frictional force is positive. This implies that friction supports the motion.

5. **(1), (2)**

Since D is the instantaneous centre of rotation and as we go from D to B velocity increases. So upper half of the ring will have greater kinetic energy than lower half.

6. **(1), (4)**

$$I_1\omega_T = I_T\,\omega_T + I_2\omega_T \qquad \text{...(i)}$$



Velocity of broken away piece decreases initially then increases.

7. **(1), (4)**

If $k^2 > Rx$, friction will act in backward direction, otherwise, in forward direction.

For given spool $K^2 = \frac{R^2}{4}$

Fig. (a): $Rx = R \cdot \frac{R}{3} = \frac{R^2}{3}$ friction forward

Fig. (b): $Rx = R \cdot \frac{R}{5} = \frac{R^2}{5}$ friction backward

In Fig. (c) as force is below the diametric plane, hence friction acts backward.

8. **(1), (2), (4)**

Let ω_1 = the initial angular velocity of the disc ω_2 = the final common angular velocity of the disc and the ring.

For the disc, $I_1 = \frac{1}{2}mr^2$

For the ring, $I_2 = mr^2$

By conservation of angular momentum,

$$L = I_1\omega_1 = (I_1 + I_2)\omega_2$$

$$\omega_2 = \frac{I_1\omega_1}{I_1 + I_2} = \frac{\omega_1}{3}$$

Initial kinetic energy = $E_1 = \frac{1}{2} I_1\,\omega_1^2$

Final kinetic energy = $E_2 = \frac{1}{2}(I_1 + I_2)\omega_2^2$

Heat produced = loss in KE

Ratio of heat produced to initial kinetic energy = $(E_1 - E_2)/E_1 = 2/3$

9. (2), (4)

The force of friction between the two surfaces in contact disappears when there is no relative (linear) motion between them. Angular momentum will not be conserved as the discs will have the final angular velocities in the opposite directions.

10. (1), (2), (3)

Let L = angular momentum

$$\tau = \frac{dL}{dt} \text{ or } dL = \tau\, dt$$

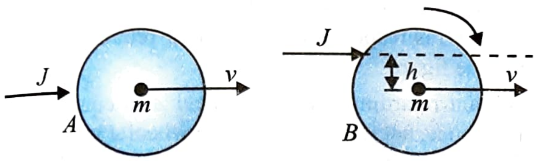
For constant torque, $\Delta L = \tau \Delta t = I\Delta\omega$

or $I\omega$ (if $\omega_i = 0$)

Rotational kinetic energy $= I\omega^2/2 = (\Delta L)^2/2I = (\tau\Delta t)^2/2I$

11. (1), (2), (4)

$J = mv$ for both. Sphere A has no angular motion. For sphere B, angular momentum imparted by $J = L$.



12. (2), (3), (4)

Let P = external force, F = force of friction between A and B, a_1 = acceleration between A and B, a_2 = acceleration beyond B

$P - F = ma_1$ and $P = ma_2$

$$a_2 > a_1$$

Let α = angular acceleration between A and B

For one rotation, $\theta = 2\pi = \frac{1}{2}\alpha T^2$

$$T = \sqrt{\frac{4\pi}{\alpha}} = \text{time of travel from } A \text{ to } B$$

Angular velocity at $B = \omega_B = \alpha T$

For one rotation to the right of B, $\theta = 2\pi = \omega_B t$

$$t = \frac{2\pi}{\alpha T} = \frac{\frac{1}{2}\alpha T^2}{\alpha T} = \frac{T}{2}$$

13. (3), (4)

In rolling without slipping, no work is done against friction. Hence, loss in gravitational potential energy of a body is equal to its total kinetic energy, i.e., linear plus rotational kinetic energies.

Also, $v = \omega r$

Total kinetic energy $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

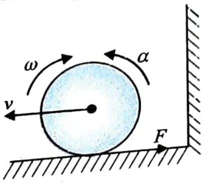
$$= \frac{1}{2}mv^2 + \frac{1}{2}(mK^2)\left(\frac{v^2}{r^2}\right) = \frac{1}{2}mv^2\left[1 + \frac{K^2}{r^2}\right]$$

where K = radius of gyration

As all the bodies have the same final total kinetic energy, their final velocities will depend upon the ratio K/r. Bodies with the smaller value of K/r will have the greater v and hence will reach the bottom earlier.

14. (1), (3), (4)

After impact, the force of friction will act in a direction opposite to that of the motion. The body will retain its initial angular motion (clockwise). The force of friction will cause linear retardation, reducing v. It will also cause an anticlockwise angular acceleration, which will reduce ω to zero and then introduce anticlockwise ω till rolling without slipping begins. F will then disappear.



15. (1), (2), (3), (4)

Angular momentum = linear momentum × perpendicular distance from the point of rotation

or $L = Jl$

Also, $I = \frac{ml^2}{3}$

$$\omega = \frac{L}{I} = \frac{Jl}{ml^2/3} = \frac{3J}{ml}$$

$$\text{Kinetic energy} = \frac{L^2}{2I} = \frac{J^2l^2}{2\frac{ml^2}{3}} = \frac{3J^2}{2m}$$

$$v_c = \omega\frac{l}{2} = \frac{3J}{2m}$$

16. (1), (3), (4)

Since, the spool rolls over the horizontal surface, instantaneous axis of rotation passes through the point of contact of spool with the horizontal surface.

About the instantaneous axis of rotation, moment produced by F is clockwise. Therefore, the spool rotates clockwise. In that case, acceleration will be rightward and thread will wind.

If rotational motion of the spool is considered about its own, then the resultant moment on it must be clockwise. But moment produced by the force is anticlockwise and its magnitude is equal to F_1/r. Hence, moment produced by the friction (about its own axis) must be clockwise and its magnitude must be greater than Fr. It is possible only when friction acts leftwards. Therefore, option (2) is correct.

17. (1), (2), (3)

Weight of the particle is balanced by normal reaction of the floor. Hence, the resultant force acting on the particle is provided by the tension in the thread.

The line of action of this tension passes through 'B' at the initial moment only. After that it does not pass through 'B'. Hence, the tension produces a moment about B. It means, angular momentum about B does not remain conserved. The tension always produces a moment about O. Magnitude of this moment is equal to TR in an anticlockwise direction. Hence, options (1), (2) and (3) are incorrect. Since neither external work is done on the particle nor energy is lost against friction, therefore KE of the ball remains constant. Hence, option (4) is correct.

18. (1), (2), (3), (4)

When the particle strikes at one end of the rod then an impulse is exerted by the particle on the rod along the direction of its original motion. An equal but opposite impulsive reaction is exerted by the rod on the particle. If system of rod and the particle is considered, then there is no external impulse on the system, hence momentum of the system remains conserved. Hence, option (1) is correct.

Since, both the impulsive reactions act at the same point, one acts on the rod and other on the particle, hence no angular impulse is produced by these impulsive reactions about any point. It means, angular momentum of the system remains conserved about every point. Hence, options (2) and (3) are correct.

19. (1), (2), (4)

There is no slipping, therefore $\tau = I\alpha$

$$T[R_2 - R_1] = [MK^2 + MR_2^{\,2}]\alpha$$

$$\alpha = \frac{200(0.6 - 0.3)}{50(0.3^2 + 0.6^2)} = 2.67 \text{ rad/s}^2$$

Using Newton's IInd law

$$T - f = MR_2\alpha$$

$$f = (T - MR_2\alpha) = \left[200 - 5 \times 0.6 \times \frac{8}{3}\right] = 120 \text{ N}$$

Now $F_{\max} = \mu_s mg = 0.1 \times 50 \times 10 = 50$ N

$$f_k = \mu_s Mg = 0.08 \times 50 \times 10 = 40 \text{ N}$$

$$Ma = -T + 40 \Rightarrow a = -3.2 \text{ m/s}^2$$

20. (2), (3), (4)

Due to force F, the plank has a tendency to slide to the right below the sphere. Hence, friction acts on the plank to the left and that on the sphere acts to the right. The friction not only acts on accelerating force on the sphere but also produces an anticlockwise moment. Therefore, the sphere experiences a rightward translational acceleration and an anticlockwise angular acceleration simultaneously.

If the angular acceleration of the sphere is α, then its centre of mass has a backward acceleration $r\alpha$ relative to the plank. Hence, its net acceleration becomes less than that of the plank. Hence, option (2) is correct. Obviously, option (1) is incorrect.

Since there is no sliding between the surfaces of the sphere and the plank, therefore, no energy is lost against friction.

Since friction, acting on the sphere, is the only force which provides motion to it, therefore KE of the sphere at any instant is equal to work done by the friction acting on it. Hence, option (3) is correct.

Since no energy is lost against friction, therefore KE of the whole system is equal to work done by force F. Hence, option (4) is also correct.

21. (1), (3)

There will be exchange of linear velocities only. However, the two spheres cannot exert torques on each other, as their surfaces are frictionless and the angular velocities of the spheres do not change.

22. (2), (3), (4)

Using law of conservation of energy for wheel A,

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}I\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2$$

$$v^2 = \frac{mg}{\frac{1}{2}\left[\frac{I}{r^2} + m\right]} \cdot h$$

$$v \propto \sqrt{h}$$

$$v_A \propto \sqrt{h} \text{ and } v_B \propto \sqrt{4h}$$

$$v_B = 2v_A$$

Also $v_A^2 = 2a_A \dfrac{h}{\sin\theta} \Rightarrow a_A = \dfrac{g\sin\theta}{1 + \dfrac{I}{mr^2}}$

Acceleration of wheel is independent of h

$$a_A = a_B$$

Also $\dfrac{h}{\sin\theta} = \dfrac{1}{2}a_A t_A^2 \Rightarrow t_A \propto \sqrt{h}$

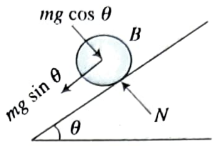
$$\frac{t_B}{t_A} = \sqrt{\frac{4h}{h}} = 2$$

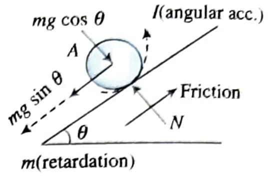
23. (1), (4)

Due to torque of friction about CM, ω eventually decreases to zero, initially there is no translation. Friction is sufficient for pure rolling therefore after sometime pure rolling begin. There is no external force in x direction therefore momentum is conserved along x direction.

24. (2), (4)

If we consider free body diagram of the wheel B, the retarding force on wheel B is equal to $mg \sin\theta$. Therefore, its retardation is equal to $mg \sin\theta$ and no moment is produced by these forces about its centroidal axis. Hence, the wheel B continues to rotate with a constant angular velocity and its translational velocity decreases during up the plane motion. Hence, at highest point it has a rotational KE.





While during ascending motion friction acts on wheel A. That is the friction which is responsible for angular retardation of wheel A as shown in the figure.

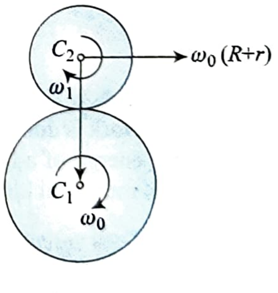
The resultant retarding force on wheel A is equal to ($mg \sin\theta$ – friction). Hence, net retarding force on wheel A is less than $mg \sin\theta$. Therefore, it continues to move up the plane for longer time and comes to rest after moving more distance than wheel B.

Wheel A comes to rest when its total kinetic energy becomes equal to zero. It means, at highest points its potential energy will be equal to total initial KE, while at highest point, potential energy of wheel B is equal to its initial translational KE up. It means, maximum height ascended by wheel A is greater than that by B.

When wheel A descends, it tries to slide down the plane due to component $mg \sin\theta$ of its weight. Hence, the friction remains acting up the plane and produces a moment about centroidal axis. Hence, wheel experiences an angular acceleration in anti-clockwise direction.

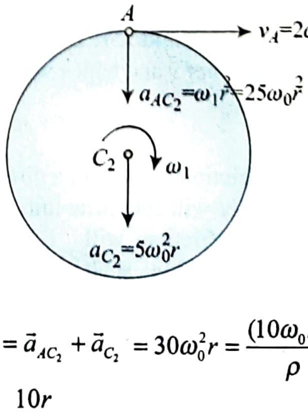
Since, all the remaining forces remains same, therefore, magnitude of friction also remains same.

25. (2), (3), (4)



$$\omega_1 r = \omega_0(R + r) \Rightarrow \omega_1 = 5\omega_0$$

$$\text{KE} = \frac{1}{2}m(\omega_0(R+r)^2) + \frac{1}{2}\left(\frac{mr^2}{2}\right)\omega_1^2 = \frac{75}{4}mr^2\omega_0^2$$



$$\vec{a}_A = \vec{a}_{AC_2} + \vec{a}_{C_2} = 30\omega_0^2 r = \frac{(10\omega_0 r)^2}{\rho}$$

$$\rho = \frac{10r}{3}$$

26. (1), (4)

$$Mg\frac{L}{2}=\frac{1}{2}\left(\frac{ML^2}{3}\right)\omega_0^2$$

$$\omega_0=\sqrt{\frac{3g}{2L}}$$

$$e=\frac{v-\omega(L-r)}{(L-r)\omega_0}=1$$

Conservation of angular momentum

$$\frac{ML^2}{3}\omega_0=\frac{ML^2}{3}\omega+mv(L-r)$$

27. (1), (4)

By conservation of linear momentum both will get same velocity of centre of mass

$$\therefore \quad \frac{1}{2}mv_{cm}^2=mgh$$

$$h_1=h_2$$

But by conservation of angular momentum about centre of mass System 2 have angular velocity also

$$\therefore \quad K_1<K_2 \qquad \left[\text{where } K=\frac{1}{2}mv_{cm}^2+\frac{1}{2}I\omega^2\right]$$

Linked Comprehension Type

For Problems 1–3

1. (1) Since acceleration is same for all the three spheres, they cover equal distances in equal intervals of time in all the cases (A), (B) and (C).

   Hence (1)

2. (3) From passage, for case (C);

$$\mu_{\min}=\frac{\tan\theta\left(\frac{k^2}{R^2}\right)}{\left(1+\frac{k^2}{R^2}\right)} \rightarrow \text{(pure rolling)}$$

Putting the values of k for different objects given in the table (in passage) we get:

$$\mu_{\min(\text{Ring})}=\frac{\tan\theta}{2}$$

$$\mu_{\min(\text{Disc})}=\frac{\tan\theta}{3},\ \mu_{\min(\text{Solid sphere})}=\frac{2}{3}\tan\theta$$

$$\mu_{\min(\text{Hollow sphere})}=\frac{2}{7}\tan\theta$$

$\Rightarrow \mu_{\min\text{ (Ring)}}$ is greater than either of $\mu_{\min\text{ (Disc)}}$, $\mu_{\min\text{ (Solid Sphere)}}$, $\mu_{\min\text{ (Hollow sphere)}}$

Therefore, the pure rolling of ring will confirm pure rolling of all other bodies.

3. (3) As given in the equation of case (B);

$$\mu NR = Mk^2\alpha \text{ and } N = Mg\cos\theta$$

As; θ, M, R, μ are same for all. α will be least for that object for which k and hence I is maximum.

Therefore α for ring $(k = R)$ and hence ω for ring at the bottom is minimum.

Also, $Mg\sin\theta-\mu N=Ma$

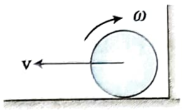
Since M, μ, θ, N are same for all objects, they have same linear acceleration and hence same linear velocity and hence same $\frac{1}{2}Mv_{cm}^2$.

$$\therefore \quad \text{KE}=\left(\frac{1}{2}Mv_{cm}^2+\frac{1}{2}I_{cm}\omega^2\right) \text{ is least for the ring.}$$

4. (2) For ring $a=\frac{g\sin\theta}{2}$ (for pure rolling) is less than that of disc.

For Problems 5–6

5. (2), 6. (1)



Just after collision

$$v=\omega r$$

Velocity of lowest potential $= v+\omega r = 2v$ to the left.

Change in angular momentum is due to translational part only, as ω does not change due to collision.

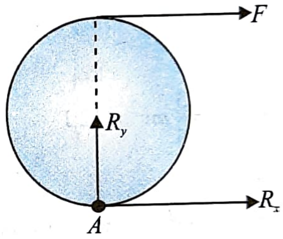
For Problems 7–9

7. (4) Using torque equation about point A

$$\Rightarrow 2FR=I_A\alpha$$

$$\Rightarrow 2FR=\frac{3}{2}MR^2\left(\frac{a}{R}\right)\Rightarrow a_t=\frac{4}{3}\frac{F}{m}$$

8. (2) $R_y = mg$



9. (3) Torque about the CM,

$$(F-R_x)R=I\alpha$$

$$\Rightarrow (F-R_x)R=\frac{Mr^2}{2}\frac{a_t}{r}$$

$$\Rightarrow (F-R_x)R=\frac{2F}{3}$$

$$\Rightarrow F-\frac{2F}{3}=R_x \Rightarrow R_x=\frac{F}{3}$$

For Problems 10–12

10. (4), 11. (1), 12. (4)

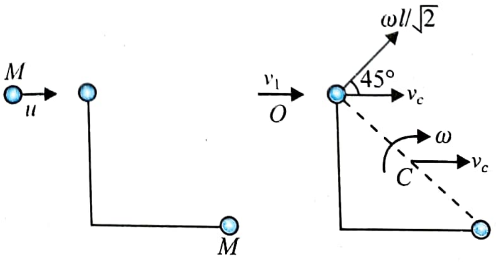
Applying conservation of momentum,

$$Mv_1+2Mv_c=Mu$$

$$v_1+2v_c=u \qquad \text{...(i)}$$

$$e=1=\frac{v_c+\frac{\omega l}{2}\cos 45^\circ-v_1}{u-0}$$

$$u=v_c+\frac{\omega l}{2}-v_1 \qquad \text{...(ii)}$$



Applying conservation of angular momentum about C,

$$Mv_1 \frac{l}{2} + 2M\left(\frac{l}{\sqrt{2}}\right)^2 \omega = Mu\frac{l}{2}$$

$$v_1 + 2\omega l = u \quad \text{...(iii)}$$

Solving Eqs. (i), (ii) and (iii), we get

$$v_1 = -\frac{u}{7};\ \omega = \frac{4u}{7l}$$

Angle rotated, $\theta = \pi$

$$t = \frac{\theta}{\omega} = \frac{7\pi l}{4u}$$

For Problems 13–15

13. (3) Applying conservation of linear momentum,

$$(2 + 1 + 2)\, v_c = 2 \times 10 + 1 \times 10$$

$$v_c = 6 \text{ m/s}$$

14. (4) Applying conservation of angular momentum about the centre of the rod,

$$\left[\frac{2 \times (4)^2}{12} + 2 \times (1)^2 + 1 \times (1)^2\right] \omega = 2 \times 10 \times 1 - 1 \times 10 \times 1$$

$$\omega = \frac{30}{17} \text{ rad/s}$$

15. (4) Linear momentum will decrease and angular momentum will increase.

For Problems 16–18

16. (1), 17. (2), 18. (1)

$$\text{KE} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{3}{4}mv^2 = 12 \text{ J}$$

$$\vec{a}_{A/C} = \vec{a}_A - \vec{a}_C$$

$$\Rightarrow \vec{a}_A = \vec{a}_{A/C} + \vec{a}_C = \omega^2 r\hat{i} + \alpha r\hat{j} + a\hat{i} = 60\hat{i} + 10\hat{j}$$

$$a_A = \sqrt{3600 + 10} = 10\sqrt{37} \text{ m/s}^2$$

$$\vec{a}_B = \omega^2 r\hat{j} + \alpha r(-\hat{i}) + a\hat{i} = 10\hat{i} + 40\hat{j}$$

$$a_B = 10\sqrt{17} \text{ m/s}^2$$

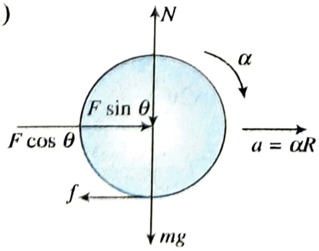
For Problems 19–21

19. (2), 20. (1), 21. (3)

$$F\cos\theta - f = ma \quad \text{...(i)}$$

$$fR = \frac{1}{2}mR^2\alpha \Rightarrow fR = \frac{1}{2}mR^2\frac{a}{R}$$

$$\Rightarrow f = \frac{ma}{2} \quad \text{...(ii)}$$



From (i) and (ii)

$$f = \frac{F\cos\theta}{3}$$

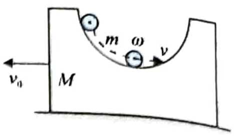
$$\text{Now } a = \frac{2f}{m} = \frac{2F\cos\theta}{3m}$$

$$f \le f_l \Rightarrow \frac{F\cos\theta}{3} \le \mu N$$

$$\Rightarrow \frac{F\cos\theta}{3} \le \mu(mg + F\sin\theta) \Rightarrow F \le \frac{3\mu mg}{\cos\theta - 3\mu\sin\theta}$$

For Problems 22–24

22. (4) As no external force is acting on the system (block + cylinder) in the horizontal direction, the linear momentum of the system remains conserved in the horizontal direction.



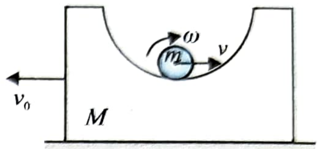
As the cylinder reaches the bottommost point of the track, its centre of mass covers a horizontal distance of 0.50 m $(R - r)$ w.r.t. block towards the right. Let the block move by x in backward direction, then

$$Mx = m(0.5 - x)$$

$$x = \frac{2 \times 0.5}{2 + 3} = 0.2 \text{ m}$$

23. (4) v and v_0 are the velocities w.r.t. ground: $\omega = \frac{v + v_0}{r}$

24. (4) $mv = Mv_0$



$$mg(R - r) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_0^2$$

$$\Rightarrow v_0 = \frac{4}{\sqrt{11}} \text{ m/s}$$

$$v = \frac{6}{\sqrt{11}} \text{ m/s}$$

For Problems 25–27

25. (4) Acceleration of the bottom of the cylinder = acceleration of the plank = 2 m/s² towards the right

Acceleration of the top of the cylinder = acceleration of B = 6 ms^{-2} towards the left

Let linear acceleration of the cylinder be a towards the left and angular acceleration of it α in an anticlockwise sense. Writing constraint, we get

$$a + R\alpha = 6 \text{ and } R\alpha - a = 2$$

$$\Rightarrow \alpha = 1 \text{ rad s}^{-2},\ a = 2 \text{ m/s}^2.$$

26. (2) For block B; $m_B g - T = m_B 6$ ...(i)

For cylinder; $T = m_{\text{cyl}} 2$ ...(ii)

Solving Eqs. (i) and (ii), $m_{\text{cyl}} : m_B = 2 : 1$

27. (1) The acceleration of the top part of the cylinder (thread) with respect to CM of the cylinder is

$$a_{AO} = \alpha R = 1 \times 4 = 4 \text{ m/s}^2$$

Length of the thread $= 20 + \frac{1}{2}(R\alpha)t^2 = 28$ m.

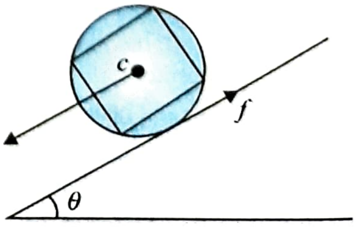
For Problems 28–30

28. (1), 29. (3), 30. (2)

$$I = \left\{\frac{M(R\sqrt{2})^2}{12} + M\left(\frac{R}{\sqrt{2}}\right)^2\right\}4 + mR^2$$

$$= \frac{8}{3}MR^2 + mR^2 = 20 \text{ kg m}^2$$

$$(4M + m)g\sin\theta - f = (4M + m)a \quad \text{...(i)}$$



$$fR = I\frac{a}{R} \qquad ...(ii)$$

Solving Eqs. (i) and (ii),

$$a = \frac{7g}{24}$$

$$f = 20\,a = \mu \times 28g\cos 30°$$

$$\mu = \frac{5}{12\sqrt{3}}$$

For Problems 31–33

31. (3) By conservation of angular momentum on the man–table system,

$$L_i = L_f$$

$$0 + 0 = I_m\,\omega_m + I_t\,\omega_t$$

$$\Rightarrow \omega_t = -\frac{I_m\,\omega_m}{I_t} \text{ where } \omega_m = \frac{v}{r} = \frac{1}{2}\text{ rad/s}$$

$$\Rightarrow \omega_t = -\frac{100\,(2)^2 \times \left(\frac{1}{2}\right)}{4000}$$

$$\Rightarrow \omega_t = -\frac{1}{20}\text{ rad/s}$$

Thus, the table rotates clockwise (opposite to the man) with angular velocity 0.05 rad/s.

32. (4) If the man completes one revolution relative to the table, then

$$\theta_{mt} = 2\pi$$

$$\Rightarrow 2\pi = \theta_m - \theta_t$$

$$2\pi = \omega_m t - \omega_t t \text{ (where } t \text{ is the time taken)}$$

$$t = \frac{2\pi}{\omega_m - \omega_t} = \frac{2\pi}{0.5-(-0.05)} = \frac{2\pi}{0.55}\text{ s}$$

Angular displacement of the table is

$$\theta_t = \omega_t t = -0.05 \times \left(\frac{2\pi}{0.55}\right) = -\left(\frac{2\pi}{11}\right)\text{ radians}$$

The table rotates through $2\pi/11$ radians clockwise.

33. (1) If the man completes one revolution relative to the earth, then

$$\theta_m = 2\pi$$

$$\text{Time} = \frac{2\pi}{\theta_m} = \frac{2\pi}{0.5}$$

During this time, the angular displacement of the table,

$$\theta_t = \omega_t\,(\text{time}) = -0.05 \times \left(\frac{2\pi}{0.5}\right) = -\frac{\pi}{5}\text{ radians}$$

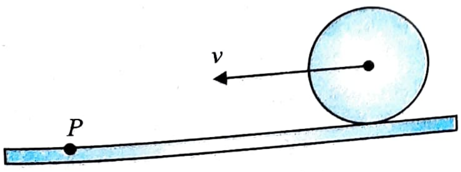
$$\theta_t \equiv 36° \text{ in clockwise direction}$$

For Problems 34–35

34. (2), 35. (2)

In reference frame of truck, the angular momentum is conserved about P, i.e.,

$$MvR = Mv_0R + \frac{2}{5}MR^2\omega_0 \qquad ...(i)$$



Rolling constraint

$$v = R\omega \qquad ...(ii)$$

On solving Eqs. (i) and (ii), we get $v = \frac{5}{7}v_0$

Note that v_0 is the speed in truck frame; in ground frame, the velocity is $v_0 - (5v_0/7) = 2v_0/7$ towards the right.

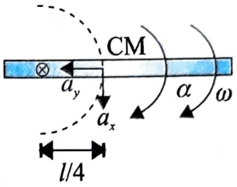
The answer is independent of M or R.

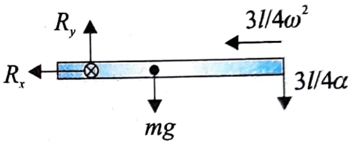
For Problems 36–38

36. (1), 37. (4), 38. (3)

From Conservation of energy, we get

$$\frac{mgl}{4} = \frac{1}{2}m\left[\frac{l^2}{12} + \frac{l^2}{16}\right]\omega^2;\ \omega^2 = \frac{24g}{7l}$$





From Newton's law,

$$\frac{mgl}{4} = \left(\frac{ml^2}{12} + \frac{ml^2}{16}\right)\alpha \qquad ...(i)$$

$$mg - R_y = m\left(\frac{l}{4}\alpha\right) \qquad ...(ii)$$

$$R_x = \frac{ml}{4}\omega^2 \qquad ...(iii)$$

$$\alpha = \frac{12g}{7l};\ \vec{a}_{\text{CM}} = -\frac{\omega^2 l}{4}\hat{i} - \frac{l}{4}\alpha\hat{j} = -\frac{6g}{7}\hat{i} - \frac{3g}{7}\hat{j}$$

$$\vec{a}_B = -\omega^2\left(\frac{3l}{4}\right)\hat{i} - \left(\frac{3l}{4}\right)\alpha\hat{j} = \frac{-18g}{7}\hat{i} - \frac{9g}{7}\hat{j}$$

$$\vec{R}_x = -\frac{6mg}{7}\hat{i};\ \vec{R}_y = +\frac{4mg}{7}\hat{j}$$

For Problems 39–42

39. (1), 40. (2), 41. (1), 42. (3)

Conservation of linear momentum,

$$mv_0 = M\frac{v_0}{8} - m\frac{v_0}{4};\ 8\,m = M - 2m$$

$$\Rightarrow 10\,m = M$$

$$\Rightarrow \frac{M}{m} = 10$$

Conservation of angular momentum about point of collision.

$$0 = \frac{M(8l)^2}{12}\,\omega - M\frac{v_0}{8}\,l$$

$$\Rightarrow \omega = \frac{3v_0}{128\,l}$$

Using coefficient of restitution equation,

$$(v_2 - v_1) = e(u_1 - u_2)$$

$$\left[\left(\frac{v_0}{8} + \omega l\right) - \left(-\frac{v_0}{4}\right)\right] = e[v_0 - 0]$$

$$\Rightarrow e = \frac{51}{128}$$

Velocity at end A, $v_A = \frac{v_0}{8} + \omega(4l) = \frac{28}{128} v_0$

$$\Rightarrow v_A = \frac{7}{32} v_0$$

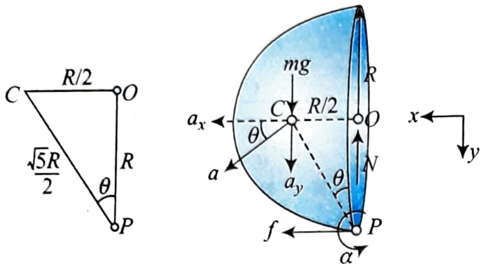
Velocity at end B, $v_B = \frac{v_0}{8} - \omega(4l) = \frac{4}{128} v_0 = \frac{1}{32} v_0$

For Problems 43–44

43. (2) $mg - N = ma_y$

$$mg - N = ma \sin\theta$$

Torque about 'P'



$$mg\frac{R}{2} = \frac{5}{3}mR^2\alpha \Rightarrow \alpha = \frac{3}{10}\frac{g}{R}$$

$$a = \left(\frac{\sqrt{5}R}{2}\right)\alpha = \frac{3\sqrt{5}}{20}g$$

$$\text{So, } N = mg - ma\sin\theta = mg - m\left(\frac{3\sqrt{5}}{20}g\right)\cdot\frac{1}{\sqrt{5}}$$

$$\Rightarrow N = \frac{17}{20}mg$$

In x-direction, $f = ma_x$

For no sliding, $f \le \mu N$

$$ma\cos\theta \le \mu N \quad \text{or} \quad \mu \ge \frac{6}{17}$$

Hence $\mu_{\min} = \frac{6}{17}$

44. (1) For slipping,

$$f = ma_x = \mu N \qquad \text{...(i)}$$

$$mg - N = ma_y \qquad \text{...(ii)}$$

from (i) and (ii), $N\left(\frac{1}{2} - \mu\right) = \left(\frac{5}{12}m\alpha R\right)$ ...(iii)

$$\alpha r \sin\theta = a_y$$

$$\Rightarrow a_y = \frac{\alpha R}{2}$$

$$\frac{mg - N}{m} = \frac{N}{2}\left(\frac{1}{2} - \mu\right) \times \frac{12}{5m}$$

$$N = \frac{5mg}{8 - 6\mu}$$

From $\mu = \frac{3}{17} \Rightarrow N = \frac{85}{118}mg$

Matrix Match Type

1. i. → b., d.; ii. → a.; iii. → a., b., c., d.; iv. → b., d.

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

But as in rolling, $v = r\omega$

$$mgh = \frac{1}{2}mv^2\left[1 + \frac{I}{mr^2}\right] \qquad \left[\text{Putting } \beta = 1 + \frac{I}{mr^2}\right]$$

$$= \frac{1}{2}\beta mv^2$$

$$\Rightarrow v = \sqrt{\frac{2gh}{\beta}}$$

Solving for acceleration, we have $a = \frac{g\sin\theta}{\beta}$

and time taken, $t = \frac{1}{\sin\theta}\sqrt{\beta\left(\frac{2h}{g}\right)}$

2. i. → c.; ii. → c.; iii. → a., b., d.; iv. → a., b., d.

$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

For a solid cylinder: $I = \frac{1}{2}MR^2$

So $a_1 = \frac{2}{3}g\sin\theta$

For a thin shell: $I = MR^2$

$$a_2 = \frac{g\sin\theta}{2}$$

So, $a_1 > a_2$, hence the solid cylinder reaches earlier. Now $v = \sqrt{2al}$, acceleration for the solid cylinder is more, so v on reaching the bottom is more for the solid cylinder.

F_{tran} is maximum and KE_{rot} is minimum for the solid cylinder. KE_{tran} is minimum and KE_{rot} maximum for the thin cylinder.

3. i. → d.; ii. → b., c.; iii. → d.; iv. → a., c.

We know that angular momentum = linear momentum × perpendicular distance. So about O, angular momentum will remain constant. But about E it will keep on changing. It is maximum about E when the particle is at A, because then the perpendicular distance is maximum.

We know that angular velocity (ω) = v/r. So about O, angular velocity will remain constant. But about E it will keep on changing. It is minimum about E when the particle is at A, because then the perpendicular distance (r) is maximum.

4. i. → a.; ii. → b., d.; iii. → a.; iv. → b., d.

- **a.** Speed of point P changes with time.
- **b.** Acceleration of point P is equal to $\omega^2 x$ (ω = angular speed of disc and $x = OP$). The acceleration is directed from P towards O.
- **c.** The angle between acceleration of P (constant in magnitude) and velocity of P changes with time. Therefore, tangential acceleration of P changes with time.
- **d.** The acceleration of the lowest point is directed towards the centre of the disc and remains constant with time.

5. i. → a., b., c.; ii. → a., b., c.; iii. → a., b.; iv. → a., b., c.

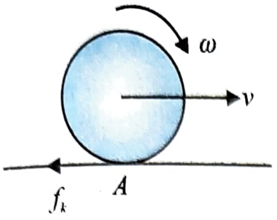
Since all forces on the disc pass through the point of contact with the horizontal surface, the angular momentum of the disc about the point on the ground in contact with disc is conserved. Also the angular momentum of the disc in all cases is conserved about any point on the line passing through the point of contact and parallel to the velocity of the centre of mass.

The KE of the disc is decreased in all cases due to work done by friction. From the calculation of velocity of the lowest point on disc, the direction of friction in case (i), (ii) and (iv) is towards the left and in case r it is towards the right.

The direction of frictional force cannot change in any given case

6. i. → c.; ii. → a., b., c., d., e.; iii. → a., b., c., d., e.; iv. → e.

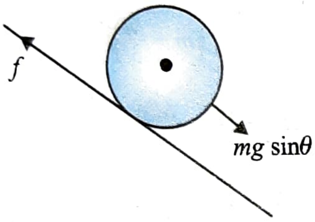
a. When the sphere rolls on the rough surface, its angular momentum about contact point *A* remains constant because torque of friction f_k about this point is zero.



b. Here no friction will act on the sphere. So here everything will remain the same.

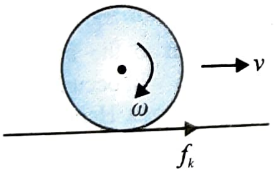
c. Here also no friction will act, so everything will remain the same.

d. Here KE will increase. Potential energy will decrease but the total mechanical energy will remain constant. Angular momentum will not be conserved, because $mg \sin\theta$ will provide the torque about contact point.

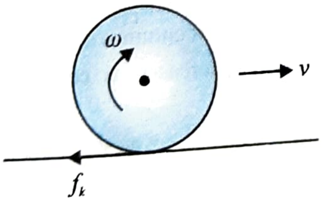


7. i. → a., b., c.; ii. → a.; iii. → d.; iv. → d.

a. v increases, ω decreases, hence translational work by friction is positive and rotational work is negative. But overall work will be negative because the friction is kinetic.



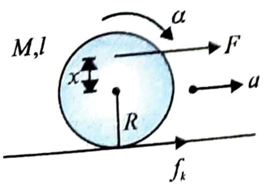
b. v decreases, ω increases, overall work by friction will be negative.



c. This is pure rolling with constant velocity. No friction will act.

d. Here F will pass through the contact point. It will not move. Hence, work by friction is zero.

8. i. → a., b., d.; ii. → a., b., c.; iii. → a., b., d.; iv. → a., b., c.



$$F + f = ma$$

$$Fx - fR = I\alpha,\ \alpha = \frac{a}{R}$$

Solve to get $f = \frac{F(MxR - I)}{I + MR^2}$

$$f = 0, \text{ if } x = \frac{I}{MR}$$

$$f > 0, \text{ if } x > \frac{I}{MR}$$

So friction will act in the forward direction.

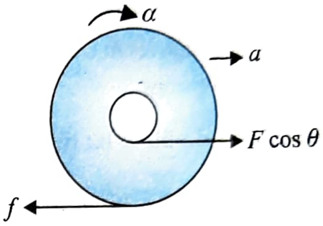
$F < 0$ if $x < I/MR$, so friction will act in the backward direction.

Also solved to get $a = \frac{FR(x+R)}{I + MR^2}$

A is positive for all cases, hence body will accelerate in the forward direction in all cases.

It means α will be in the clockwise direction. Hence, bodies will rotate in the clockwise direction.

9. i. → b., c.; ii. → a., c.; iii. → c.



$$F \cos\theta - f = ma$$

$$fR - Fr = \frac{Ia}{R}$$

$$F \cos\theta - F\frac{r}{R} = a\left(m + \frac{I}{R^2}\right)$$

If $\cos\theta > r/R$, then a is positive

$$a = \frac{F\left(\cos\theta - \frac{r}{R}\right)}{\left(m + \frac{I}{R^2}\right)}$$

If $\cos\theta < r/R$, then a is negative.

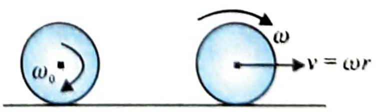
In (i), since a = positive, α should be clockwise hence f is towards the left.

Similarly for others.

10. i. → a., b., d.; ii. → a., b.; iii. → a., b., c.; iv. → a., b.

In each case, the friction force will act in the forward direction. This friction will increase translational velocity and decrease angular velocity.

Apply conservation of angular momentum about the lowest point,



$$I\omega_0 = I\omega + mvr$$

$$I\omega_0 = \frac{Iv}{r} + mvr$$

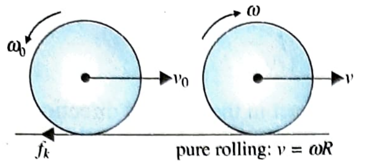
$$v = \frac{I\omega_0 R}{I + mr^2} = \frac{\omega_0 r}{1 + \frac{mr^2}{I}}$$

I is minimum for the solid sphere. So v is minimum for the solid sphere.

Similarly, v will be maximum for the ring. From $v = u + at$, time taken by the ring will be maximum, because acceleration is the same for all.

11. i. → a.; ii. → a.; iii. → a.; iv. → c.

Here angular momentum will be conserved about bottommost point, because no external torque acts about this point.



$$\vec{L}_i = \vec{L}_f \Rightarrow Mv_0R - I_{cm}\,\omega_0 = MvR + I_{cm}\omega$$

$$\Rightarrow Mv_0R - \frac{MR^2}{2}\omega_0 = MvR + \frac{MR^2}{2}\frac{v}{R}$$

$$\Rightarrow v = \frac{2}{3}\left[v_0 - \frac{\omega_0 R}{2}\right]$$

If $v > 0$, finally cylinder moves in forward direction.

If $v < 0$, finally cylinder moves in backward direction.

If $v = 0$, finally cylinder stops.

12. i. → a., c., d.; ii. → b., c., d.; iii. → b.; iv. → b., c., d.

i. Here $x > \frac{I}{mR}$, so friction acts towards right.

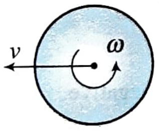
ii. Here $x = 0$, so friction acts towards left.

iii. Here α will be anticlockwise, bottommost point tends to move forward, so friction acts in backward direction.

iv. Similar to 2nd part.

13. i. → a., b.; ii. → b., c.; iii. → a., b.; iv. → a., b.

For (i)



$\frac{2v}{R}$ Angular momentum about $O = mVR + I\omega \circlearrowleft$. Hence final angular momentum should also be in $\circlearrowleft$.

For (ii): Initial angular momentum $= mvR - I\omega \circlearrowleft$ for ring $L_i = mvR - mR^2 \times \frac{2v}{R} = mvR \circlearrowright$; final angular momentum is to be $\circlearrowright$; for solid sphere $L_i = mvR - \frac{2}{5}mR^2\frac{2v}{R} = \frac{mvR}{5} \circlearrowleft$

For (iii): $L_i = mVR + I\omega \circlearrowleft$

For ring $L_i = mVR + MR^2 \times \frac{v}{2R} = \frac{3mVR}{2} \circlearrowleft$

For sphere $L_i = mVR + \frac{2mR^2}{5}\frac{V}{2R} = \frac{6mVR}{5} \circlearrowleft$

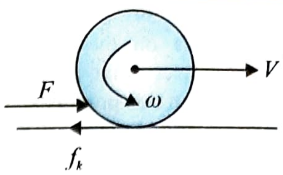
For (iv): $L_i = mVR - I\omega \circlearrowleft$

For ring $L_i = mVR - mR^2\frac{V}{2R} = \frac{mVR}{2} \circlearrowleft$

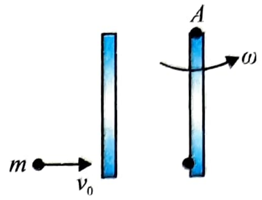
For sphere $L_i = mVR - \frac{2}{5}mR^2\frac{V}{2R} = \frac{4mVR}{5} \circlearrowleft$

14. i. → d.; ii. → b., d.; iii. → a., b., c.; iv. → b., c., d.

a. Immediately the sphere will gain velocity, so momentum increases.



b. Here angular momentum about the hinged point will remain constant.



Applying conservation of angular momentum about A,

$$mv_0L = \left(mL^2 + \frac{ML^2}{3}\right)\omega$$

Final momentum, $\omega = \frac{mv_0}{\left(m + \frac{M}{3}\right)^2}$

$$m\omega L + M\omega\frac{L}{2} = \left(m + \frac{M}{2}\right)\frac{mv_0}{\left(m + \frac{M}{3}\right)}$$

This is greater than mv_0.

c. Here no external force or external torque acts. So all remain conserved, as the collision is elastic.

d. Here also momentum increases. As the collision is elastic, so energy is also conserved. Angular momentum is conserved about the hinged point.

15. i. → a., b., c., d.; ii. → a., d.; iii. → a., b., c., d.; iv. → b., c., d.

a. Since impulse is given to the body, so its momentum will increase and translation will occur. The impulse J does not pass through the centre of mass. Hence, the dumb-bell will rotate and its angular momentum will increase.

b. Here J passes through centre of mass. So rotation does not occur and angular momentum remains zero.

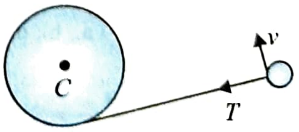
c. This situation is similar to part (a).

d. Here body is hinged. So translation does not occur, rotation will be there so angular momentum will increase.

Linear momentum increases as centre of mass gains some velocity.

16. i. → b., c.; ii. → a., b., c.; iii. → a., d.; iv. → a., d.

a. Angular momentum will not be conserved as T will provide torque about C. Kinetic energy will remain conserved as tension is perpendicular to velocity and it does not do any work. Obviously, the total mechanical energy also remains conserved.



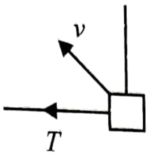
b. Angular momentum remains conserved as no external torque acts. Mechanical energy and kinetic energy of (rod + sleeve) system will remain the same because no work is done by external forces and there is no dissipating force in the system.

c. Angular momentum will remain conserved, because there is no external force (friction is zero because of ice).

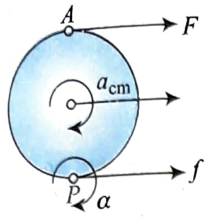
Kinetic energy and mechanical energy will decrease because of work done by internal forces (due to muscles) between the icekaters.

d. Angular momentum will remain conserved, because torque of T will be zero.

T will do positive work on the block due to which kinetic energy and mechanical energy of the block will increase.



17. (2)



Point P is ICR, applying torque equation about P

$$F \times 2R = I_P\alpha \Rightarrow 2FR = (mk^2 + mR^2)\alpha$$

$$\Rightarrow \quad \alpha = \frac{2F}{mR\left(1+\frac{k^2}{R^2}\right)}$$

Hence acceleration of CM; $a_{cm} = \alpha R = \frac{2F}{m\left(1+\frac{k^2}{R^2}\right)}$

From FBD: $F + f = ma_{cm}$

which gives $f = F\left[\frac{1-\frac{k^2}{R^2}}{1+\frac{k^2}{R^2}}\right]$

For solid sphere: $\frac{k^2}{R^2} = \frac{2}{5}$; $a_{cm} = \frac{10}{7}\frac{F}{m}$

and $f = F\left[\frac{1-\frac{2}{5}}{1+\frac{2}{5}}\right] = \frac{3F}{7}$

Similarly, For ring: $\frac{k^2}{R^2} = 1$

$a_{cm} = \frac{F}{m}$ and $f = 0$

For hollow sphere; $\frac{k^2}{R^2} = \frac{2}{3}$

$a_{cm} = \frac{6}{5}\frac{F}{m}$ and $f = \frac{F}{5}$

For disc; $\frac{k^2}{R^2} = \frac{1}{2}$

$a_{cm} = \frac{4}{3}\frac{F}{m}$ and $f = \frac{F}{3}$

Numerical Value Type

1. (2) Applying conservation of energy, we get

$$mgs(\sin\alpha - \sin\beta) = \frac{1}{2}I\omega^2 + 2\left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)$$

where $\omega = v/r$ and $I = mr^2/2$

Putting values and solving, we get $v = 2$ m/s.

2. (0) Let f be the friction force acting on the disc in the backward direction. Then $2F - f = ma$ and $(F + f)r = I\omega$

$$\Rightarrow \quad (F + f)r = \frac{1}{2}mr^2\frac{a}{r}.$$

∴ Solving the above equations, we get $f = 0$.

3. (6) Since $10 < mg$, the cylinder rolls on ground without lifting up

KE = Workdone = $F \times$ Displacement $= 10 \times \frac{3}{5} = 6$ J

Note: Point of application of F will move upward by same distance as the cylinder moves on the ground.

4. (5) $a_c = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta$

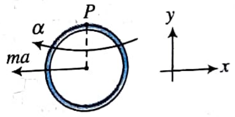
$$a_P = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{2}$$

$$a_{rel} = a_C - a_P = \frac{g\sin\theta}{6} = \frac{10\times\frac{1}{2}}{6} = \frac{5}{6}$$

$$s_{rel} = \frac{1}{2}\times\frac{5}{6}\times(2\sqrt{3})^2 = 5\text{ m}$$

5. (2) The pseudo force acting at the CM of the ring is $\vec{F}_{ps} = -ma\hat{i}$ which produces a torque about P given as

$$\vec{\tau}_{ps} = -(ma)(R)\hat{k}$$



This produces an angular acceleration

$$\vec{\alpha} = \frac{\vec{\tau}_{ps}}{I_P} = \frac{-maR\hat{k}}{2mR^2} \qquad (\because I_P = mR^2 + mR^2 = 2mR^2)$$

$$= -\frac{a}{2R}\hat{k} = \frac{4}{2\times1} = 2\text{ rad/s}^2$$

6. (4) FBD: The forces acting on the body are $F \rightarrow$, $mg \downarrow$ and $N \uparrow$ and let us assume that f is backward (←).

Force equation: $F - f = ma$ ...(i)

$N - mg = 0$ ...(ii)

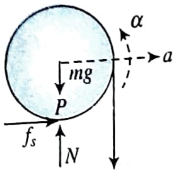
Torque equation: $f \cdot R = \frac{2}{5}mR^2\alpha$ ...(iii)

Condition for rolling $a = R\alpha$ ...(iv)

Putting a from Eq. (i), α from Eq. (iii) in Eq. (iv), we have

$$\frac{F-f}{m} = R\cdot\frac{5f}{2mR} \Rightarrow f = \frac{2}{7}F = 4\text{ N}$$

7. (1) FBD: Let the static friction f_s act towards right. It pushes the body with an acceleration a, say and rotate the body with an angular acceleration α, say. The other forces acting on the body are $N\uparrow$ and $mg\uparrow$.



Force equation: $f_s = ma$ ...(i)

$N = mg$ ...(ii)

Torque equation:

$\tau_C = f_s R = mk^2\alpha$ ...(iii)

Rolling: $a_P = a_0$ where $a_P = a + R\alpha$

$$a + R\alpha = a_0 \qquad \text{...(iv)}$$

Substituting a from Eq. (i), α from Eq. (iii) in Eq. (iv), we have

$$\frac{f_s}{m} + R\left(\frac{f_s R}{mk^2}\right) = a_0$$

$$f_s = \frac{ma_0}{1+\dfrac{R^2}{k^2}} = \frac{4\times 1}{(1+3)} = 1\text{ N}$$

8. **(7)** $\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh$

$$\frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v^2}{R^2}\right) = Mgh$$

$$\frac{7}{10}Mv^2 = Mgh$$

$$h = \frac{7v^2}{10g} = \frac{7(10)^2}{10(10)} = 7\text{ m}$$

9. **(6)** Total kinetic energy of loop $= \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$

$$= \frac{1}{2}(MR^2\omega^2) + \frac{1}{2}Mv^2 = Mv^2 = 8\text{ J (given)}$$

Total kinetic energy of disc $= \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$

$$= \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}Mv^2 = \frac{3}{4}Mv^2 = \frac{3}{4}(8) = 6\text{ J}$$

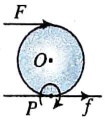
10. **(5)** $K = K_R + K_T = \frac{1}{2}\left[mR^2 + m\frac{(2R)^2}{12}\right]\left[\frac{V}{R}\right]^2 + \frac{1}{2}(2m)V^2$

$$= \frac{5}{3}mv^2 = \frac{5}{3}(3)(1)^2 = 5\text{ J}$$

11. **(20)** $F + f = ma \qquad \text{...(i)}$

Torque equation about point of contact

$$F2R = I_P\alpha$$



$$F2R = \left(\frac{2}{5}mR^2 + mR^2\right)\alpha$$

$$F = \frac{7}{10}mR\alpha \qquad \text{...(ii)}$$

For no sliding case $a = R\alpha \qquad \text{...(iii)}$

From (ii) and (iii), $a = \frac{10F}{7m} \qquad \text{...(iv)}$

From (i) and (iv), $F + f = m\cdot\frac{10}{7}\frac{F}{m} \Rightarrow f = \frac{3}{7}F$

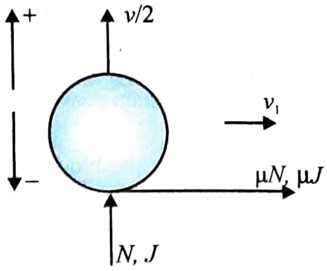
i.e., friction will act in backward direction. For no sliding $f \le \mu N$

$$\frac{3}{7}F \le \mu mg \Rightarrow F \le \frac{7}{3}\mu mg$$

$$F \le \frac{7}{3}\times\frac{2}{7}\times 3\times 10 \Rightarrow F \le 20\text{ N}$$

12. **(2)** $J = \int N\,dt = m\frac{v}{2} - (-mv) = \frac{3}{2}mv \qquad \text{...(i)}$

$$\mu JR = \int \mu(N\,dt)\,R = \left(\tfrac{2}{5}mR^2\omega - 0\right) = \tfrac{2}{5}mR^2\omega \qquad \text{...(ii)}$$



From Eqs. (i) and (ii), we get

$$\frac{3}{2}mv\,R\mu = \frac{2}{5}mR^2\omega \qquad \text{...(iii)}$$

Let V and V_1 be the speeds of the plank and the sphere, respectively, in the horizontal direction.

$$\mu J = \int \mu N dt = MV = mV_1 \qquad \text{...(iv)}$$

From Eqs. (i) and (iv), $\mu\left(\frac{3}{2}\right)mv = MV$

$$V = \frac{3}{2}\frac{\mu mv}{M} = \frac{3}{2}\frac{4}{15}\frac{mR\omega}{M} = \frac{2}{5}\frac{m}{M}R\omega$$

and $V_1 = \frac{2}{5}R\omega = 2\text{ m/s}$

13. **(4.00)** $fr\Delta t = I_{cm}\omega \qquad \text{...(i)}$

$$-f\Delta t = m(0 - v_0) \qquad \text{...(ii)}$$

$$\omega r = v_1 \qquad \text{...(iii)}$$

$$\Rightarrow \quad mv_0 r = \frac{2}{5}mr^2\omega$$

$$\Rightarrow \quad \omega = \frac{5v_0}{2r} \Rightarrow v_1 = \frac{5v_0}{2}$$

$$\Rightarrow \quad v_0 = \frac{2v_1}{5} \Rightarrow v_0 = \frac{2\times 10}{5} = 4\text{ m/s}$$

14. **(5.00)** From COAM about CM,

We get, $v_0\frac{l_0}{2} = \frac{(l_0 + x)}{2}v,$

where x = maximum elongation = 0.4 m

natural length = 2.4 m

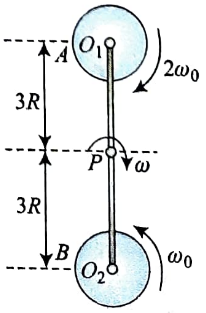
v = speed of the sphere in CM frame at maximum elongation

From conservation of energy,

$$2\times\frac{1}{2}mv_0^2 = \frac{1}{2}(2m)\left(\frac{\sqrt{2}v_0}{2}\right) + 2\frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

Solving, we get, $v_0 = 5\text{ m/s}$

15. **(2.00)** The angular velocities of both the disc will remain unchanged



From conservation of mechanical energy

$$\Delta K + \Delta U = 0 \Rightarrow \Delta K = -\Delta U$$

$$\left(\frac{1}{2}I_P\omega^2-0\right)=-(Mg3R-2Mg3R)$$

$$I_P=\frac{M(6R)^2}{12}+M(3R)^2+2M(3R)^2=30MR^2$$

$$\Rightarrow 3MgR=15MR^2\omega^2 \text{ or } \omega=\sqrt{\frac{g}{5R}}$$

On substituting the given values we get $\omega=\sqrt{\frac{10}{5\times0.5}}=2$ rad/s

16. **(20.00)** The centre of mass of the ring rolls in a circle of radius $(3R/4)$.

$\Rightarrow$ Velocity of centre of mass at the topmost point,

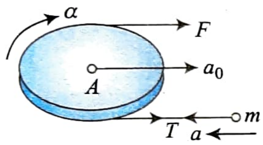
$$v=\sqrt{g\times\frac{3R}{4}}$$

$$\Rightarrow \omega=\sqrt{\frac{3gR}{4}}\Big/\frac{R}{4}=\sqrt{\frac{12g}{R}}=2\sqrt{\frac{3g}{R}}$$

On substituting the given values we get

$$\omega=2\sqrt{\frac{3\times10}{0.3}}=20 \text{ rad/s}$$

17. **(5.00)** For no slipping condition of thread acceleration of point A on the circumference of the disc = acceleration of the small block.



$$\Rightarrow R\alpha=a_0+a \quad \text{...(i)}$$

Other equations are

$$T=ma \quad \text{...(ii)}$$

$$F+T=ma_0 \quad \text{...(iii)}$$

and $(F-T)R=\frac{1}{2}mR^2\cdot\alpha$

$$\Rightarrow F-T=\frac{1}{2}mR\alpha \quad \text{...(iv)}$$

Using (i) in (iv) $F-T=\frac{1}{2}m(a+a_0)$

Using (ii) in this $F=\frac{3}{2}ma+\frac{1}{2}ma_0$ ...(v)

From (ii) and (iii) $F=ma_0-ma$ ....(vi)

Solving (v) and (vi) $a=\frac{F}{4m}$

Hence $a=\frac{20}{4\times1}=5 \text{ m/s}^2$

18. **(1.20)** If we take 'plate and particle' as system. There is no external torque about the axis passing through O. Hence angular momentum of the system should be conserved.

Let ω = angular speed of square plate when the particle moves out from point B. Let it is in clockwise sense.

Angular momentum conservation about rotation axis passing through O,

$$mv_A\frac{l}{2}(-\hat{k})=mv_B\frac{l}{2}(\hat{k})+I_O\omega(-\hat{k})$$

$$-1\times2\times1=1\times v_B\times1-4\omega$$

$$\Rightarrow \omega=\frac{v_B+2}{4} \quad \text{...(i)}$$

Conserving mechanical energy

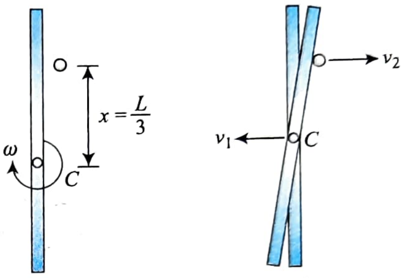
$$\frac{1}{2}\times1\times2^2=\frac{1}{2}\times1\times v_B^2+\frac{1}{2}\times4\times\omega^2$$

$$v_B^2+4\omega^2=4 \quad \text{...(ii)}$$

From (i) and (ii), $v_B^2+4\left(\frac{v_B+2}{4}\right)^2=4$

$$5v_B^2+4v_B-12=0\Rightarrow v_B=\frac{6}{5}\text{ m/s}=1.20\text{ m/s}$$

19. **(3.00)** Let the velocity of particle after collision be v_1 and that of centre of the stick be v_2 (as shown)



Before collision After collision

Momentum conservation gives: $mv_1=Mv_2$ ....(i)

Angular momentum conservation about C:

$$\frac{ML^2}{12}\omega=mv_1\frac{L}{3}\Rightarrow v_1=\frac{ML\omega}{4m} \quad \text{...(ii)}$$

Collision is elastic, i.e., $e=1$

$$\frac{v_1-(-v_2)}{\omega\frac{L}{3}-0}=1\Rightarrow v_1+v_2=\frac{\omega L}{3}$$

Substituting for v_1 and v_2 from (i) and (ii)

$$\frac{M\omega L}{4m}+\frac{\omega L}{4}=\frac{\omega L}{3} \quad \text{...(iii)}$$

$$\Rightarrow \frac{m}{M}=3$$

Archives

JEE Main

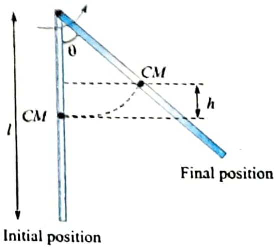
Single Correct Answer Type

1. **(4)** The uniform rod of length l and mass m can swing about an axis passing through the end. When it is given angular velocity ω it starts rotating in vertical plane. When the centre of mass is raised through h, the increase in potential energy is mgh.

This is equal to the decrease in kinetic energy.

Using conservation of mechanical energy $\Delta K+\Delta U=0$

$$\left(0-\frac{1}{2}I\omega^2\right)+(mgh)=0$$



$$\Rightarrow mgh=\frac{1}{2}\left(m\frac{l^2}{3}\right).\omega^2$$

$$\therefore h=\frac{l^2.\omega^2}{6g}$$

2. (3) The position vector of the particle from the origin at any time t can be calculated by using $\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$$\Rightarrow \vec{r} = v_0\cos\theta t\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\hat{j}$$

∴ Velocity vector at any time can be calculated by using $\vec{v} = \vec{u} + \vec{a}t$,

$$\Rightarrow \vec{v} = v_0\cos\theta\hat{i} + (v_0\sin\theta - gt)\hat{j}$$

The angular momentum of the particle about the origin is

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$\vec{L} = m(\vec{r} \times \vec{v})$$

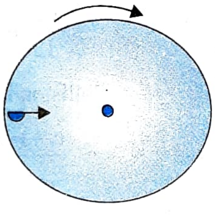
$$= m\Big[v_0\cos\theta t\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\hat{j} \times (v_0\cos\theta\hat{i} + (v_0\sin\theta - gt)\hat{j})\Big]$$

$$= m\Big[(v_0^2\cos\theta\sin\theta t - v_0gt^2\cos\theta)\hat{k} + (v_0^2\sin\theta\cos\theta t - \frac{1}{2}gt^2v_0\cos\theta)(-\hat{k})\Big]$$

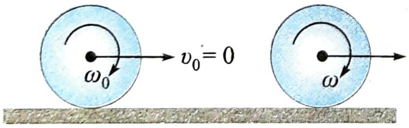
$$= m\Big[v_0^2\sin\theta\cos\theta t\hat{k} - v_0gt^2\cos\theta\hat{k} - v_0^2\sin\theta\cos\theta t\hat{k} + \frac{1}{2}v_0gt^2\cos\theta\hat{k}\Big]$$

$$= m\left[-\frac{1}{2}v_0gt^2\cos\theta\hat{k}\right] = -\frac{1}{2}mgv_0t^2\cos\theta\hat{k}$$

3. (4) From angular momentum conservation about vertical axis passing through centre: When insect is coming from circumference to center, moment of inertia decreases and when insect is going from centre to circumference, moment of inertia increases. So angular velocity first increases and then decreases.



4. (2)



From conservation of angular momentum about any fix point on the surface. Applying conservation at point of contact,

$$mr^2\omega_0 = mvr + mr^2\omega = mvr + mr^2\left(\frac{v}{r}\right)$$

$$\Rightarrow mr^2\omega_0 = mvr + mvr = 2mvr$$

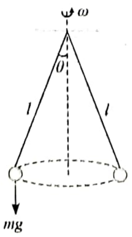
or $v = \dfrac{r\omega_0}{2}$

5. (1) Torque of weight about point of suspension.

$$\tau_{\text{weight}} = mg \times l\sin\theta$$

Direction of torque by weight is parallel to the plane of rotation of the particle.

As $\vec{\tau}$ is perpendicular to the angular momentum of the bob so the magnitude of angular momentum remains the same but direction changes.



6. (2), (4) When the particle moves along AB, $\vec{L} = mvr_\perp(-\hat{k})$

$$\vec{L}_{AB} = mg\frac{R}{\sqrt{2}}(-\hat{k})$$

When the particle moves from C to D, $\vec{L} = mvr_\perp(\hat{k})$

$$\vec{L}_{CD} = mv\left[\frac{R}{\sqrt{2}} + a\right](\hat{k})$$

When the particle moves from B to C,

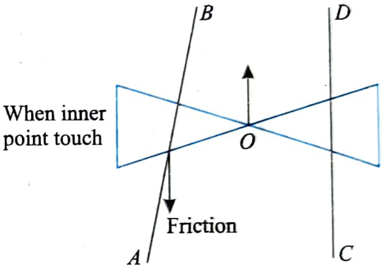
$$\vec{L}_{BC} = mvr_\perp(\hat{k}) = mv\left(\frac{R}{\sqrt{2}} + a\right)(\hat{k})$$

When the particle moves from D to A

$$\vec{L}_{DA} = mvr_\perp(-\hat{k}) = mv\frac{R}{\sqrt{2}}(-\hat{k})$$

So (2) and (4) are false.

7. (1) When this cone proceeds then on AB rail the inner point will touch the rail whose velocity in forward direction slips forward and friction acts in backward direction. Due to this friction the system turns left.



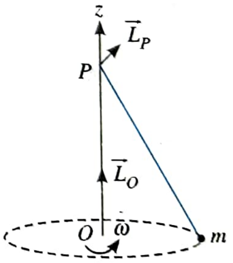
JEE Advanced

Single Correct Answer Type

1. (1) Linear momentum remains constant if net external force on the system of particles is zero.

2. (3) Magnitude and direction of $\vec{L}_O$ remain constant.

Magnitude of $\vec{L}_P$ remains constant but direction of $\vec{L}_P$ changes.



3. (4) $I_P > I_Q$

$$a_P = \frac{g\sin\theta}{1 + \frac{I_P}{mR^2}}, \quad a_Q = \frac{g\sin\theta}{1 + \frac{I_Q}{mR^2}}$$

$$a_P < a_Q \Rightarrow V = u + at \Rightarrow t \propto \frac{1}{a}$$

$$I_P > I_Q$$

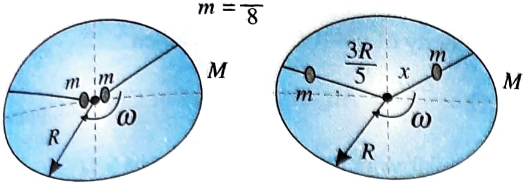
$$V^2 = u^2 + 2as \Rightarrow V \propto a \Rightarrow V_P < V_Q$$

Translational KE, $\frac{1}{2}mV^2 \Rightarrow$ TR KE$_P <$ TR KE$_Q$

$$V = \omega R \Rightarrow \omega \propto V \Rightarrow \omega_P < \omega_Q$$

4. (4) Angular momentum of ring and mass system should be conserved.

$$m=\frac{M}{8}$$



Initially Finally

$I\omega=$ constant

Initial moment of inertia of system (ring + masses)

$$I_{\text{initial}}=MR^2$$

Final moment of inertia

$$MR^2\cdot\omega=\left(MR^2+\frac{M}{8}\frac{9}{25}R^2+\frac{M}{8}x^2\right)\frac{8\omega}{9}$$

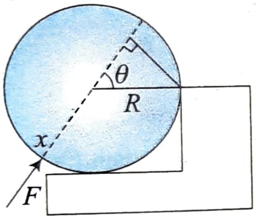
$$\frac{9}{8}MR^2=MR^2+\frac{9}{25}\frac{MR^2}{8}+\frac{M}{8}x^2$$

$$\frac{Mx^2}{8}=MR^2\left(\frac{1}{8}-\frac{9}{25\times 8}\right)$$

$$\frac{M}{8}x^2=\frac{2}{25}MR^2$$

$$\Rightarrow\ x^2=\frac{16}{25}R^2\ \Rightarrow\ x=\left(\frac{4}{5}\right)R$$

5. (3) If the force is applied normal to the circumference at point X



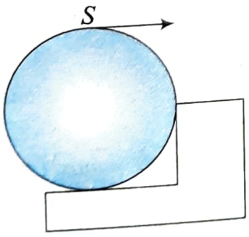
If force is applied normal to surface at point X

$$\tau=FR\sin\theta$$

$$\tau\propto\sin\theta$$

Since when θ increases, τ decreases. Thus τ depends on θ and it is not constant. Hence option (1) is incorrect.

If the force is applied tangentially at point S

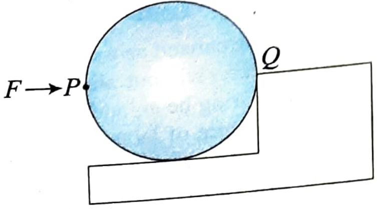


If force applied tangentially at S, then

$$\tau=F\times R\neq 0$$

Wheel can climb, so option (2) is incorrect.

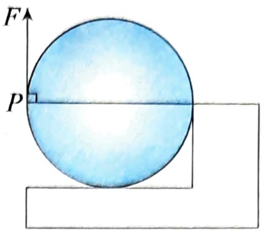
If the force is applied normal to the circumference at point P



If force is applied normal to surface at P then line of action of force will pass from Q and thus $\tau=0$.

So option (3) is correct.

If the force is applied at point P tangentially

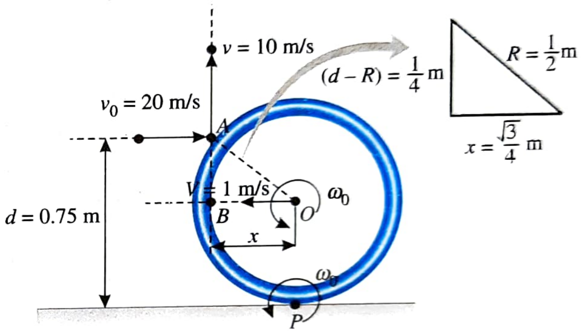


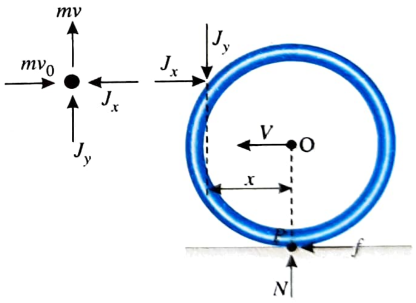
$\tau=F\times 2R=$ constant, so option (4) is incorrect.

Multiple Correct Answers Type

1. (1), (3)

The friction and normal reaction on the ring is impulsive in nature.





Considering the motion of ball,

$$mv_0-J_x=0\ \text{ or }\ J_x=mv_0$$

Hence, $J_x=0.1\times 20=2$ kg m/s

and $J_y=mv=0.1\times 10=1$ kg m/s

Now considering the motion of ring.

$$J_x=MV_f-(-MV)$$

$$2=MV_f+2\times 1\Rightarrow V_f=0$$

Hence, the linear velocity of ring becomes zero just after collision.

Conserving angular momentum of system (ring + ball) about point of contact (P).

$$I_P\omega_0(\hat{k})+mv_0d(-\hat{k})=I_0\vec{\omega}+mvx(-\hat{k})$$

$$I_0\vec{\omega}=I_P\omega_0\hat{k}+m[vx-v_0d]\hat{k}$$

As ring is rolling $\omega_0 = \frac{V}{R} = \frac{1}{1/2} = 2$ rad/s

$$2\times\left(\frac{1}{2}\right)^2 \vec{\omega} = \frac{3}{2}\times 2\times\left(\frac{1}{2}\right)^2 \times 2\hat{k} + 0.1\left[10\times\frac{\sqrt{3}}{4} - 20\times\frac{1}{4}\right]\hat{k}$$

$$2\times\left(\frac{1}{2}\right)^2 \vec{\omega} = \frac{3}{2}\hat{k} + 0.1\times\frac{10}{4}[3-2]\,\hat{k}$$

Hence, $\omega \neq 0$ and will be in counter clockwise direction. Now friction will be kinetic in nature and will act in leftward direction.

2. (1), (2), (4)

$$\vec{r} = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 3\alpha t^2 \hat{i} + 2\beta t \hat{j}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 6\alpha t \hat{i} + 2\beta \hat{j}$$

At $t = 1$

(a) $\vec{v} = 3\times\frac{10}{3}\times 1\hat{i} + 2\times J\times 1\hat{j} \;= 10\hat{i} + 10\hat{j}$

(b) $\vec{L} = \vec{r}\times\vec{p}$

$$= \left(\frac{10}{3}\times 1\hat{i} + 5\times 1\hat{j}\right)\times 0.1(10\hat{i} + 10\hat{j})$$

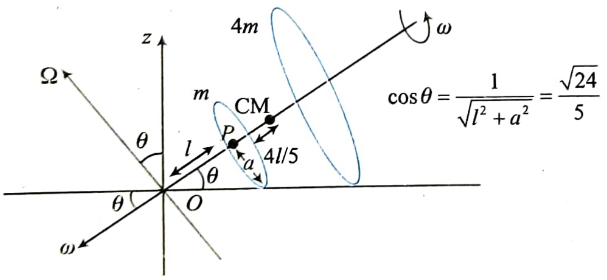
$$= -\frac{5}{3}\hat{k}$$

(c) $\vec{F} = m\times\left(6\times\frac{10}{3}\times 1\hat{i} + 2\times 5\hat{j}\right) = 2\hat{i} + \hat{j}$

(d) $\vec{\tau} = r\times\vec{F}$

$$= \left(\frac{10}{3}\hat{i} + 5\hat{j}\right)\times(2\hat{i} + \hat{j}) \;= +\frac{10}{3}\hat{k} + 10(-\hat{k}) \;= -\frac{20}{3}\hat{k}$$

3. (2), (4)



$\cos\theta = \frac{1}{\sqrt{l^2 + a^2}} = \frac{\sqrt{24}}{5}$

(a) $L_Z = L_{CM-0}\cos\theta - L_{D-CM}\sin\theta$

$$= \frac{81\sqrt{24}}{5}a^2 m\omega\times\frac{\sqrt{24}}{5} - \frac{17ma^2\omega}{2}$$

$$\times\frac{1}{\sqrt{24}} = \frac{81\times 24ma^2\omega}{25} - \frac{17ma^2\omega}{2\sqrt{24}}$$

(b) $L_{D-CM} = \frac{ma^2}{2}\omega + \frac{4m(2a)^2}{2}\omega = \frac{17ma^2\omega}{2}$

(c) $L_{CM-O} = (5m)\left[\frac{9l}{5}\Omega\right]\frac{9l}{5} = \frac{81ml^2\Omega}{5} = \frac{81ml^2}{5}\times\frac{a\omega}{l}$

$$L_{CM-O} = \frac{81mla\omega}{5} = \frac{81\sqrt{24}a^2 m\omega}{5}$$

(d) Velocity of point P : $a\omega = l\Omega$ then

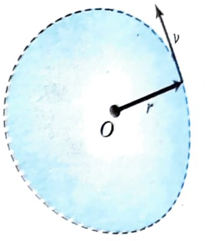
$\Omega = \frac{a\omega}{l}$ = Angular velocity of CM w.r.t. point O.

Angular velocity of CM w.r.t. z axis = $\Omega\cos\theta$

$$\omega_{CM-z} = \frac{a\omega}{l}\frac{\sqrt{24}}{5} = \frac{a\omega}{\sqrt{24}a}\frac{\sqrt{24}}{5}$$

$$\omega_{CM-z} = \frac{\omega}{5}$$

4. (2), (3)



We are given potential energy, $V = \frac{kr^2}{2}$

Differentiating P.E. function w.r.t. r

$$\frac{dV}{dr} = \frac{k}{r}\cdot 2r = kr$$

Hence force, $F = -kr$ (towards centre)

$$\left[\text{As}\quad F = -\frac{dV}{dr}\right]$$

At $r = R$, $kR = \frac{mv^2}{R}$ [Centripetal force]

$$V = \sqrt{\frac{kR^2}{m}} = \sqrt{\frac{k}{m}}R$$

Hence option (2) is correct.

We define angular momentum $L = mvR$

or $L = m\sqrt{\frac{k}{m}}R^2$

Hence option (3) is also correct.

Linked Comprehension Type

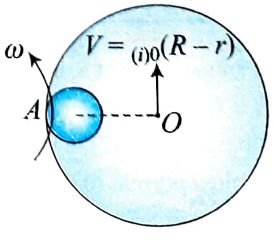
For Problems 1–2

1. (2) A will be instantaneous centre of rotation

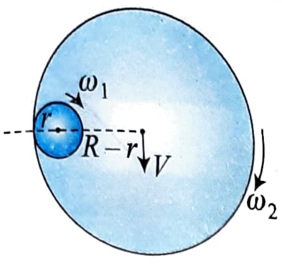
$$\omega R = \omega_0(R - r)$$

$$\omega = \omega_0\left(\frac{R - r}{R}\right)$$

$$\text{KE} = \frac{1}{2}(2mR^2)\omega^2 = m\omega_0^2(R - r)^2$$



2. (2) The motion can be visualized as a larger ring spinning around a smaller rotating disk, without slipping. Let angular speeds of smaller and larger ring be ω_1 and ω_2. In addition to rotation about its own axis centre of larger ring is moving say with a speed v. Centre of larger ring is moving in a circle of radius $(R - r)$.



So, $V=(R-r)\,\omega_2$ ...(i)

Also, no slipping at the point of contact gives

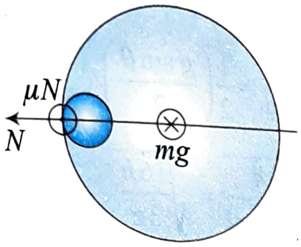
$$r\omega_1 = R\omega_2 - V$$ ...(ii)

(i) and (ii) give $\omega_1=\omega_2\rightarrow\omega_0$

Now draw the FBD of ring

$$N = ma_c = m\omega_0^2(R-r)$$ ...(iii)

Balancing forces in vertical direction for translational equilibrium, $mg=\mu N$



$$\therefore\quad mg=\mu m\omega_0^2(R-r)\quad \text{i.e. } \omega_0=\sqrt{\frac{g}{\mu(R-r)}}$$

Matrix Match Type

1. (1) (P) $\vec r(t)=\alpha t\hat i+\beta t\hat j$

$$\vec v=\frac{d\vec r(t)}{dt}=\alpha\hat i+\beta\hat j\{\text{constant}\}$$

$$\vec a=\frac{d\vec v}{dt}=0$$

$$\vec p=m\vec v(\text{ remain constant})$$

$$K=\frac{1}{2}mv^2\ \text{(remain constant)}$$

$$\vec F=-\left[\frac{\partial U}{\partial x}\hat i+\frac{\partial U}{\partial y}\hat i\right]=0$$

$\Rightarrow$ U → constant

$$E=K+U$$

$$\frac{d\vec L}{dt}=\vec\tau=\vec r\times\vec F=0$$

$$\vec L=\text{constant}$$

(Q) $\vec r=\alpha\cos(\omega t)\hat i+\beta\sin(\omega t)\hat j$

$$\vec v=\frac{d\vec v}{dt}=-\alpha\omega\sin(\omega t)\hat i+\beta\omega\cos(\omega t)\hat j$$

$$\vec a=\frac{d\vec v}{dt}=-\alpha\omega^2\cos(\omega t)\hat i-\beta\omega^2\sin(\omega t)\hat j$$

$$=-\omega^2\left[\alpha\cos(\omega t)\hat i+\beta\sin(\omega t)\hat j\right]$$

$$\vec a=-\omega^2\vec r$$

$$\vec\tau=\vec r\times\vec F=0\ \{\vec r \text{ and } \vec F \text{ are parallel}\}$$

$$\Delta U=-\int\vec F\cdot dr=+\int_0^r m\omega^2\cdot r\cdot dr$$

$$\Delta U=m\omega^2\left[\frac{r^2}{2}\right]$$

$$U\propto r^2$$

$r=\sqrt{\alpha^2\cos^2(\omega t)+\beta^2\sin^2(\omega t)}$ r is a function of time (t)

U depends on r hence it will change with time.

Total energy remains constant because force is central.

(R) $\vec r(t)=\alpha\left(\cos\omega t\hat i+\sin(\omega t)\hat j\right)$

$$\vec v(t)=\frac{d\vec r(t)}{dt}$$

$$=\alpha\left[-\omega\sin(\omega t)\hat i+\omega\cos(\omega t)\hat j\right]$$

$|\vec v|=\alpha\omega$ (Speed remains constant)

$$\vec a(t)=\frac{d\vec v(t)}{dt}$$

$$=\alpha\left[-\omega^2\cos(\omega t)\hat i-\omega^2\sin(\omega t)\hat j\right]$$

$$=-\alpha\omega^2\left[\cos(\omega t)\hat i+\sin(\omega t)\hat j\right]$$

$$\vec a(t)=-\omega^2(\vec r)$$

$$\vec\tau=\vec F\times\vec r=0$$

$|\vec r|=\alpha$ (remains constant) Force is central in nature and distance from fixed point is constant.

Potential energy remains constant. Kinetic energy is also constant (speed is constant) energy remains constant

(S) $\vec r=\alpha t\hat i+\frac{\beta}{2}t^2\hat j$

$\vec v=\frac{d\vec r}{dt}$ (Speed of particle depends on 't')

$$\vec a=\frac{d\vec v}{dt}=\beta\hat j\{\text{constant}\}$$

$$\vec F=m\vec a\ \{\text{constant}\}$$

$$\Delta U=-\int\vec F\cdot d\vec r=-m\int_0^t\beta\hat j.\left(\alpha\hat i+\beta t\hat j\right)dt$$

$$U=\frac{-m\beta^2t^2}{2}$$

$$K=\frac{1}{2}mv^2=\frac{1}{2}m\left(\alpha^2+\beta^2t^2\right)$$

$$E=K+U$$

$$=\frac{1}{2}m\alpha^2\left[\text{remains constant}\right]$$

Numerical Value Type

1. **(4)** Since net torque about centre of rotation is zero, so we can apply conservation of angular momentum of the system about center of disc

$$L_i = L_f$$

$0 = I\omega + 2mv\,(r/2)$; comparing magnitude

$$\therefore \quad \left(\frac{0.45 \times 0.5 \times 0.5}{2}\right)\omega = 0.05 \times 9 \times \frac{0.5}{2} \times 2$$

$$\therefore \quad \omega = 4$$

2. **(7)** Kinetic energy of a purely rolling disc

$$= \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{m}{2}\right)V^2 = \frac{3}{4}mV^2$$

Using conservation of energy on both discs and equating their final energies

$$\frac{3}{4}mV_1^2 + mg \cdot 30 = \frac{3}{4}mV_2^2 + mg \cdot 27$$

$$3mg = \frac{3}{4}m(V_2^2 - V_1^2)$$

$$4g = V_2^2 - 9$$

$$V_2^2 = 49$$

$$V_2 = 7$$

3. **(6)** Moment of inertia of a hollow sphere is $I_0 = \frac{2}{3}mR^2$

Here, for sphere A:

$$I_A = \int_0^R \frac{2}{3}r^2 \cdot 4\pi r^2\,drK\frac{r}{R}$$

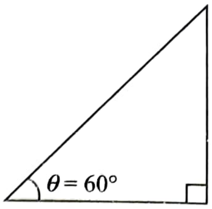
$$= \frac{8\pi K}{3R}\int_0^R r^5 dr = \frac{8\pi K}{3R}\frac{R^6}{6}$$

Also, for sphere B:

$$I_B = \int_0^R \frac{2}{3}r^2 \cdot 4\pi r^2 dr\frac{K}{R^5} \cdot r^5 = \frac{8\pi K}{3R^5} \cdot \frac{R^{10}}{10}$$

Ratio, $\frac{I_B}{I_A} = \frac{6}{10}$

4. **(0.75)** The acceleration of a rolling object in an inclined plane.



$$a_c = \frac{g\sin\theta}{1 + \frac{I_C}{MR^2}}$$

For ring $I_C = MR^2 \Rightarrow \quad a_{ring} = \frac{g\sin\theta}{2}$

For disc $I_C = \frac{MR^2}{2} \Rightarrow \quad a_{disc} = \frac{2g\sin\theta}{3}$

For ring: $\frac{h}{\sin\theta} = \frac{1}{2}\left(\frac{g\sin\theta}{2}\right)t_1^2$

$$\Rightarrow t_1 = \sqrt{\frac{4h}{g\sin^2\theta}} = \sqrt{\frac{16h}{3g}}$$

For disc: $\frac{h}{\sin\theta} = \frac{1}{2}\left(\frac{2g\sin\theta}{3}\right)t_2^2$

$$\Rightarrow t_2 = \sqrt{\frac{3h}{g\sin^2\theta}} = \sqrt{\frac{4h}{g}}$$

The difference of time of reaching the ground

$$\Rightarrow \sqrt{\frac{16h}{3g}} - \sqrt{\frac{4h}{g}} = \frac{2-\sqrt{3}}{\sqrt{10}}$$

$$\text{or} \quad \sqrt{h}\left[\frac{4}{\sqrt{3}} - 2\right] = 2 - \sqrt{3}$$

$$\Rightarrow \sqrt{h} = \frac{(2-\sqrt{3})\sqrt{3}}{(4-2\sqrt{3})} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow h = \frac{3}{4} = 0.75 \text{ m}$$

Chapter 5

Concept Application Exercises

Exercise 5.1

1. $\frac{\Delta A_1}{\Delta t_1} = \frac{\Delta A_2}{\Delta t_2}$ or $\frac{\Delta t_1}{\Delta t_2} = \frac{\Delta A_1}{\Delta A_2} = \frac{A}{2A} = \frac{1}{2}$

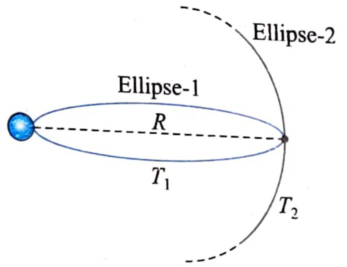
2. (a) Since, $mv_1r_1 = mv_2r_2$, $\frac{v_1}{v_2} = \frac{r_2}{r_1}$

(b) $\frac{K_1}{K_2} = \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{r_2}{r_1}\right)^2$

(c) $\frac{\left(\frac{dA}{dt}\right)_1}{\left(\frac{dA}{dt}\right)_2} = 1$

3. $\frac{T_1}{T_2} = \frac{\left(\frac{a+b}{2}\right)^{3/2}}{\left(\frac{a+c}{2}\right)^{3/2}} = \left(\frac{a+b}{a+c}\right)^{3/2}$

4. A straight path of falling planet can be imagined as the thin ellipse of time period say T_1 and semi-major axis $R/2$.



Applying Kepler's 3rd law,

$$T_1^2 \propto \left(\frac{R}{2}\right)^3 \quad ...(i)$$

and $T_2^2 \propto R^3$ ...(ii)

where $T_2 = T$

Then, the time fall is $t = \frac{T}{2^{3/2}}$

5. (a) The particle does possess angular momentum, because it is not headed straight for the origin.

(b) Its angular momentum is constant. There are no identified outside influences acting on the object.

(c) Since speed is constant, the distance traveled between t_A and t_B is equal to the distance traveled between t_C and t_D. The area of a triangle is equal to one-half its (base) width across one side times its (height) dimension perpendicular to that side.

So $\frac{1}{2}bv_0(t_B - t_A) = \frac{1}{2}bv_0(t_D - t_C)$

states that the particle's radius vector sweeps out equal areas in equal times.

Exercise 5.2

1. At interior points gravitational force is only due to the sphere as we use

(a) $F = \frac{Gmm'(x-r)}{r^3}$

(b) $F = \frac{Gmm'}{(x-r)^2}$

(c) At exterior points it is due to both sphere and shell given as

$$F = \frac{GMm'}{(x-R)^2} + \frac{Gmm'}{(x-r)^2}$$

2. (a) The distance between any of the spheres at the corners and the sphere at the center is $r = \frac{l}{2}\cos 30° = \frac{l}{\sqrt{3}}$

where l the length of one side of the equilateral triangle. The net (downward) contribution caused by the two bottom-most spheres (each of mass m) to the total force on m_4 has magnitude

$$2F_y = 2\left(\frac{Gm_4m}{r^2}\right)\sin 30° = 3\frac{Gm_4m}{l^2}$$

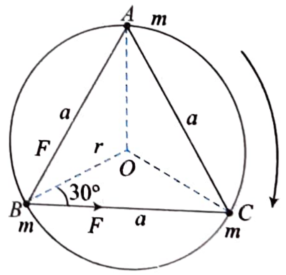
This must equal the magnitude of the pull from M, so

$$3\frac{Gm_4m}{l^2} = \frac{Gm_4m}{(l/\sqrt{3})^2}$$

which readily yields $m = M$.

(b) Since m_4 cancels in that last step, then the amount of mass in the center sphere is not relevant to the problem. The net force is still zero.

3. Consider one of the particles (say B). It is rotating in circle of radius $r = \frac{a}{\sqrt{3}}$.



Resultant force on it is towards the centre of the circle is

$$2F\cos 30° = 2\frac{Gm^2}{a^2}\frac{\sqrt{3}}{2} = \sqrt{3}\frac{Gm^2}{a^2}$$

This must be equal to necessary centripetal force

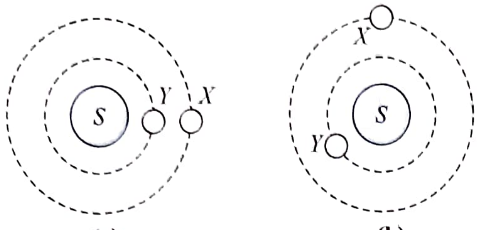
$$\therefore \quad \frac{mv^2}{r} = \frac{\sqrt{3}Gm^2}{a^2}$$

$$\Rightarrow \quad v^2 = \frac{Gm}{a} \qquad \left[\because r = \frac{a}{\sqrt{3}}\right]$$

$$\Rightarrow \quad v = \sqrt{\frac{Gm}{a}}$$

4. To find the angular displacement of planet Y, we apply Newton's second law:

$$\sum F = ma : \frac{Gm_{\text{planet}}M_{\text{star}}}{r^2} = \frac{m_{\text{planet}}v^2}{r}$$



(a) (b)

Then, using $v = r\omega$, $\frac{GM_{\text{star}}}{r} = v^2 = r^2\omega^2$

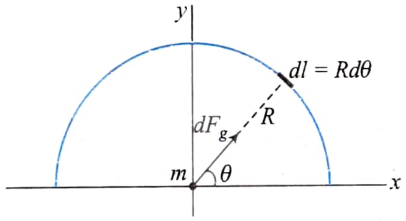
$$GM_{\text{star}} = r^3\omega^2 = r_x^2\omega_x^2 = r_y^2\omega_y^2$$

solving for the angular velocity of planet Y gives

$$\omega_y = \omega_x\left(\frac{r_x}{r_y}\right)^{3/2} = \left(\frac{90.0°}{5.00\,\text{yr}}\right)3^{3/2} = \frac{468°}{5.00\,\text{yr}}$$

So, given that there are 360° in one revolution we convert 468° to find that planet Y has turned through 1.30 revolutions.

5. (a) The gravitational force exerted on m due to a mass element dM from the thin rod is



$$dF_g = \frac{Gm(dM)}{R^2}$$

By symmetry, the force is along the y-direction. With

$$dM = \lambda dl = \left(\frac{M}{\pi R}\right)Rd\theta = \frac{M}{\pi}d\theta$$

where $\lambda = \frac{M}{\pi R}$ is the mass density (mass per unit length), we have

$$dF_{g,y} = dF_g\sin\theta = \frac{Gm}{R^2}\left(\frac{Md\theta}{\pi}\right)\sin\theta = \frac{GMm}{\pi R^2}\sin\theta d\theta$$

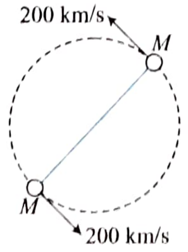
Integrating over θ gives

$$F_{g,y} = \int_0^{\pi}\frac{GMm}{\pi R^2}\sin\theta d\theta = \frac{GMm}{\pi R^2}\int_0^{\pi}\sin\theta d\theta = \frac{2GMm}{\pi R^2}$$

(b) If the rod were a complete circle, by symmetry, the net force on the particle would be zero.

6. Applying Newton's second law, $\sum F = ma$ yields $F_g = ma_c$ for each star:

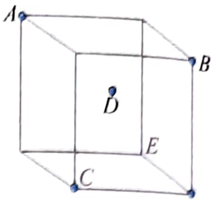
$$\frac{GMM}{(2r)^2} = \frac{Mv^2}{r} \text{ or } M = \frac{4v^2r}{G}$$



We can write r in terms of the period, T, by considering the time and distance of one complete cycle. The distance traveled in one orbit is the circumference of the stars' common orbit, so $2\pi r = vT$.

Therefore, $M = \frac{4v^2r}{G} = \frac{4v^2}{G}\left(\frac{vT}{2\pi}\right)$

$$M = \frac{2v^3T}{\pi G} = \frac{2(200\times10^3)^3(20d)(86400)}{\pi(6.67\times10^{-11})} = 1.32\times10^{32}\,\text{kg}$$



7. (a) The net force on mass at D will be zero if an identical mass is placed at E. In this case the resultant force due to masses at A and B will be vertically upward and that due to masses at C and E will be vertically down.

(b) Required force = Force equal and opposite to that applied by the mass placed at $E = \frac{Gm^2}{(\sqrt{3}a/2)^2} = \frac{4Gm^2}{3a^2}$

8. If the lead sphere were not hollowed the magnitude of the force it exerts on m would be

$$F_1 = \frac{GMm}{d^2}$$

Part of this force is due to material that is removed. We calculate the force exerted on m by a sphere that just fills the cavity, at the position of the cavity, and subtract it from the force of the solid sphere.

The cavity has a radius $r = R/2$. The material that fills it has the same density (mass to volume ratio) as the solid sphere, that is, $\frac{M_c}{r^3} = \frac{M}{R^3}$, where M_c is the mass that fills the cavity. The common factor $4\pi/3$ has been cancelled. Thus, $M_c = \left(\frac{r^3}{R^3}\right)M = \left(\frac{R^3}{8R^3}\right)M = \frac{M}{8}$

The centre of the cavity is $d - r = d - R/2$ from m, so the force it exerts on m is $F_2 = \frac{G(M/8)m}{(d-R/2)^2}$

The force of the hollowed sphere on m is

$$F = F_1 - F_2 = GMm\left(\frac{1}{d^2} - \frac{1}{8(d-R/2)^2}\right)$$

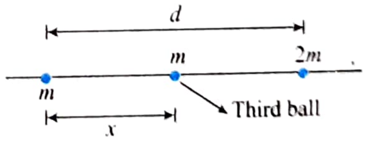
$$= \frac{GMm}{d^2}\left(1 - \frac{1}{8(1-R/2d)^2}\right)$$

9. There are two possibilities:

(1) the net force is towards right,

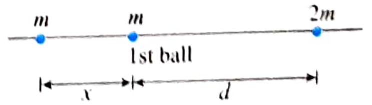
(2) the net force is towards left

When net force is towards right:



$$\frac{Gm(2m)}{d^2} + \frac{Gmm}{x^2} = \frac{6Gm^2}{d^2} \Rightarrow \frac{1}{x^2} = \frac{4}{d^2} \Rightarrow x = \frac{d}{2}$$

When the net force is to the left:



$$\frac{Gmm}{x^2} - \frac{Gm(2m)}{d^2} = \frac{6Gm^2}{d^2} \text{ or } \frac{1}{x^2} = \frac{8}{d} \Rightarrow x = \frac{d}{2\sqrt{2}}$$

ercise 5.3

1. Spherical shell behaves as a point mass for $r > R$ and hence

$$E = \frac{GM}{r^2}$$

It means $a_A = E_A = \frac{GM}{9R^2}$

And $a_B = E_B = \frac{GM}{R^2}$

and inside $E = 0$.

$\Rightarrow a_C = E_C = 0$

2. Let r be the distance between the two spheres.

Here, $r = 10$ m

Suppose that gravitational field is zero at a distance x from the sphere of mass 64 kg. Then,

$$\frac{G \times 64}{x^2} = \frac{G \times 289}{(r-x)^2}$$

or $8(r-x) = 17x$

or $x = \frac{8}{25} r = \frac{8 \times 10}{25} = 3.2$ m (from the 64 kg sphere)

3. With usual notation, $R = 6000$ km, $h = 2000$ km

If g and g_h denote the values of acceleration due to gravity on the surface and at a height h, then

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2} = \frac{(6000)^2}{(6000+2000)^2} = \frac{36 \times 10^6}{64 \times 10^6}$$

or $\frac{g_h}{g} = \frac{9}{16}$

Thus, the weight at a height of 2000 km $= \left(\frac{9}{16}\right)(120 \text{ kg}) = 67.5$ kg wt

4. Let g_h and g_d denote the acceleration due to gravity at points 2 km above and the same distance below the Earth's surface.

Now, $g_h = g\left(\frac{1-2h}{R}\right) = g\left(1 - \frac{4}{R}\right)$ $(h = 2 \text{ km})$

and $g_d = g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{2}{R}\right)$ $(d = 2 \text{ km})$

(Here, h is the height above the Earth's surface and d is the depth below the surface.)

Thus, $\frac{g_h}{g_d} = \frac{g(1-4/R)}{g(1-2/R)} = \frac{R-4}{R-2}$

as $(R-4) < (R-2)$, $\frac{R-4}{R-2} < 1$, $\frac{g_h}{g_d} < 1$

or $g_h < g_d$ or $g_d > g_h$ or $mg_d > mg_h$

Thus, the weight of the body 2 km below the Earth's surface is more than its weight 2 km above its surface.

5. $\Delta W = N_e - N_P$

$$= m(g_0 - R\omega^2) - mg_0$$

$$= -mR\omega^2 = \frac{4\pi^2 mR}{T^2} = \frac{4 \times 9.81 \times 1 \times 6400 \times 10^3}{(86400)^2}$$

$$= \frac{4 \times 9.8 \times 6.4}{(8.64)^2} \times 10^{-2} = 3.36 \times 10^{-2} \text{ N}$$

6. Since we are not sure as to whether the height will be too small compared to R, so let us use the general equation, we are given that

$$g' = \frac{3}{4} g$$

$\therefore \frac{3}{4} = \left(\frac{R}{R+h}\right)^2$

$\Rightarrow \frac{R+h}{R} = \frac{2}{\sqrt{3}}$ (taking only the +ve root)

$\Rightarrow \frac{h}{R} = \frac{2-\sqrt{3}}{\sqrt{3}} = \left(\frac{2-\sqrt{3}}{\sqrt{3}}\right)\frac{\sqrt{3}}{\sqrt{3}}$ (Rationalizing the denominator)

or $\frac{h}{R} = \frac{2\sqrt{3}-3}{3}$ or $h = (2\sqrt{3}-3)\frac{R}{3}$

which is the required height.

7. Since the height above the earth's surface h is too small compared to R.

Here $g' = g\left(1 - \frac{2h}{R}\right)$

Now, given $h = \frac{R}{100}$

$\therefore g' = g\left(1 - \frac{2}{100}\right) = \frac{98}{100} g$ or $\frac{g'}{g} = \frac{98}{100}$

$$\% \text{ change} = \frac{\text{change}}{\text{original}} \times 100 = \left(\frac{g'-g}{g}\right) \times 100$$

$$= \left(\frac{g'}{g} - 1\right) \times 100 = \left(\frac{98}{100} - 1\right) \times 100 = -2\%$$

$\therefore$ The percentage decrease (which is denoted by the minus sign), in the value of acceleration due to gravity is 2%.

8. Given $g' = \frac{2}{3} g$

we have $\frac{2}{3} g = g\left(1 - \frac{d}{R}\right)$

$\Rightarrow 1 - \frac{d}{R} = \frac{2}{3} \Rightarrow \frac{d}{R} = 1 - \frac{2}{3} = \frac{1}{3}$

$\therefore d = \frac{R}{3}$

9. Let M be mass of the moon. Then, mass of the earth is equal to $81M$. If r is distance between the earth and the moon, then $r = 3.8 \times 10^5$ km $= 3.8 \times 10^8$ m

Suppose that gravitational field is zero at a distance x from the earth.

Then, $\frac{G \times (81M)}{x^2} = \frac{GM}{(r-x)^2}$

or $r - x = \frac{x}{9}$

or $x = \frac{9r}{10} = \frac{9 \times 3.8 \times 10^8}{10} = 3.42 \times 10^8$ m (from the earth)

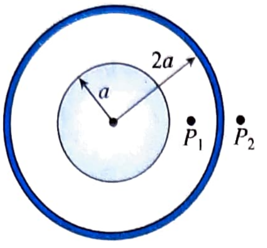
Gravitational field due to the earth at a distance x,

$$E_{\text{grav}} = \frac{G \times 81M}{x^2} = \frac{gR^2}{x^2} \quad (\because GM_e = gR^2)$$

$$=\frac{9.8\times(6.4\times10^{6})^{2}}{(3.42\times10^{8})^{2}}=3.43\times10^{-3}\,\mathrm{N\,kg^{-1}}$$

Obviously, gravitational field due to the moon at a distance x will also be equal to $3.43\times10^{-3}\,\mathrm{N\,kg^{-1}}$ (in magnitude).

10. The situation is shown in figure.



The point P_1 is at a distance $\frac{3}{2}a$ from the centre and P_2 is at a distance $\frac{5}{2}a$ from the centre.

As P_1 is inside the cavity of the thin spherical shell, the field here due to shell is zero. The field due to the solid sphere is

$$g=\frac{GM}{\left(\frac{3}{2}a\right)^{2}}=\frac{4GM}{9a^{2}}$$

This is also the resultant field. The direction is towards the centre. The point P_2 is outside the sphere as well as the shell. Both may be replaced by single particles of the same mass at the centre. The field due to each of them is

$$g=\frac{GM}{\left(\frac{5}{2}a\right)^{2}}=\frac{4GM}{25a^{2}}$$

The resultant field is $g=2g'=\dfrac{8GM}{25a^{2}}$ towards the centre.

11. Acceleration inside the earth at a distance r from the centre

$$g_1=\frac{GM}{R^3}\cdot r=\frac{GM}{R^2}\left(\frac{R-h_1}{R}\right)=g\left(1-\frac{h_1}{R}\right)$$

$$\Delta g_1=\left(\frac{dg_1}{dh_1}\right)\Delta h_1=-\frac{g}{R}\Delta h_1$$

$$\Delta h_1=+1\,\mathrm{km}$$

$$\therefore\quad \Delta g_1=-\frac{g}{R}(1\,\mathrm{km}) \qquad \text{...(i)}$$

[This is independent of depth h_1]

At a distance r from the centre outside the earth

$$g_2=\frac{GM}{r^2}=\frac{GM}{R^2}\frac{R^2}{r^2}=g\frac{R^2}{(R+h_2)^2}$$

$$\therefore\quad \Delta g_2=\left(\frac{dg_2}{dh_2}\right)(\Delta h_2)=\frac{-2gR^2}{(R+h_2)^3}\Delta h_2$$

$$\Delta h_2=1\,\mathrm{km}$$

$$\Delta g_2=-\frac{2gR^2}{(R+h_2)^3}(1\,\mathrm{km}) \qquad \text{...(ii)}$$

Since, $\Delta g_1=\Delta g_2$

$$\therefore\quad -\frac{g}{R}(1\,\mathrm{km})=-\frac{2gR^2}{(R+h_2)^3}(1\,\mathrm{km})$$

$$\therefore\quad (R+h_2)^3=2R^3$$

$$\Rightarrow\quad 1+\frac{h_2}{R}=(2)^{1/3}$$

$$h_2=R\,[2^{1/3}-1]$$

Exercise 5.4

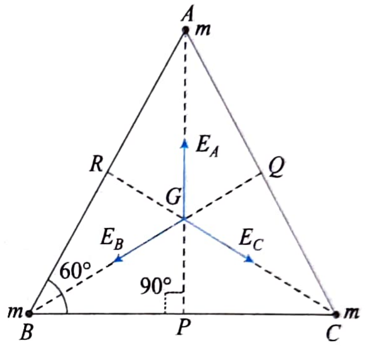
1. (a) As a consequence of Kepler's second law of planetary motion, a planet moves faster, when it is close to the sun and moves slower, when away from the sun. Therefore, its linear speed keeps on changing.

   (b) As the planet moves under the effect of a pure radial force, its angular momentum remains constant.

   (c) As the linear speed of the planet varies, its kinetic energy also keeps on changing.

   (d) As the distance of the planet from sun keeps on changing from one position to other position. The potential of sun and earth system will not be constant.

2. Let ABC be the equilateral triangle of each side equal to l. Three mass points, each of mass m, lie at the three vertices as shown in figure.



If G is the centroid of the triangle, then $AG=BG=CG$

Now, the centroid G divides the normal AP in the ratio 2:1.

$$AG=\frac{2}{3}AP=\frac{2}{3}AB\sin 60^\circ=\frac{2}{3}l\times\frac{\sqrt{3}}{2}=\frac{l}{\sqrt{3}}$$

The gravitational field at the point G is equal to the force on a unit mass placed at the point G due to the three equal mass points lying at distances $AG=BG=CG=l/\sqrt{3}$ from it. The three forces will be equal in magnitude and will be directed as shown. The symmetry shows that the resultant force i.e., the gravitational field at the point G will be zero. Now, the gravitational potential at the point G

$$V_G=\frac{Gm}{AG}+\left(-\frac{Gm}{BG}\right)+\left(-\frac{Gm}{CG}\right)=-\frac{3Gm}{l/\sqrt{3}}$$

$$\Rightarrow\quad V_G=-3\sqrt{3}\frac{Gm}{l}$$

3. Let M be the mass and R be radius of the earth. Then, gravitational potential energy of the object of mass m at the surface of the earth,

$$U_i=-\frac{GMm}{R} \qquad \text{...(i)}$$

and the potential energy of the object, when taken to a height equal to $2R$,

$$U_f=\frac{GMm}{R+h}=-\frac{GMm}{R+2R}=-\frac{GMm}{3R} \qquad \text{...(ii)}$$

Therefore, required work done,

$$U_f-U_i=-\frac{GMm}{3R}-\left(-\frac{GMm}{R}\right)=\frac{2GMm}{3R}$$

Since $GM=gR^2$, we have

$$U_f-U_i=\frac{2\times gR^2\times m}{3R}=\frac{2}{3}mgR$$

4. $$U=-\frac{G(3m)(m)}{l}-\frac{G(m)(2m)}{2l}-\frac{G(3m)(2m)}{\sqrt{5}l}$$

$$=-\frac{Gm^2}{l}\left(3+1+\frac{6}{\sqrt{5}}\right)=-\left(4+\frac{6}{\sqrt{5}}\right)\frac{Gm^2}{l}$$

5. $$W_{ext}=\Delta U=(U_{\text{final}}-U_{\text{initial}})$$

$$=0-\left(-\frac{2Gm^2}{l}-\frac{2Gm^2}{l}-\frac{Gm^2}{l}\right)=\frac{5Gm^2}{l}$$

6. Let u be the speed, with which the body is thrown from the surface of the earth so as to reach a height h $(= R)$ above the surface of the earth. Then,

KE + PE of the body on the surface of the earth

= PE of the body at height h $(= R)$

$$\text{or}\quad \frac{1}{2}mu^2+\left(-\frac{GMm}{R}\right)=-\frac{GMm}{R+h}=-\frac{GMm}{R+R}=-\frac{GMm}{2R}$$

$$\text{or}\quad \frac{1}{2}mu^2=-\frac{GMm}{2R}+\frac{GMm}{R}=\frac{GMm}{2R}$$

$$\text{or}\quad u=\sqrt{\frac{GM}{R}}=\sqrt{gR}$$

7. Let r be the distance between the centres of the earth and the moon. Initially the body lies at a distance $r/2$ from the centre of either of them.

Therefore, initial PE of the body,

$$U_i=-\frac{GM_1m}{r/2}-\frac{GM_2m}{r/2}=-\frac{2G(M_1+M_2)m}{r}$$

Therefore, energy required by the body to escape the gravitational pull

$$\Delta U=0-U_i=0-\left[-\frac{2G(M_1+M_2)m}{r}\right]=\frac{2G(M_1+M_2)m}{r}$$

If v_e is escape velocity, then

$$\frac{1}{2}mv_e^2=\frac{2G(M_1+M_2)m}{r}\quad\text{or}\quad v_e=\sqrt{\frac{4G(M_1+M_2)}{r}}$$

8. Initially, both the masses are at rest. Let v_1 and v_2 be the velocities of the masses M_1 and M_2, when they are at a distance r from each other. It follows that when the bodies are infinite distance apart, they must be at rest. Therefore, according to the principle of conservation of linear momentum,

$$M_1v_1+M_2(-v_2)=0\quad\text{or}\quad v_2=\frac{M_1}{M_2}v_1 \qquad \text{...(i)}$$

Also, when the bodies are infinite distance apart, their potential energy should also be zero. Therefore, according to the principle of conservation of energy,

$$\Delta K+\Delta U=0$$

$$\Rightarrow\left(\frac{1}{2}M_1v_1^2+\frac{1}{2}M_2v_2^2-0\right)+\left(-\frac{GM_1M_2}{r}\right)=0 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$v_1=\sqrt{\frac{2G}{(M_1+M_2)r}}M_2\quad\text{and}\quad v_2=\sqrt{\frac{2G}{(M_1+M_2)r}}M_1$$

Now, the velocity of approach of the bodies,

$$v=v_1-(-v_2)=v_1+v_2$$

$$v=\sqrt{\frac{2G}{(M_1+M_2)r}}M_2+\sqrt{\frac{2G}{(M_1+M_2)r}}M_1$$

$$=\sqrt{\frac{2G(M_1+M_2)}{r}}$$

9. The PE of the spaceship–Venus system,

$$U_1=-\frac{GM_Vm}{R_V}=-\frac{6.6\times10^{-11}\times5.0\times10^{24}\times800}{6.0\times10^6}$$

$= -4.4 \times 10^{10}$ J

and PE of the spaceship–sun system,

$$U_2=-\frac{GM_sm}{r}=-\frac{6.6\times10^{-11}\times2\times10^{30}\times800}{1.0\times10^{11}}$$

$= -1.05 \times 10^{12}$ J

Therefore, total potential energy of the spaceship,

$U = U_l + U_2 = -4.4 \times 10^{10} + (-1.05 \times 10^{12}) = -1.1 \times 10^{12}$ J

Hence, energy required to rocket the spaceship out of the solar system $= 0 - U = 0 - (-1.1 \times 10^{12}) = 1.1 \times 10^{12}$ J

Exercise 5.5

1. If a satellite of mass m is revolving in a circular orbit of radius r, with speed v around the earth of mass M, the centripetal force is provided by the gravitational pull, i.e.,

$$\frac{mv^2}{r}=\frac{GMm}{r^2}$$

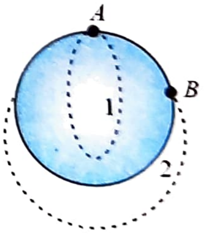
or $mv^2r = GMm$

or $(mvr)v = GMm$ = constant

or Lv = constant

where $mvr = L$ is the angular momentum of the satellite. As the air friction causes the retarding torque, it will decrease the angular momentum L of the satellite. Because of this the velocity of the satellite v increases in order to keep Lv = a constant.

2. If the container is thrown in the direction opposite to the motion of the satellite, it will begin to move along an ellipse (say 1) lying inside the circular orbit of the satellites. The period of revolution will be slightly less than that of the satellites. Therefore, they can meet at the point of contact of the orbits (circular orbit of satellite and the elliptical orbit of the container) only after a great number of revolution. On the other hand, if the container is thrown in the direction of the satellite, the container begins to move along an ellipse like 2 in the figure. The velocity u should be such that during one revolution of the container, satellite B also makes one revolution and in addition covers the distance AB. So the container should be thrown in the direction of the first satellite.



3. Initial total energy of each satellite $= -\frac{GMm}{2r}$

Hence, total energy of both satellites

$$2\times\left(-\frac{GMm}{2r}\right)=-\frac{GMm}{r}$$

When satellites travelling in opposite directions collide, the final velocity v is given by $(2mv) = mv - mv = 0$

$\Rightarrow$ $v = 0$

Therefore, the wreckage comes to rest and it will fall freely under gravity. After collision, total energy of satellite is

$$2\left(-\frac{GMm}{r}\right)=\left(-\frac{2GMm}{r}\right)$$

4. In elliptical orbit, total energy

$$E = -\frac{GMm}{2a} = \text{constant}$$

At position r, if v is orbital speed of satellite, then

$$\text{KE} = \frac{1}{2}mv^2 \text{ and PE} = \frac{GMm}{r}$$

$$\therefore \quad \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a} \Rightarrow v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

5. Here, $R = 6.5 \times 10^6$ m ; $x = 500$ km $= 5 \times 10^5$ m

(i) If v is orbital velocity of the satellite, then acceleration of the satellite,

$$a = \frac{v^2}{R+x} \quad \text{(centripetal acceleration)}$$

Since $v = \sqrt{\frac{gR^2}{R+x}}$; we have

$$a = \frac{gR^2}{(R+x)^2} = \frac{9.8\times(6.5\times10^6)^2}{(6.5\times10^6+5\times10^5)^2} = 8.45 \text{ m s}^{-2}$$

(ii) The acceleration due to gravity at any point along the orbit,

$$g' = \frac{gR^2}{(R+x)^2}$$

It follows that $g' = a = 8.45$ m s^{-2}

6. Let M be the mass of the earth. If E is total energy of the satellite in the orbit and U_0 is its PE on the surface of the earth, then energy required to place the satellite in the orbit,

$$\Delta E = E - U_0 = -\frac{GMm}{2r} - \left(-\frac{GMm}{R}\right) = GMm\left[\frac{1}{R} - \frac{1}{2r}\right]$$

Now, $g = \frac{GM}{R^2}$ or $GM = gR^2$

$$\therefore \quad \Delta E = mgR^2\left[\frac{1}{R} - \frac{1}{2r}\right] = mg\left[R - \frac{R^2}{2r}\right]$$

7. The work done in shifting from earth's surface to the orbit is

$$W_1 = \Delta U = \frac{GMm}{R} - \frac{GMm}{R+h} = \frac{GMmh}{(R+h)R}$$

The work done in putting the satellite into the orbit is

$$W_2 = \frac{1}{2}mv^2; \; v = \sqrt{\frac{GM}{R+h}}$$

$$W_2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{R+h}}\right)^2 = \frac{GMm}{2(R+h)}$$

Hence $\frac{W_1}{W_2} = \frac{2h}{R}$

8. Here, $R = 6{,}400$ km $= 64 \times 10^6$ m; $g = 9.8$ m s^{-2}

The orbital velocity of the satellite orbiting close to the earth's surface,

$$v = \sqrt{gR} = \sqrt{9.8\times6.4\times10^6} = 7.92\times10^3 \text{ m s}^{-1}$$

$= 7.92$ km s^{-1}

Also, escape velocity on the surface of the earth,

$$v_e = \sqrt{2gR} = \sqrt{2\times9.8\times6.4\times10^6}$$

$= 11.2 \times 10^3$ m s^{-1} $= 11.2$ km s^{-1}

Therefore, additional velocity to be imparted to the satellite,

9. The period of revolution of a satellite revolving in an orbit of radius r is given by $T = \frac{2\pi}{R}\sqrt{\frac{r^3}{g}}$.

Thus, $T^2 \propto r^3$ or $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$ or $T_2 = \left(\frac{r_2}{r_1}\right)^{3/2} \times T_1$

Here, $r_1 = R + 6R = 7R$; $T_1 = 24$ h

and $r_2 = R + 2.5R = 3.5\,R$

$$T_2 = \left(\frac{3.5R}{7R}\right)^{3/2} \times 24 = 6\sqrt{2}\text{ h}$$

10. (a) Orbital velocity of satellite is

$$v_0 = \sqrt{\frac{GM}{r}} = \frac{1}{2}\sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \; r = 2R = R + h \Rightarrow h = R = 6400\text{ km}$$

(b) Using conservation of energy, we have

$$-\frac{GMm}{2R} + 0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \; v = \sqrt{\frac{GM}{R}} = \sqrt{gR} = 7.92\text{ km/s}$$

Exercises

Single Correct Answer Type

1. **(3)** Equal time is taken to cover equal area.

2. **(4)** Here, angular momentum is conserved. According to it,

$$I_1\omega_1 = I_2\omega_2$$

or $MR_1^2\omega_1 = MR_1^2\omega_2$ or $R_1v_1 = R_2v_2$

At point P_4, the value of R is minimum and hence the velocity is maximum or KE is maximum.

3. **(4)** Change in energy = 1/(2 $(GMm)/R$) = $1/2Mv_e^2$

So it is independent of angle as gravitational field is conservative in nature.

4. **(2)** From the surface of the earth, the escape velocity is $\sqrt{(2GM)/R}$.

From the satellite, the escape velocity is calculated as follows.

By conservation of energy,

$$\frac{1}{2}mv_2^2 - \frac{GMm}{2R} = 0$$

$$\Rightarrow \; v_2^2 = \sqrt{\frac{GM}{R}} \Rightarrow v_2 < v_1$$

5. **(1)** Conserving angular momentum,

$$mv_1r_{\text{min}} = mv_2r_{\text{max}} \text{ or } \frac{v_1}{v_2} = \frac{r_{\text{max}}}{r_{\text{min}}}$$

6. **(3)** At point P, we have $I_1 - I_2 = 0$ (because the gravitational field inside a shell is zero). Hence, $I_1 = I_2$.

7. **(4)** The gravitational field intensity at a point inside the spherical shell is zero.

Note: There is NO gravitational field inside a spherical shell.

8. **(3)** Note that there must be some point where the gravitational field of the earth is balanced by the gravitational field of Mars.

9. (1) $U_g = -\frac{GM_1M_2}{R}$

$$\Rightarrow U_f - U_i = -\frac{GMm}{2R} - \frac{GMm}{R}$$

$$\Delta U = \frac{GMm}{2R} = \frac{mgR}{2} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

10. (3) $\frac{mv^2}{R} = \frac{GMm}{R^2}$ ...(i)

and $\frac{mv'^2}{R'} = \frac{GMm}{R'^2}$ ...(ii)

Dividing Eq. (i) by Eq. (ii), $\frac{v^2}{v'^2} = \frac{R'}{R}$

$$\Rightarrow \frac{v'}{v} = \sqrt{\frac{R}{R'}} = \sqrt{\frac{2}{3}} \quad \left[\because R' = \frac{3R}{2}\right]$$

$$\Rightarrow v' = \sqrt{\frac{2}{3}}v$$

11. (4) $g_E = (GM)/R^2$

$$g_M = \frac{GM'}{R'^2} = \frac{GM/10}{R^2/4} = \frac{2}{5}\frac{GM}{R^2} = \frac{2}{5}g_E$$

$$W_M = \frac{2}{5}W_E = 200 \times \frac{2}{5} = 80 \text{ N}$$

12. (2) $F_G = \frac{GMm}{R}$ (as $F_G \propto \frac{1}{R}$ given)

So $\frac{mv^2}{R} = \frac{GMm}{R} \quad \Rightarrow v^2 \propto R^0$

13. (3) $F_G = \frac{Gm^2}{4R^2} \Rightarrow \frac{Mv^2}{R} = \frac{Gm^2}{4R^2}$

$$\therefore \; v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$

14. (4) $g = \frac{GM}{R^2}$ ...(i)

$$g' = \frac{Gm}{\left(\frac{90}{100}R\right)^2} = \frac{100}{81}\frac{GM}{R^2} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$g' = \frac{100}{81}g \quad \Rightarrow \frac{g'}{g} = \frac{100}{81}$$

$$\frac{g'}{g} - 1 = \frac{100}{81} - 1$$

$$\therefore \; \Delta g = \frac{19}{81}g = 23\% \text{ of } g$$

So increase by more than 19% of g.

15. (1) i. g' = acceleration due to gravity on the Moon

$$= G\frac{4}{3}\pi R'\rho' = G\frac{4}{3}\pi \times \frac{R}{4} \times \frac{2}{3}\rho = \frac{g}{6}$$

Now, $h_m = \frac{u^2}{2g}$ or $h_m \propto \frac{1}{g}$

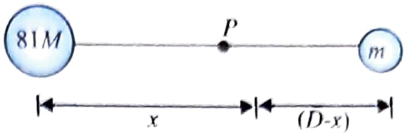
Hence $h'_m g' = h_m g$

or $h'_m = 6h_m = 6 \times 0.5 = 3$ m

ii. $t = \frac{2u}{g}$ or $t \propto \frac{1}{g}$

$$\therefore \; t'g' = tg \text{ or } \frac{t'}{t} = \frac{g}{g'} = \frac{6}{1}$$

16. (4)



$$\frac{81GM}{x^2} = \frac{GM}{(D-x)^2}$$

$$\frac{9}{x} = \frac{1}{(D-x)}$$

$$D = \frac{9}{10}x$$

$$x = \frac{9}{10}D$$

17. (2) $F = (GMm)/R^2 = mg$

As the mass remains the same and radius, i.e., distance from the centre is also the same, therefore g will remain the same, i.e., 10 m s^{-2}.

18. (3) $\frac{m_1v_1^2}{r_1} = m_1\frac{GM}{r_1^2}, \quad \frac{m_2v_2^2}{r_2} = m_2\frac{GM}{r_2^2}$

$$\frac{v_1^2}{v_2^2} = \frac{r_2}{r_1}; r_2 < r_1 \Rightarrow v_1^2 < v_2^2$$

Hence, $v_1 < v_2$.

19. (3) $v_A = \sqrt{\frac{GM}{r_A}}$ and $v_B = \sqrt{\frac{GM}{r_B}}$

Given $r_B = 3r_A$

Now $F_A = \frac{mv_A^2}{r_A} = \frac{m}{r_A}\frac{GM}{r_A} = \frac{GMm}{r_A^2}$

$$F_B = \frac{GMm}{r_B^2}$$

$$\frac{F_B}{F_A} = \frac{r_A^2}{r_B^2} = \frac{1}{9}$$

20. (3) Total energy, $E = \frac{1}{2}mv^2 - \frac{GmM}{r}$

$$= \frac{GmM}{2r} - \frac{GMm}{r} = -G\frac{mM}{2r}$$

$$r = 2R + R = 3R$$

$$E = -\frac{GmM}{6R}$$

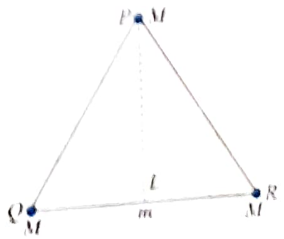
Potential energy $= -\frac{GMm}{R}$

Minimum energy required $= -\frac{1}{6}\frac{GMm}{R} - \left(\frac{-GMm}{R}\right) = \frac{5}{6}\frac{GMm}{R}$

$$= \frac{5}{6}mgR$$

21. (2) Force on m due to masses at Q and R is zero. So, the net force is due to the mass at P.

Hence, $F = (GMm)/(PL)^2$



Now, $PL = l \sin 60° = l \frac{\sqrt{3}}{2}$

$$F = \frac{4GMm}{3l^2}$$

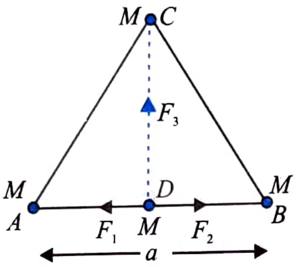
22. (2) $U_p = -\sum \frac{GMm}{r}$

$$= -GMm\left[\frac{1}{l \sin 60°} + \frac{1}{l/2} + \frac{1}{l/2}\right]$$

$$= -\frac{2}{\sqrt{3}} \frac{GMm}{l} [1 + 2\sqrt{3}]$$

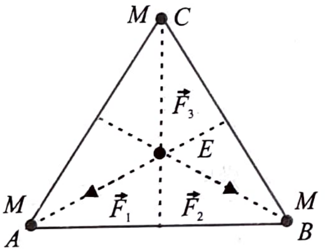
23. (3) $I = \frac{F}{m} = \frac{4GMm/(3l^2)}{m} = \frac{4GM}{3l^2}$

24. (2) i. Gravitational force on the particle placed at the midpoint D of side AB of length a is $= \vec{F}_1 + \vec{F}_2 + \vec{F}_3$. But $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude and opposite in direction.



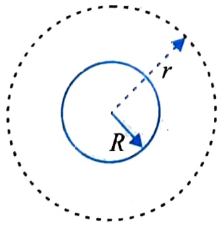
$$F = F_3 = \frac{4GMm}{3a^2} = \frac{4GM^2}{3a^2} \qquad \left[\because CD^2 = \frac{3a^2}{4}\right]$$

ii. Gravitational force on the particle placed at the point of intersection of three medians $= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$, since the resultant of $\vec{F}_1$ and $\vec{F}_2$ is equal and opposite to $\vec{F}_3$.



25. (3) Gravitational field inside a shell is zero, but for points outside it, the shell behaves as if whole of its mass is concentrated at its centre. Hence, for a point lying in between the shells there will be a field only due to the inner shell ($= GM_2/r^2$) because the point will be an external point for the inner shell but internal for the outer shell.

26. (4) $\frac{GMm}{r^2} = m\omega_0^2 r$



$\Rightarrow GM = \omega_0^2 r^3$

$\Rightarrow gR^2 = \omega_0^2 r^3$

$\Rightarrow g = \frac{\omega_0^2 r^3}{R^2}$

27. (1) From the principle of conserving angular momentum, we have

$mvR = mv'r$ ...(i)

[v' = speed when spaceship is just touching the planet]

From conserving of energy, we have

$\frac{1}{2} mv^2 = \frac{1}{2}mv'^2 - \frac{GMm}{r}$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$R = \frac{r}{v}\left[v^2 + \frac{2GM}{r}\right]^{1/2}$$

28. (4) Let mass of the cavity $= M'$

Density of the sphere $= \frac{M}{\frac{4}{3} \neq R^3}$

Mass of the cavity cut out $= M' = \frac{4}{3}\pi \frac{R^3}{8} \times \frac{M}{\frac{4}{3}\pi R^3}$

$\therefore \quad M' = \frac{M}{8}$

$\Rightarrow F_{\text{net}} = F_{Mm} - F_{M'm}$

$$= \frac{GMm}{4R^2} - \frac{GM'm}{\left(\frac{5}{2}R\right)^2} = \frac{GMm}{4R^2} - \frac{GMm}{50R^2}$$

$$F_{\text{net}} = \frac{23}{100} \frac{GMm}{R^2}$$

29. (4) Let the mass of the particle be m

PE at a distance of $R' = (GMm)/R'$

PE at a distance of $R_e = -(GMm)/R_e$

Decrease in PE = Increase in KE

$\Rightarrow -\frac{GMm}{R'} + \frac{GMm}{R_e} = \frac{1}{2}mv^2$

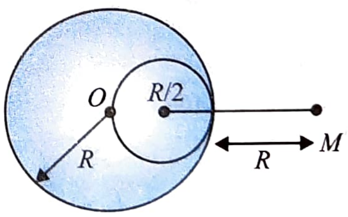
$$v^2 = 2GM\left[\frac{1}{R_e} - \frac{1}{R'}\right]$$

$\Rightarrow v^2 = \frac{2GM}{R_e}\left(1 - \frac{R_e}{R'}\right) \Rightarrow v = \sqrt{\frac{2MR_e}{R_e^2}\left(1 - \frac{R_e}{R'}\right)}$

$\therefore \quad v = \sqrt{2gR_e\left(1 - \frac{R_e}{R'}\right)}$

30. (4) $F_1 = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2}$

Mass of the cavity $= M/8$ if mass of the sphere $= M$ as volume of the cavity is 1/8th of the sphere.



$$F_2 = \frac{GMm}{4R^2} - \frac{GMm}{8\left(\frac{3}{2}R\right)^2}$$

$$= \frac{GMm}{R^2}\left[\frac{1}{4} - \frac{1}{18}\right] = \frac{GMm}{R^2}\left[\frac{9-2}{36}\right]$$

$$= \frac{GMm}{R^2}\frac{7}{36}$$

$\therefore \quad \frac{F_1}{F_2} = \frac{36}{4 \times 7} = \frac{9}{7}$

31. (2) $m_1 = \frac{4}{3}\pi R^3\rho$, $m_2 = \frac{4}{3}\pi(2R)^3\rho$, distance between the centres of the two spherical objects, $r = 3R$.

$$F = \frac{GM_1 m_2}{r^2} = G\left(\frac{4}{3}\pi R^3\rho\right)\left(8 \times \frac{4}{3}\pi R^3\rho\right) \times \frac{1}{(3R)^2}$$

$$= \frac{128}{81} G\pi^2 R^4 \rho^2$$

(2) $\frac{GMm}{r_0} = \frac{1}{2}mv^2 - \frac{GMm}{R}$

or $\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{r_0}$

or $\frac{v^2}{2} = \frac{GM}{R} - \frac{GM}{r_0}$

or $v^2 = 2GM\left[\frac{1}{R} - \frac{1}{r_0}\right]$

or $v = \sqrt{2GM\left(\frac{1}{R} - \frac{1}{r_0}\right)}$

(2) $T = \frac{2\pi R}{v}$

$E = \frac{1}{2}mv^2 = \frac{GMm}{R^{n-1}}$

or $v = \left[\frac{2GM}{R^{n-1}}\right]^{1/2}$

$\therefore\ T = \frac{2\pi R}{\sqrt{2GM/R^{n-1}}} = \frac{2\pi}{\sqrt{2GM}} \times R^{(n+1)/2}$

$\Rightarrow T \propto R^{(n+1)/2}$

(1) In the first case, $v_e = \sqrt{\frac{2GM}{R}}$ or $50 = \sqrt{\frac{2GM}{R}}$

In the second case, $v_e = \left[\frac{2G(4M)}{R}\right]^{\frac{1}{2}} = 2\sqrt{\frac{2GM}{R}}$

$= 2 \times 50 = 100$ km s^{-1}

(3) At height h above the surface of the earth, therefore

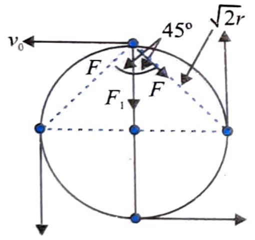
$$g' = g\left(1 - \frac{2h}{R}\right) \Rightarrow \Delta g_1 = \frac{2h}{R}g$$

At depth d below the surface of the earth

$$g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \Delta g_2 = \frac{d}{R}g$$

$$\Delta g_1 = \Delta g_2 \Rightarrow d = 2h$$

36. (1)



Centripetal force = net gravitational force

$\Rightarrow mv_0^2 = 2F\cos 45° + F_1 = \frac{2Gm^2}{(\sqrt{2})^2}\frac{1}{\sqrt{2}} + \frac{Gm^2}{4r^2}$

$\frac{mv_0^2}{r} = \frac{Gm^2}{4r^2}[2\sqrt{2} + 1]$

$\Rightarrow v_0 = \left[\frac{Gm(2\sqrt{2}+1)}{4r}\right]^{\frac{1}{2}}$

37. (3) Since $T^2 \propto r^3$

$\Rightarrow \quad \frac{r_1^3}{r_2^3} = \frac{T_1^2}{T_2^2} = \frac{1}{64}$

$\Rightarrow \quad \frac{r_1}{r_2} = \frac{1}{4} \Rightarrow r_2 = 4 \times 10^4$ km

$v_1 = \frac{2\pi r_1}{T_1}$ and $v_2 = \frac{2\pi r_2}{T_2}$

$\Rightarrow \quad v_1 = \frac{2\pi \times 10^4}{1}$ and $v_2 = \frac{2\pi \times 4 \times 10^4}{8}$ (in kmph)

$\Rightarrow \quad v_1 = 2\pi \times 10^4$ kmh^{-1} and $v_2 = \pi \times 10^4$ kmh^{-1}

So, speed of S_2 with respect to S_1 is $\pi \times 10^4$ kmh^{-1} when they are closest.

38. **(3)** As the sphere is having non-uniform mass density, so nothing can be predicted about the variation of gravitational field intensity.

39. **(2)** If a body is projected from the surface of the earth with a velocity v and reaches a height h, then according to law of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{mgh}{1 + h/R}$$

Here $v = kv_e = k\sqrt{2gR}$

Therefore, $\frac{1}{2}mk^2\, 2gR = \frac{mg(r-R)}{1 + \frac{(r-R)}{R}}$

$$k^2R\left[1 + \frac{r-R}{R}\right] = r - R \text{ or } k^2r = r - R$$

or $r = \frac{R}{1-k^2}$

40. **(4)** $P = \rho_{\text{atm}}gh = \frac{m}{\frac{4}{3}\pi[(R+h)^3 - R^3]}\frac{GM}{R^2}h$

$$\Rightarrow P = \frac{m}{\frac{4}{3}\pi R^3\left[\left(1+\frac{h}{R}\right)^3 - 1\right]} \times \frac{G\frac{4}{3}\pi R^3 l_0}{R^2} \times h$$

$$= \frac{m}{\frac{4}{3}\pi R^3\left[1+\frac{3h}{R} - 1\right]} \times G\frac{4}{3}\pi R l_0 h \qquad \left(\text{Here } M = \frac{4}{3}\pi R^3 l_0\right)$$

Hence, $P = \frac{mG\rho}{3R}$

$\therefore\ m \propto \frac{PR}{\rho}$

41. **(2)** Because the gravitational force is the mutual force, hence the position of centre of mass remains unaffected.

$\therefore\ M_1R_1 = M_2R_2$ or $\frac{R_1}{R_2} = \frac{M_2}{M_1}$

42. **(1)** At a distance x from the centre of earth, the gravitational force

$$F = \frac{GMm}{x^2}$$

$$dW = Fdx = \frac{GMm}{x^2}dx$$

We get $W = \int_R^{R+h} \frac{GMm}{x^2}dx = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$

$= gR^2m\left[\frac{1}{R} - \frac{1}{R+h}\right]$

Therefore, the gain in potential energy is

$$gR^2m\left[\frac{1}{R} - \frac{1}{2R}\right] = \frac{mgR}{2}$$

43. **(4)** $\frac{mv^2}{r} = \frac{GMm}{r^{5/2}}$ $\qquad \left[\because F \propto \frac{1}{r^{5/2}}\right]$

or $r\omega^2 = \frac{GM}{r^{5/2}}$

or $r\frac{4\pi^2}{T^2} = \frac{GM}{r^{5/2}}$

or $T^2 \propto r^{7/2}$ or $T^2 \propto r^{3.5}$

44. **(2)** Potential energy of mass m when it is midway between masses M_1 and M_2 is

$$U = -\frac{GM_1m}{d/2} - \frac{GM_2m}{d/2} = -\frac{2Gm}{d}(M_1 + M_2)$$

According to law of conservation of energy,

$$\frac{1}{2}mv_e^2 = \frac{2Gm}{d}(M_1 + M_2)$$

Therefore, escape velocity,

$$v_e = \sqrt{\frac{4G(M_1 + M_2)}{d}}$$

45. (2) Energy of the skylab in the first orbit is

$$-\frac{GMm}{2(2R)} = -\frac{GMm}{4R}$$

Total energy required to place the skylab into the orbit of radius $2R$ from the surface of earth is

$$-\frac{GMm}{4R} - \left(-\frac{GMm}{R}\right) = \frac{3GMm}{4R}$$

$$= \frac{3gR^2m}{4R} = \frac{3}{4}mgR$$

Energy of the skylab in the second orbit $= -(GMm)/6R$.

Energy needed to shift the skylab from the first orbit to the second orbit is

$$-\frac{GMm}{4R} - \frac{GMm}{6R} = \frac{GMm}{R} \times \frac{2}{24} = \frac{mgR}{12}$$

46. (1) Energy of each satellite in the orbit $= \frac{-GMm}{2r}$.

Total energy of the system before collision,

$$E_i = E_1 + E_2 = 2E = -2 \times \frac{GMm}{2r} = -\frac{GMm}{r}$$

As the satellites of equal mass are moving in the opposite directions and collide inelastically, the velocity of the wreckage just after the collision is

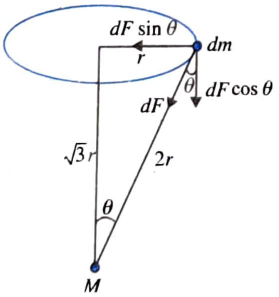
$$mv - mv = 2mV, \text{ i.e., } V = 0$$

The energy of the wreckage just after the collision will be totally potential and will be

$$E_f = \frac{GM(2m)}{r} = -\frac{2GMm}{r}$$

As after collision the wreckage comes to standstill in the orbit, it will move along the radius towards the earth under gravity.

47. (3) $dF = G\frac{Mdm}{4r^2}$



$$F = \Sigma dF \cos\theta$$

$$= \Sigma \frac{GMdm}{4r^2}\cos\theta$$

$$= \frac{GM}{4r^2} \times \frac{\sqrt{3}r}{2r}\Sigma dm$$

$$= \frac{\sqrt{3}GMm}{8r^2}$$

Alternative solution:

The gravitational field due to the ring at a distance $\sqrt{3}r$ is given by

$$E = \frac{Gm(\sqrt{3}r)}{[r^2 + (\sqrt{3}r)^2]^{3/2}} \text{ or } E = \frac{\sqrt{3}Gm}{8r^2}$$

The required force is EM, i.e., $(\sqrt{3}Gm)M/8r^2$.

Note: Follow the central argument in this solution: The gravitational field at any point on the ring due to a sphere is equal to the field due to a single particle of mass M located at the centre of the sphere.

48. (2) Gravitational potential at a point on the surface of earth is $-\frac{GM}{R}$.

If the earth is assumed to be a solid sphere, then the gravitational potential at the centre of the earth is $-\frac{3GM}{2R}$.

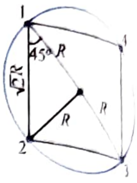
Decrease in gravitational potential is $\frac{R}{2} \times \frac{GM}{R^2} = \frac{Rg}{2}$

Loss in potential energy is $\frac{R}{2} \times \frac{GM}{R^2} \times m$

Now, gain in kinetic energy = loss in potential energy.

Therefore, $\frac{1}{2}mv^2 = \frac{1}{2}mgR$ or $v = \sqrt{gR}$

49. (4) $F_{12} = \frac{GM^2}{2R^2}$



$$F_{14} = \frac{GM^2}{2R^2}$$

The resultant of these two forces is $(\sqrt{2}GM^2)/2R^2$. Now, $F_{12} = (GM^2)/(4R^2)$.

The combined resultant of all the forces is

$$\frac{\sqrt{2}GM^2}{2R^2} + \frac{GM^2}{4R^2} \text{ or } \frac{GM^2}{R^2}\left[\frac{\sqrt{2}}{2} + \frac{1}{4}\right]$$

Equating this with centripetal force, we get

$$\frac{Mv^2}{R} = \frac{GM^2}{R^2}\left[\frac{2\sqrt{2}+1}{4}\right]$$

$$\text{or } v^2 = \frac{GM}{R}\left[\frac{2\sqrt{2}+1}{4}\right] \quad \text{or } v = \sqrt{\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)}$$

50. (2) a_1 = acceleration of first

$$= \frac{Gm_1m_2}{r^2} \times \frac{1}{m_1} = \frac{Gm_2}{r^2} = \frac{6.67 \times 10^{-11} \times 100}{(100)^2}$$

$$= 6.67 \times 10^{-13} \text{ m s}^{-1}$$

a_2 = acceleration of second

$$= \frac{Gm_1}{r^2} = \frac{6.67 \times 10^{-11} \times 100}{(100)^2} = 6.67 \times 10^{-13} \text{ m s}^{-1}$$

Net acceleration of approach

$$a = a_1 + a_2 = 2 \times 6.67 \times 10^{-13} \text{ m s}^{-1}$$

Now, $s = \frac{1}{2}at^2$

$$1 \times 10^{-2} = \frac{1}{2} \times 6.67 \times 10^{-13} \times 2 \times t^2$$

Solving, we get $t = 1.41$ days.

and to solve we get $h + R = R/(1 - K^2)$.

51. (4) Let ρ = density of sphere, R = radius of sphere, r = radius of the spherical cavities.

Mass of the complete sphere $= \frac{4}{3}\pi R^3\rho = M$

Mass of the removed sphere $= \frac{4}{3}\pi r^3\rho = m$

Here $m = \frac{Mr^3}{R^3} = \frac{M(1)^3}{(4)^3} = \frac{M}{64}$

Now $\vec{I}_R = \vec{I} + \vec{I}_P + \vec{I}_Q$

Here $I = 0$, also $\vec{I}_P = -\vec{I}_Q \Rightarrow I_R = 0$

52. (1) At the centre of the cavities, if the smaller spheres are not removed, then

$$I_R = \frac{GM}{R^3} \times \frac{GM}{64} \times 2 = \frac{GM}{32}$$

Also, $\vec{I}_R = \vec{I} + \vec{I}_P + \vec{I}_Q$

$$\vec{I} = \vec{I}_R - \vec{I}_P - \vec{I}_Q$$

At P, we have $I_P = 0$ and $I_Q = \frac{Gm}{2} = G\left(\frac{M}{64}\right) \times \frac{1}{4^2}$

Hence, $I = \frac{31GM}{1024}$

(3) The circle lies outside the bigger sphere.

Hence, $V_R = V + V_P + V_Q$

or $V = V_R + V_P + V_Q$

$$= -\frac{Gm}{6} - \left(-\frac{Gm}{r}\right) \times 2$$

where $r = \sqrt{6^2 + 2^2} = \sqrt{40}$

$$\therefore \; V = -\frac{GM}{2}\left[\frac{1}{3} - \frac{1}{32\sqrt{10}}\right]$$

4. (1) As angular momentum is conserved, hence

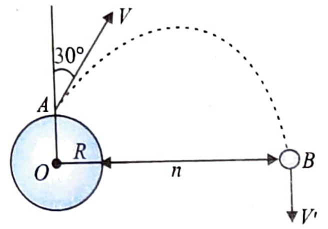
$$I_1\omega_1 = I_2\omega_2$$

or $MR_1^2\omega_1 = MR_2^2\omega_2$ or $MR_1v_1 = MR_2v_2$ $\left(\because \omega = \frac{v}{R}\right)$

or $R_1v_1 = R_2v_2$ or $VR = vr$

$$\therefore \; V = \frac{vr}{R}$$

5. (2) Conservation of angular momentum of the body about O yield the following:

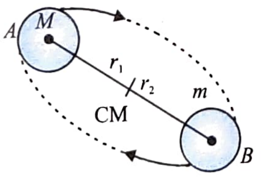


$(mv \sin 30°)\, R = mV'(R + h)$

$\frac{V}{2}R = \frac{V}{4}(R + h)$ $\left[\therefore V' = \frac{V}{4}\right]$

Therefore, $h = R$

56. (4) If the binary stars are rotating about their common centre of mass, then they have to be on the same line all the time, otherwise the centre of mass will be changing. Their angular velocities have to be the same although in the same time, the smaller mass will describe a bigger circle.



But as $\omega = \frac{2\pi}{T}$, so $T_A = T_B$

57. (3) If x_1 and x_2 are the distances covered by the two bodies, then

$x_1 + x_2 = 9R$.

Also, $Mx_1 = 5Mx_2 \Rightarrow x_2 = \frac{x_1}{5}$

$x_1 + \frac{x_1}{5} = 9R \Rightarrow x_1 = 7.5R$

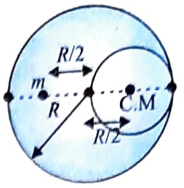
58. (1) Let the minimum speed imparted to the particle of mass m so that it just reaches the surface of the earth is v. Applying the principle of conservation of energy,

$$\frac{1}{2}mv^2 + \left(-\frac{3}{2}\frac{GM}{R}m\right) = -\frac{GM}{R}m + 0$$

Solving, we get $v = \sqrt{\frac{GM}{R}}$.

59. (2) Mass of sphere $M \propto R^3$. If mass of complete sphere is M, then, the mass of cavity will be $M/8$.

Using the figure, F = force due to whole sphere–force due to cavity.



$$F = \frac{GMm}{R^3}\left(\frac{R}{2}\right) - \frac{G(M/8)m}{(R)^2}$$

$$= \frac{GMm}{R^2}\frac{1}{2} - \frac{G(M/8)m}{R^2}$$

$$= \frac{GMm}{R^2}\left[\frac{4}{8} - \frac{1}{8}\right] = \frac{3}{8}\frac{GMm}{R^2} = \frac{3}{8}mg$$

60. (3) Given $8 = \frac{2\pi}{\omega_1 + \omega_2} = \frac{2\pi}{\frac{2\pi}{T_1} + \frac{2\pi}{T_2}}$

$\Rightarrow T_1 = 24$ h for the earth

$\Rightarrow T_2 = 12$ h (T_2 being the time period of the satellite, it will remain the same as the distance from the centre of the earth remains constant)

$$\Rightarrow T = \frac{2\pi}{\omega_2 + \omega_1} = \frac{2\pi}{\frac{2\pi}{T_1} - \frac{2\pi}{T_2}} = 24 \text{ h}$$

61. (3) $M_A = \sigma 4\pi R_A^2$, $M_B = \sigma 4\pi R_B^2$

where σ is surface density.

$$\Rightarrow V_A = \frac{-GM_A}{R_A}, V_B = \frac{-GM_B}{R_B}$$

$$\frac{V_A}{V_B} = \frac{M_A}{M_B}\frac{R_B}{R_A} = \frac{\sigma 4\pi R_A^2}{\sigma 4\pi R_B^2} \times \frac{R_B}{R_A} = \frac{R_A}{R_B}$$

Given $\frac{V_A}{V_B} = \frac{M_A}{M_B} = \frac{3}{4}$

Then $R_B = \frac{4}{3}R_A$

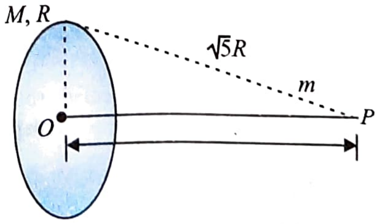
For new shell of mass M and radius R,

$$M = M_A + M_B = \sigma 4\pi R_A^2 + \sigma 4\pi R_B^2$$

$$\sigma 4\pi R^2 = 4\pi(R_A^2 + R_B^2)$$

Then $\frac{V}{V_A} = \frac{M R_A}{R M_A} = \frac{\sigma 4\pi[R_A^2 + R_B^2]}{(R_A^2 + R_B^2)^{\frac{1}{2}}} \times \frac{R_A}{\sigma 4\pi R_A^2} = \frac{\sqrt{R_A^2 + R_B^2}}{R_A} = \frac{5}{3}$

62. (4)



Gravitational potential at 'P', $V_P = \frac{-GM}{\sqrt{5}R}$

Gravitational potential at 'O', $V_O = -\frac{GM}{R}$

Using work energy theorem,

$$W = \Delta K \Rightarrow m[V_P - V_O] = \frac{1}{2}mv^2$$

$$m = \left[\frac{GM}{R} - \frac{GM}{\sqrt{5}R}\right] = \frac{1}{2}mv^2$$

$$\sqrt{\frac{2GM}{R}\left[1 - \frac{1}{\sqrt{5}}\right]} = v$$

63. (3) During total eclipse, total attraction due to the Sun and the Moon,

$$F_1 = \frac{GM_sM_e}{r_1^2} + \frac{GM_mM_e}{r_2^2}$$

When the Moon goes on the opposite side of the earth, the effective force of attraction is

$$F_2 = \frac{GM_sM_e}{r_1^2} - \frac{GM_mM_e}{r_2^2}$$

Change in force, $\Delta F = F_1 - F_2 = \dfrac{2GM_mM_e}{r_2^2}$

Change in acceleration of the earth,

$$\Delta a = \frac{\Delta F}{M_e} = \frac{2GM_m}{r_2^2}$$

Average force on the earth,

$$F_{av} = \frac{F_1 + F_2}{2} = \frac{GM_sM_e}{r_1^2}$$

Average acceleration of the earth,

$$a_{av} = \frac{F_{av}}{M_e} = \frac{GM_s}{r_1^2}$$

Percentage change in acceleration is

$$\frac{\Delta a}{a_{av}} \times 100 = \frac{2GM_m}{r_2^2} \times \frac{r_1^2}{GM_s} \times 100 = 2\left(\frac{r_1}{r_2}\right)^2 \frac{M_m}{M_s} \times 100$$

64. (2) Using Kepler's third law, the semi-major axis of the comet's orbit is given by

$$T^2 = \frac{4\pi^2}{GM_s} \times a^3$$

$$a = \left[\frac{T^2 \times GM_s}{4\pi^2}\right]^{\frac{1}{3}}$$

$$= \left[\frac{(90 \times 24 \times 3600) \times 6.67 \times 10^{-11} \times 2 \times 10^{30}}{4\pi^2}\right]^{\frac{1}{3}}$$

$$= 5.89 \times 10^{10}\ \text{m}$$

So, the major axis of the comet's orbit would be $2a = 1.178 \times 10^{11}$ m

As $2a <$ mean distance between the earth and the Sun, so the collision is not possible.

65. (3) As we decrease the magnitude of mechanical energy of the spacecraft–earth system, it means we are increasing the energy of the spacecraft–earth system as the total energy of the bounded system is negative.

As we change the energy, the circular orbit of the spacecraft will become elliptical. Let a be the semi-major axis of this new elliptical orbit.

$$E_{\text{final}} = -\frac{GM_m}{2a}$$

$$E_{\text{initial}} = \frac{GM_m}{2r}, \text{ where } r = 7000 \text{ km}$$

$$\therefore \quad \frac{GM_m}{2r} = \frac{GM_m}{2a}$$

$$a = \frac{r}{0.9} = 1.11\,r$$

$r_{\max} = 2a$ [where $r_{\max}$ is the distance corresponding to aphelion]

$= 2.22r - r = 1.22r$

Required greatest height, $h = r_{\max} - R_e = 2140$ km

66. (2) $E = \dfrac{m}{2} \times \dfrac{6GM_e}{5r} - \dfrac{GM_em}{2r} = -\dfrac{2}{3}\dfrac{GM_em}{r}$

which is the total energy of the earth–satellite system.

So, semi-major axis of the elliptical orbit is $a = \dfrac{5r}{4}$.

Speed of the satellite at the apogee position is

$$v_A = \frac{v_P \times r}{2a - r} = \frac{2}{3}\sqrt{\frac{6GM_e}{5r}}$$

For orbit to change to a circle of radius $3r/2 = (2a - r)$, the rocket has to be fired when the satellite is at the apogee position.

New orbital speed is $v_0 = \sqrt{\dfrac{GM_e}{3r/2}} = \sqrt{\dfrac{2GM_e}{3r}}$.

Required change in the orbital speed is

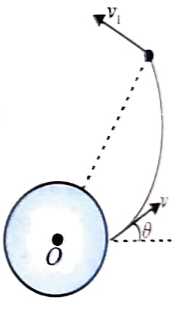
$$\Delta v = v_A - v_0 = 0.085\sqrt{\frac{GM_e}{R}}.$$

67. (1) As all the points on the periphery of either ring are at the same distance from point P, the potential at point P due to the whole ring can be calculated as $V = -(GM)/\left(\sqrt{R^2 + x^2}\right)$ where x is the axial distance from the centre of the ring. This expression is independent of the fact whether the distribution of mass is uniform or non-uniform.

So, at P, $V = -\dfrac{GM}{\sqrt{2}R} - \dfrac{G \times 2M}{\sqrt{5}R} = -\dfrac{GM}{R}\left[\dfrac{1}{\sqrt{2}} + \dfrac{2}{\sqrt{5}}\right]$.

68. (1) Let the speed at the maximum height be v_1, then from energy conservation,

$$\frac{mv^2}{2} - \frac{GMm}{R_e} = \frac{mv_1^2}{2} - \frac{GMm}{5R_e/4}$$



From angular momentum conservation,

$$mvR_e \sin\theta = mv_1 \times \frac{5R_e}{4}$$

Using $v = \sqrt{\dfrac{gR_e}{2}}$ and solving the equation, we get $\theta = \sin^{-1}\left(\dfrac{\sqrt{5}}{4}\right)$.

69. (2) Even though the distribution of the mass is unknown, we can find the potential due to the ring on any axial point because from any axial point the entire mass is at the same distance (whatever would be the nature of distribution).

Potential at A due to the ring is $V_A = \dfrac{GM}{\sqrt{2}R}$

Potential at B due to the ring is $V_B = \dfrac{GM}{\sqrt{5}R}$

$$dU = U_f - U_i = U_B - U_A = m_0(V_B - V_A)$$

$$= \frac{GMm_0}{R}\left[-\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{2}}\right]$$

$$W_{gr} = -W_{ext}$$

$$W_{gr} = -dU = -W_{ext}$$

$$W_{ext} = dU = \frac{GMm_0}{R}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right]$$

70. (1) Here time period of the satellite w.r.t. an observer on the equator is 24 h and the satellite is moving from west to east, so angular velocity of the satellite w.r.t. earth's axis of rotation (considered as fixed) is $\omega = \dfrac{2\pi}{T_s} + \dfrac{2\pi}{T_e}$, where T_s and T_e are time periods of satellite and earth, respectively.

$$\omega = \frac{\pi}{6}(\text{h}^{-1}) = 1.45 \times 10^{-4} \text{ rad s}^{-1}$$

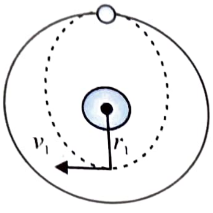
$$\text{From, } v = \sqrt{\frac{GM}{r}}$$

$$r\omega = \sqrt{\frac{GM}{r}}$$

$$r^{\frac{3}{2}} = \sqrt{\frac{GM}{r}}$$

$$r = 2.66 \times 10^7 \text{ m} = 2.66 \times 10^4 \text{ km}$$

4. (1) As the velocity of the particle is less than the orbital velocity of the satellite, the particle goes in the elliptical orbit of the semi-major axis less than r.



Let r_1 be the minimum distance and v_1 be the velocity of the particle at this position, then

$$m_0 \times \sqrt{\frac{2}{3}} \times v_0 r = m_0 v_1 r_1,$$

where m_0 is the mass of the particle and v_0 is the orbital speed equal to $\sqrt{GM/r}$.

$$v_1 r_1 = \sqrt{\frac{2}{3}}\, v_0 r$$

From energy conservation,

$$\frac{m_0 \times \frac{2}{3} v_0^2}{2} - \frac{Gm_0}{r} = \frac{m_0 v_1^2}{2} - \frac{GMm_0}{r_1}$$

Solving the equations, we get $r_1 = r/2$.

Multiple Correct Anwers Type

1. **(1), (2), (3), (4)**

For all the points, $E_P > E_K$ (numerically). So, the total energy is negative. Thus, the system is a bound system corresponding to all the points.

2. **(2), (4)**

If universal gravitational constant G starts to decrease, then gravitational force between the Sun and the earth will also start to decrease. Therefore, the earth will try to follow a path of larger radius. Hence, its period of revolution round the Sun will increase. Therefore, duration of the year will increase. But rotational motion of the earth about its own axis will remain unchanged; hence, period of rotation will remain unchanged or length of day will remain unchanged. Hence, option (1) is incorrect, (2) is correct and (3) is incorrect.

Since the radius of the circular path of the earth will increase or the earth will follow a spiral path of increasing radius, therefore its PE will increase but PE is always negative, so the magnitude of PE will decrease. Hence, KE will also decrease. Therefore, option (4) is correct.

3. **(2), (3)**

For an earth satellite moving in a circular orbit, the centripetal force required for its circular motion is provided by the gravitational force exerted by the earth on it.

It means the resultant force on the astronaut is equal to the gravitational force exerted by the earth on him. Hence, no reaction is exerted by the floor of the satellite on him.

In other words, his acceleration towards the earth's centre (centripetal acceleration) is exactly equal to acceleration caused by the gravitational force.

Hence, options (2) and (3) are correct.

4. **(1), (3), (4)**

Statement (1) is incorrect; there is no method by which a body can be shielded from the gravitational field of another body. Statement (2) is correct. Statement (3) is incorrect. As the earth rotates about its axis, a body on the surface of the earth also rotates with it. Since the body is in a rotating (non-inertial) frame, it experiences an outward centrifugal force against the inward force of gravity. As a result, the acceleration due to gravity decreases due to rotation. Statement (4) is incorrect, the forces are equal in magnitude but opposite in direction. Hence, choices (1), (3) and (4) are wrong.

5. **(3), (4)**

Linear velocity or orbital velocity is $v = \sqrt{GM/r}$ where r is the distance of the satellite from the centre of the earth. Therefore, v decreases as r is increased.

Also $v = r\omega$, where ω is the angular velocity.

Therefore, $\omega = \frac{v}{r} = \frac{1}{r}\sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{r^3}}$.

Thus, ω decreases with increase in r. The time period of the satellite is given by $T = 2\pi\sqrt{\frac{r^3}{GM}}$

Thus, T increases as r is increased. Hence, the correct choices are (3) and (4).

6. **(1), (2), (3)**

(1) For a proper explanation, the earth–Moon system will be regarded as an isolated system. At a particular point in space, the gravitational force of attraction of the earth on astronaut will be balanced by the gravitational force of attraction of the Moon.

(2) $V = -GM/R$. As R decreases, GM/R increases or $-GM/R$ decreases.

(3) In case of the spherical shell, plot of I versus r is discontinuous.

7. **(1), (2), (3), (4)**

(1) and (2) Factual statement.

(3) $g = \frac{GM}{R^2}$ or $M = \frac{gR^2}{G}$

(4) The weight of a body on the surface of earth is not determined by the orbital motion of earth around Sun.

8. **(2), (3)**

When a planet is on the major axis of the orbit, gravitational force on the planet is normal to its motion. So, no work is done. As that energy remains the same, no work is done in complete revolution.

9. **(1), (3)**

If $r > R$, $g' = g\frac{R^2}{(R+h)^2} = g\frac{R^2}{r^2}$

If $r < R$, $g' = g\left[\frac{R-d}{R}\right]$

where d is the depth below the surface of earth.

If $d = R$, $g = 0$.

Further, due to rotation: $g' = g - R\omega^2 \cos^2\theta$

If $\omega = 0$, then g increases.

10. **(1), (3), (4)**

From relation $|E| = \frac{dV}{dr}$

when $V = 0$, $E = 0$; when $V =$ constant, $E = 0$; when $V \neq 0$, $E \neq 0$.

So, options (1), (3) and (4) are correct.

11. (1), (2), (3), (4)

i. If $v < v_e$ and the body is projected vertically upwards, the body will rise up to that height where its velocity becomes zero. After that it will fall freely due to gravity following a straight line path.

ii. If $v < v_e$ but the body is projected at some angle w.r.t. vertical direction, the body will reach up to a certain height (where vertical component of the velocity becomes zero) and then fall down following a parabolic path (this will be a case of projectile motion).

iii. If $v = v_0 = \sqrt{GM/R}$, then the orbit will be circular.

But if $v_0 < v < v_e$ (escape velocity), then the orbit will be an ellipse.

So, all the options are correct.

12. (1), (4)

(1) For Moon, escape velocity is 2.4 km s^{-1}.

(4) $v_0 = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{M} \times \frac{4}{3}\pi R^3 \rho}$

or, $v_0 \propto \sqrt{\rho}$

13. (1), (3)

If the speed of the orbiting satellite is made $\sqrt{2}$ times, then the satellite will escape because its velocity becomes equal to escape velocity. This implies that the speed is increased by 41% and the KE is doubled.

14. (2), (3)

If radius of an orbiting satellite is decreased, then its PE gets decreased. As ME = KE + PE, so ME also decreases.

$$\left[\text{PE} = -\frac{GMm}{r},\ \text{KE} = +\frac{GMm}{2r} \text{ and ME} = -\frac{GMm}{2r}\right]$$

15. (3), (4)

If two satellites of different masses are revolving in the same orbit, then they have the same time period and speed because

$$v_0 = \sqrt{\frac{GM}{r}}$$

and $$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Angular momentum (= mvr) and energy = – $(GMm)/2r$ both depend on the mass of the satellites. So, options (3) and (4) are correct.

16. (1), (2), (3)

Choices (1), (2) and (3) are correct. When the particle exerts a force on another particle; the second particle exerts an equal and opposite force on the first particle. Choice (4) is incorrect because although G is constant everywhere, the value of g varies with height and depth and also from place to place.

17. (2), (3)

Statement (1) is incorrect; the magnitude of the force is 6.67×10^{-11} N. Statement (2) is correct. If the escape velocity for a planet is high, gases are not able to escape. Statement (3) is also correct. Statement (4) is incorrect; force of friction arises due to electrical forces. Hence, correct choices are (2) and (3).

18. (3), (4)

Linear speed is $v = r\omega$. At the equator, the radius of the earth r is maximum. Therefore, $\omega = \frac{v}{r}$ is minimum. Also,

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{24 \times 60 \times 60} = 7.3 \times 10^{-5} \text{ rad s}^{-1}$$

Hence, the correct choices are (3) and (4).

19. (2), (3), (4)

Distances of the two satellites from the centre of the earth are $r_1 = 2R$ and $r_2 = 8R$, respectively (R = earth's radius). Their potential energies are

$$V_1 = -\frac{GmM}{r_1} \text{ and } V_2 = -\frac{GmM}{r_2}$$

Their ratio is $\frac{V_1}{V_2} = \frac{r_2}{r_1} = \frac{8R}{2R} = 4$

The kinetic energy of a satellite can be obtained from the relation

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

or $$K = \frac{1}{2}mv^2 = \frac{GmM}{2r}$$

$$K_1 = \frac{GmM}{2r_1} \text{ and } K_2 = \frac{GmM}{2r_2}$$

The ratio of their kinetic energies is

$$\frac{K_1}{K_2} = \frac{r_2}{r_1} = \frac{8R}{2R} = 4$$

Their total energies are

$$E_1 = -\frac{GmM}{r_1} + \frac{GmM}{2r_1} = -\frac{GmM}{2r_1}$$

and $$E_2 = -\frac{GmM}{r_2} + \frac{GmM}{2r_2} = -\frac{GmM}{2r_2}$$

Their ratio is $\frac{E_1}{E_2} = \frac{r_2}{r_1} = \frac{8R}{2R} = 4$

Hence, the correct choices are (2), (3) and (4).

20. (2), (4)

The acceleration due to gravity is $g = \frac{GM}{R^2}$.

The new value of g would be $g' = \frac{G(0.99M)}{(0.99R)^2} = 1.01\frac{GM}{R^2}$

Thus, g would increase by about 1%. The new escape velocity would be $v'_e = \sqrt{\frac{2 \times 0.99M \times G}{0.99R}} = \sqrt{\frac{2MG}{R}} = v_e$

Thus, the escape velocity will remain unchanged. The potential energy of a body of mass m on the earth's surface would be

$$-\frac{Gm\,(0.99M)}{(0.99R)} = -\frac{GmM}{R}$$

Thus, the potential energy will also remain unchanged. Hence, the correct choices are (2) and (4).

21. (1), (3), (4)

Work done is independent of the path chosen and depends only on the initial and final positions of the object. Also the work done on any closed path in a gravitational field will be zero. Since, every point on the surface of the earth is at the same potential, no work is done for points on the surface of the earth. Hence, the correct choices are (1), (3) and (4).

22. (2), (3)

$$W = U_2 - U_1 = \frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right) = -GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$= gR^2m\left[\frac{h}{R(R+h)}\right] = \frac{mgRh}{R+h}$$

For $h << R,\ W = mgh$

For $h = R,\ W = \frac{mgR}{2}$

23. **(1), (3), (4)**

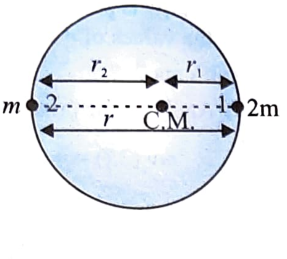
(1) $g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho$ or $\rho = \frac{3g}{4\pi GR}$

(2) Acceleration due to gravity varies from place to place, hence, not correct.

(3) $v_e' = \sqrt{\frac{2G(2M)}{\frac{R}{2}}} = 2\sqrt{\frac{2GM}{R}} = 2 \times 11.2$ km s^{-2} $= 22.4$ km s^{-1}

(4) Factual statement.

24. **(1), (4)**

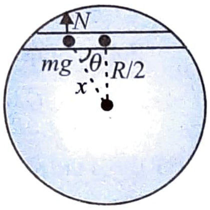


$$r_2 = \frac{2mr}{m+2m} = \frac{2r}{3}$$

$$T_2^2 = \frac{4\pi^2 r_2^3}{Gm} \Rightarrow T_2^2 = \frac{32\pi^2 r^3}{27Gm}$$

$T_2 \propto r^{\frac{3}{2}}$ and $T_2 \propto m^{-\frac{1}{2}}$

25. **(2), (3)**



Net force towards the centre of the earth

$$= mg' = \frac{mGMx}{R^3} = \frac{mgR^2x}{R^3}$$

$$= (mgx)/R$$

Normal force, $N = mg' \sin\theta$

Thus, pressing force,

$$N = \frac{mgx}{R}\frac{R}{2x}$$

$N = \frac{mg}{2}$ is constant and independent of x.

Hence, option (2) is correct.

Tangential force, $F = ma = mg' \cos\theta$

$$a = g' \cos\theta = \frac{gx}{R}\frac{\sqrt{\frac{R^2}{4} - x^2}}{x}$$

$$\Rightarrow a = \frac{gx}{R}\sqrt{R^2 - 4x^2}$$

Curve is parabolic and at $x = \frac{R}{2}$, $a = 0$.

Hence, option (3) is correct.

26. **(1), (3)**

The centre of mass of the double star system remains stationary and both the stars revolve around in circular orbits which are concentric with the centre of mass.

The distance of centre of mass from the heavier star is equal to

$$\left(\frac{Mr + 2M.0}{M + 2M}\right) = \frac{r}{3}$$

Hence, the heavier star revolves in a circle of radius $r/3$ while the lighter star in a circle of radius $2r/3$. Hence, option (1) is incorrect.

To calculate period of revolution of a double star system, concept of reduced mass can be used.

The reduced mass of the system is equal to $\frac{(M)(2M)}{(M+2M)} = \frac{2M}{3}$.

Hence, the period of revolution will be equal to $\frac{2\pi}{\sqrt{\frac{2GM}{3}}} r^{\frac{3}{2}}$

where r is distance between two stars. Hence, option (2) is correct.

Kinetic energy of a star will be equal to $1/2\, mv^2$ where v is speed of the star which is equal to (radius of its circular orbit) $\times\, \omega$.

Hence, KE of heavier star is $E_1 = \frac{1}{2}(2M)\left(\frac{r}{3}\omega\right)^2$ and that of lighter star, $E_2 = \frac{1}{2}M\left(\frac{2r}{3}\omega\right)^2$.

It means, KE of the lighter star is twice that of the heavier star. Hence, option (3) is incorrect.

27. **(1), (2), (3), (4)**

From Kepler's law, $T^2 \propto R^3$

So, $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{\frac{3}{2}}$

$$R_2 = \left(\frac{T_2}{T_1}\right)^{\frac{2}{3}} \times R_1 = \left(\frac{24}{6}\right)^{\frac{2}{3}} \times 1.28 \times 10^4 \text{ km} = 3.22 \times 10^4 \text{ km}$$

Orbital velocity of S_1 is $v_1 = \frac{2\pi R_1}{T_1} = \frac{2\pi \times 1.28 \times 10^4}{4}$

$$v_1 = 0.64\,\pi \times 10^4 \text{ km}$$

Orbital velocity of S_2 is $v_2 = \frac{2\pi R_2}{T_2} = \frac{2\pi \times 3.22 \times 10^4}{24}$

$$v_2 = 0.64\pi \times 10^4 \text{ km}$$

At $t = 12$ h the two satellites are closest to each other and after every 24 h they come at the same position relative to each other. It is clear that direction of v_2 w.r.t. v_1 is changing continuously in both magnitude and direction.

Angular velocity of S_2 w.r.t. S_1 at $t = 12$ h is

$$\omega = v_2/R_2 = 0.468\pi \text{ rad s}^{-1}$$

28. **(1), (2), (4)**

Just before collision, the total energy of two satellites is

$$E = -\frac{GMm}{2r} - \frac{GMm}{2r} = -\frac{GMm}{r}$$

Let the orbital velocity be v, then from momentum conservation,

$$mv - mv = 2m \times v_1$$

$$v_1 = 0$$

As velocity of combined mass just after collision is zero, the combined mass will fall towards the earth. At this instant, the total energy of the system only consists of the gravitational potential energy given by

$$U = \frac{GM \times 2m}{r}$$

29. **(2), (3)**

By energy conservation, $v_A = \sqrt{\frac{4}{3}\frac{GM}{R}}$

For making the path elliptical velocity required at greatest approach point.

(i) For aphelion point $v_R = \sqrt{\frac{GM}{3R}}$

(solving by using angular momentum and energy conservation so energy change

$$= \frac{1}{2}m\left(\frac{GM}{3R}\right) - \frac{1}{2}m\left(\frac{4GM}{3R}\right) = -\frac{1}{2}\frac{GMm}{R}$$

(ii) For perihelion point $v_R = \sqrt{\frac{GM}{R}}$

$$\Rightarrow \Delta E = \frac{1}{2}m\left(\frac{GM}{R}\right) - \frac{1}{2}m\left(\frac{4GM}{3R}\right) = -\frac{1}{6}\frac{GMm}{R}$$

30. (1), (2), (3)

(1) By the conservation of energy

$$0-\frac{GMm}{2R}=\frac{1}{2}mv^2-\frac{GMm}{R}$$

So, $v=\sqrt{gR}$

(2) Let escape velocity at this orbit is v_e

$$\frac{1}{2}mv_e^2-\frac{GMm}{2R}=0 \Rightarrow v_e=\sqrt{\frac{GM}{R}}$$

(3) Total energy of satellite $E=-\frac{GMm}{2(R+h)}$

Total energy in the new orbit

$$E_2=-\frac{GMm}{2(R+R/2)}=-\frac{GMm}{3R}$$

Total energy in the old orbit

$$E_1=-\frac{GMm}{2\cdot(2R)}=-\frac{GMm}{4R}$$

Energy spent = total energy of satellite in the new orbit - total energy of satellite in old orbit

$$\text{Energy spent }=E_2-E_1=\left(-\frac{GMm}{3R}\right)-\left(-\frac{GMm}{4R}\right)=-\frac{GMm}{12R}$$

31. (1), (2), (3), (4)

As initially the particles are at an infinite separation hence total energy is zero and by conservation of energy during motion of particles total energy will remain zero. At a separation r the potential energy of the particles can be given as $-\frac{4Gm^2}{r}$ hence the kinetic energy magnitude must be same for total energy to be zero and by kinetic energy we can calculate the velocity of approach as no external force or torque is present in the system total angular momentum of the system will remain conserved.

32. (1), (2), (3)

As the orbital speed of a satellite in a circular orbit around earth varies inversely proportional to the square root of the orbit radius, in this case as the orbit radius is minimum option (1), (2) and (3) are correct.

33. (2), (4)

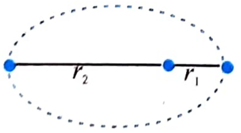
As value of g is directly proportional to M/R^2 and as M is same for both we use ρR^3 = constant hence we get $R_1:R_2=2:1$ thus $g_1:g_2=1:4$ and escape velocity is $\sqrt{2gR}$.

The ratio of escape velocities will be $1:\sqrt{2}$.

Linked Comprehension Type

For Problems 1–3

1. (2)



Gravitational PE at perihelion = $-GMm/r_1$ as r_1 is minimum Therefore, PE is minimum.

2. (3) Total energy = constant.

3. (3) As Pluto moves away, the displacement has component opposite to the air force, hence work done is negative.

For Problems 4–6

4. (1) For the system to be bounded one, total energy of the system must be negative. So, object having total energy E_1 is bounded and other is unbounded.

5. (4) Total energy = KE + PE = negative, so the system is bounded.

At $r=r_0$, total energy = PE and KE = 0.

For $r>r_0$, total energy = PE and KE = negative, which is not possible, so r_0 is the maximum distance of the object from the earth's centre.

6. (1) The object having total energy E_2 is not bounded as its total energy is positive and it is never an intersecting PE curve, so all the values of r are possible for it.

For Problems 7–8

7. (3) Applying the principle of conservation of angular momentum,

$$mv_1(2R)=mv_2(4R)$$

$$v_1=2v_2 \quad \text{...(i)}$$

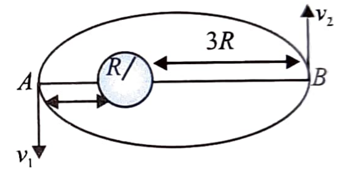
From conservation of energy,

$$\frac{1}{2}mv_1^2-\frac{GMm}{2R}=\frac{1}{2}mv_2^2-\frac{GMm}{4R} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_2=\sqrt{\frac{GM}{6R}},\ v_1=\sqrt{\frac{2GM}{3R}}$$

8. (1) If r is the radius of curvature at point A,



$$\frac{mv_1^2}{r}=\frac{GMm}{(2R)^2}$$

$$r=\frac{4v_1^2R^2}{GM}=\frac{8R}{3}\text{ (putting value of }v_1)$$

For Problems 9–10

9. (1), 10. (3)

$$E_P=\frac{GM}{(4a)^2}=\frac{GM}{16a^2} \quad \text{and} \quad E_{P}=\frac{GM}{(6a)^2}+\frac{GM}{5a^2}=\frac{61GM}{900a^2}$$

For Problems 11–15

11. (2) From work–energy theorem, $dK=-dU+W_{\text{air friction}}$

$W_{\text{air friction}}$ is negative, so

$dK=-dU+$ (a negative quantity)

As K increases, it means U decreases by an amount greater than magnitude of $W_{\text{air friction}}$.

12. (1) We have $v=\sqrt{\frac{GM}{R}}$

Taking log on both the sides and then differentiating,

$$\frac{dv}{v}=\frac{-1\times dR}{2R}$$

$$\frac{\Delta v}{v}=\frac{\Delta R}{2R} \quad [\because \Delta R=-dR]$$

$$\Delta v=\frac{\Delta R}{2}\sqrt{\frac{GM}{R^3}}$$

(3) $g' = g - \omega^2 R_e \cos^2 \lambda$

$$\frac{dK}{K} = \frac{2dv}{v} = \frac{2\Delta v}{v} \qquad [\therefore dv = \Delta v \text{ as } v \text{ is increasing}]$$

$$= \frac{2\Delta R}{2R} = \frac{\Delta R}{R}$$

$$dK = K \times \frac{\Delta R}{R} = \frac{GMm}{2R} \times \frac{\Delta R}{R} = \frac{GMm}{2R^2} \times \Delta R$$

$$\Delta K = dK \qquad [\therefore \text{KE is increasing}]$$

14. (2) $\Delta U = U_f - U_i = -\left[\frac{GMm}{R - \Delta R}\right] - \left[-\frac{GMm}{R}\right]$

$$= -\left[GMm\left(\frac{1}{R - \Delta R} - \frac{1}{R}\right)\right] = -\frac{GMm}{R^2}\Delta R$$

15. (4) From the work–energy theorem,

$$dK = -dU + W_{\text{air friction}}$$

$$\frac{GMm}{2R^2} \times \Delta R = -\left[-\frac{GMm}{R^2}\Delta R\right] + W_{\text{air friction}}$$

$$W_{\text{air friction}} = -\frac{GMm}{2R^2} \times \Delta R$$

For Problems 16–20

16. (3) As it is not known that by how much the velocity has changed, which will cause the change in orbital radius and hence time speed, so we can use either rocket system to carry out the docking in the minimum possible time, though the extent of firing rockets may differ.

17. (2) Time period of A and B in a given circular orbit is

$$T = (2\pi r^{3/2})/\sqrt{GM}$$

where $r = 9600$ km, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, $M = 5.98 \times 10^{24}$ kg, which gives

$$T = 9357.79 \text{ s} = 9358 \text{ s}$$

Initially, total energy of A is $E = -\frac{GMm}{2r} = -6.65 \times 10^9$ J

18. (4) Speed of A before firing the rocket,

$$v_i = \sqrt{GM/r} = 6445.82 \text{ m s}^{-1}$$

After firing the rocket speed of A is

$$v_f = 0.995\, v_i = 6413.6 \text{ m s}^{-1}$$

Energy of the satellite after firing,

$$E_f = 2E + \frac{mv_f^2}{2} = -6.714 \times 10^9 \text{ J}$$

as due to firing potential energy does not change.

From $E = -\frac{GMm}{2a}$, the new semi-major axis is given by

$$a = -\frac{GMm}{2E_f} \Rightarrow a = 9505.3 \text{ km}$$

New time period, $T' = \frac{2\pi a^{32/2}}{\sqrt{GM}} = 9219.67$ s

19. (1) Initially, the time periods of the satellite and the spacecraft were same, equal to 9358 s and after firing rockets of A, it acquires slower and faster orbit and A will come to initial position in time period of 9220 s while B will take 9358 s to come to its initial position or (9358 – 100) s to come to the original position of A as 100 s is the initial time gap between the two.

It means now A leads B by (9358 – 100 – 9220) = 38 s.

20. (2) To bring back the satellite in the same orbit, we have to make the speed of the satellite as initial one for which we have to increase v_f by such amount so that it becomes equal to v_i for which we have to fire the rocket in backward direction with the same extent.

For Problems 21–22

21. (3) $\because W = \Delta KE \quad \because W_{\text{person}} + W_{\text{field}} = 1/2\, mv^2$

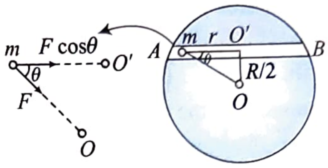
$$\Rightarrow W_{\text{person}} + U_1 - U_2 = 1/2\, mv^2$$

$$\Rightarrow -3 + (0 - V_A) \times 1 = 1/2 \times 1 \times 2^2 \Rightarrow V_A = -5 \text{J/kg}$$

22. (1) $|\vec{E}| = \frac{\Delta V}{\Delta r} = \frac{8 - 4}{(1)\sin 30°} = 8 \text{ N kg}^{-1}$

For Problems 23–24

23. (4) Force due to gravitational field at a distance r, in radial direction.



$$F = \frac{4\pi}{3} G\rho r m$$

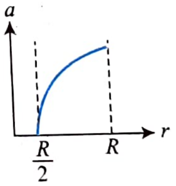
Component of force along AB

$$= F\cos\theta = \frac{4\pi}{3} Gm \frac{M_e}{\frac{4}{3}\pi R^3} r \frac{\sqrt{r^2 - \frac{R^2}{4}}}{r}$$

$$F'_{AB} = m\frac{GM_e}{R^3}\sqrt{r^2 - \left(\frac{R^2}{4}\right)}$$

$$a_{AB} = \frac{GM_e}{R^3}\sqrt{r^2 - \left(\frac{R^2}{4}\right)}$$

Hence graph should be parabola.



24. (1) Acceleration of the object, $a_{AB} = \frac{GM_e}{R^3} x = \omega^2 x$

Angular frequency, $\omega = \sqrt{\frac{GM_e}{R^3}}$

Hence time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^3}{GM_e}}$

Matrix Match Type

1. **i. → a., b.; ii. → c.; iii. → a., b.; iv. → c.**

i. Acceleration due gravity is different at different points and it does not depend upon direction of projection.

ii. Angular momentum of the earth remains constant about the Sun.

iii. Escape velocity depends upon g and independent of direction of projection.

iv. Gravitational potential will remain constant.

2. i. → a., c.; ii. → a., b., c., d.; iii. → a., b., c., d.; iv. → a., c., d.

i. A geostationary satellite has circular orbit over equator.

ii. Total energy will remain constant in any orbit.

iii. Angular momentum will remain constant in any orbit.

iv. In circular orbit, speed remains the same, whereas in elliptical orbit speed changes.

3. i.→ a.; ii. → a., c., d.; iii. → a., b., c., d.; iv. → a.

As rocket has been fired the speed of the satellite increases in all these three cases which leads to the increase in kinetic energy and hence total energy becomes less negative or we can say the semi-major axis increases.

Due to thrust exerted by firing of the rocket in a direction perpendicular to the plane of the orbit, the plane of the orbit changes.

Here speed decreases and hence the kinetic energy, the total energy and the semi-major axis decrease.

4. i. → a.; ii. → f.; iii. → c.

Angular momentum of the particle = $m(v_0 + v)a$

$$= \sqrt{\frac{5}{4}}\, m v_0 a \qquad \left[v_0 = \sqrt{\frac{GM_e}{a}}\right]$$

Total energy of the particle $= \frac{1}{2}m(v_0 + v)^2 - \frac{GM_e m}{a}$

$$= \frac{1}{2} \times \frac{5}{4} m v_0^2 - \frac{GM_e m}{a}$$

$$= \frac{5}{8}\frac{GM_e m}{a} - \frac{GM_e m}{a} = -\frac{3GM_e m}{a}$$

At any distance r, TE $= \frac{1}{2} mu^2 - \frac{GM_e m}{a}$

But angular momentum conservation gives

$$mur = \sqrt{\frac{5GM_e}{4a}}$$

$$\Rightarrow u = \sqrt{\frac{5}{4}\frac{GM_e a}{r^2}a}$$

TE at any distance $r = \frac{1}{2} m \frac{5}{4} \frac{GM_e a}{r^2} - \frac{GM_e m}{r}$

But through conservation of total energy, we have

$$\frac{1}{2} m \frac{5}{4}\frac{GM_e a}{r^2} - \frac{GM_e m}{r} = -\frac{3GM_e m}{a}$$

On solving, we get

$$3r^2 - 8ar + 5a^2 = 0$$

$$(r - a)(3r - 5a) = 0$$

$$R = a,\ r = \frac{5a}{3}$$

Minimum distance = a

Maximum distance = $\frac{5a}{3}$

5. i. → a., c.; ii. → a., c.; iii. → b., c.; iv. → a., c.

i. At centre of thin spherical shell, $V \neq 0, E = 0$.

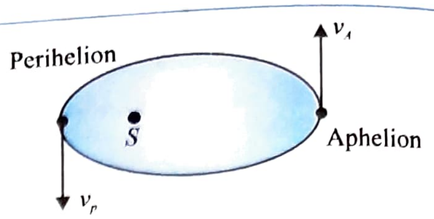
ii. At centre of solid sphere, $V \neq 0, E = 0$.

iii. At centre of spherical cavity inside solid sphere, $V \neq 0, E \neq 0$.

iv. At centre of two point masses, $V \neq 0, E = 0$.

6. i. → b., d.; ii. → a., c., d.; iii. → a., d.; iv. → b., c.

Speed of the planet is maximum at perihelion and minimum at aphelion. So, KE is maximum at perihelion and minimum at aphelion.



PE and KE of the planet are dependent on both mass of planet and on semi-major axis.

7. i. → c., d.; ii. → d.; iii. → a.; iv. → b., c.

Gravitational potential is defined as the work done in carrying a unit mass from a point outside the field (zero potential) to a point inside the field. So when reference point is altered the potential will vary and the potential difference will remain the same. Also the energy is not dependent on the angle by any means and depends only on the two points (initial and final) in any conservative field like gravitational field.

So i. → c., d.

Escape velocity is given by $\sqrt{GM/R}$ and is dependent on the mass of the planet and does not depend on the angle of projection.

So ii. → d.

Acceleration due to gravity is given by $g = GM/R^2 = 4/3\,\pi R_{\rho} G$ and so its ratio in two planets depends on their densities when their mass is not the same.

So iii. → a.

As the Kepler's law, the angular momentum will remain conserved for keeping the area swept in equal time to remain the same. The same concept was proposed by Bohr's model of atom.

So iv. → b., c.

8. (3) $g' = g - R\omega^2 \cos^2 \phi$

$\phi = 90°$ at pole, there is no effect of rotation of earth at pole.

At equator, g' will increase, when ω decreases.

T will increase if ω decreases.

r will increase from Kepler's third law

$E = -\frac{GMm}{2r}$, E will increase with increase in r.

9. (3) Potential

Sphere: $V = -\frac{GM}{2R^3}(3R^2 - r^2)$ for $r \le R$

$V = -\frac{GM}{r}$ for $r > R$

Shell: $V = -\frac{GM}{R}$ for $r \le R$

$V = -\frac{GM}{r}$ for $r > R$

Field

Sphere: $E_g = \frac{GM}{R^3} \cdot r$ for $r \le R$

$E_g = \frac{GM}{r^2}$ for $r > R$

Shell: E_g = zero for $r < R$

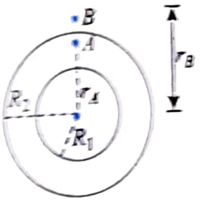
$E_g = \frac{GM}{r^2}$ for $r \ge R$

Inside a shell, $V = -\frac{GM}{R}$ and $E = 0$

Outside the shell, $V = -\frac{GM}{r}$ and $E = \frac{GM}{r^2}$

$$V_A = -\frac{GM_1}{R_2} - \frac{GM_2}{r_A}$$

$$V_B = -\frac{GM_1}{r_B} - \frac{GM_2}{r_B}$$



$$V_B > V_A$$

Now V is negative $E_B = \frac{GM_1}{r_B^2} + \frac{GM_2}{r_B^2}$

$$E_A = \frac{GM_1}{r_A^2}$$

erical Value Type

(6) $F = mr\omega^2 = 10^{24} \times 1.5 \times 10^8 \times 10^3 \times (2 \times 10^{-7})^2 = 6 \times 10^{21}$ N

(2) $g = \frac{GM}{R^2} = G\frac{4}{3}\pi R^3\rho$

$$g = \frac{4}{3}G\pi R\rho \Rightarrow g \propto R\rho$$

$$g' \propto R'\rho' \Rightarrow \rho' = 2\rho$$

Given, $\frac{g}{g'} = 1$

$$\frac{R}{R'} = 2$$

(3) Given $R' = 3R$, $M = \frac{4}{3}\pi R^3\rho$

$$g' = GM/R'^2$$

Putting $M = \frac{4}{3}\pi R'^3\rho$,

$$g' = \frac{G}{R'^2}\frac{4}{3}\pi R'^3\rho = \frac{27}{9}\frac{G}{R^2}\frac{4}{3}\pi R^3\rho = 3\frac{GM}{R^2} = 3g$$

4. (3) $F = -K/r^2$ (negative sign is for attractive force)

Potential energy $U = -\int F\,dr = \int \frac{K}{r^2}dr = -\frac{K}{r}$

Conservation of energy gives (let at other extreme position $r = b$)

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2}mv_1^2 - \frac{K}{a} = \frac{1}{2}mv_2^2 - \frac{K}{b} \quad \text{...(i)}$$

where $v_1 = \sqrt{\frac{K}{2ma}}$

Conservation of angular momentum gives

$$mv_1a = mv_2b$$

$$v_2 = \frac{a}{b}v_1 = \frac{a}{b}\sqrt{\frac{K}{2ma}}$$

Therefore, from Eq. (i)

$$\Rightarrow \frac{1}{2}m\frac{K}{2ma} - \frac{K}{a} = \frac{1}{2}m\left(\frac{a}{b}\right)^2\frac{K}{2ma} - \frac{K}{b}$$

$$-\frac{3K}{4a} = \frac{aK}{4b^2} - \frac{K}{b} \Rightarrow b^2 - \frac{4a}{3}b + \frac{a^2}{3} = 0$$

Hence, $b = a/3 \Rightarrow a/b = 3$

5. (64) $W_{ext} = \Delta U = U_f - U_i$

$$= \left(\frac{-GM}{(R+h)}\right) - \left(\frac{-GM}{R}\right)$$

$$\Delta U = \frac{mgh}{1+\frac{h}{R}} = \frac{mgR}{1+\frac{R}{h}} = \frac{mgR}{2}$$

$$\Delta U = \frac{2 \times 10 \times 6.4 \times 10^6}{2} \Rightarrow \Delta U = 6.4 \times 10^7 \text{ joule}$$

$$\Rightarrow W = \Delta U = 64 \text{ MJ}$$

6. **(2)** Let m = moon's mass; M_s = sun's mass; M_e = earth's mass; r_{ms} = the centre-to-centre distance from the sun to the moon; r_{me} = the centre-to-centre distance from the earth to the moon; F_{ms} = gravitational force exerted on the moon by the sun; and F_{me} = gravitational force exerted on the moon by the earth.

Then $\frac{F_{ms}}{F_{me}} = \frac{M_S r_{me}^2}{M_e r_{ms}^2}$

Putting the values, $\frac{F_{ms}}{F_{me}} = \left(\frac{M_S}{M_e}\right)\left(\frac{r_{me}}{r_{ms}}\right)^2$

$$= 3.2 \times 10^5\left[\frac{1}{400}\right]^2 = \frac{32}{16} = 2$$

7. **(4)** $g = \frac{GM}{R^2}$ (if R increases, g decreases)

Taking logarithms on both the sides, we get

$$\log g = \log G + \log M - 2\log R$$

Differentiating, we get $\frac{dg}{g} = 0 + 0 - \frac{2dR}{R}$

$$\frac{dg}{g} = -2\left(\frac{-2}{100}\right) = \frac{4}{100}$$

% increase in $g = \frac{dg}{g} \times 100 = \frac{4}{100} \times 100 = 4\%$

8. **(2)** From the law of conservation of energy,

$$(U+K)_{\text{surface}} = (U+K)_{\text{at }\infty}$$

$$-\frac{GMm}{R} + \frac{1}{2}mu^2 = 0 + \frac{1}{2}mv^2$$

$$-\frac{2GM}{R} + u^2 = v^2$$

$$-v_0^2 + u^2 = v^2 \qquad \left(\because v_e = \sqrt{\frac{2GM}{R}}\right)$$

$$v^2 = -(11.2)^2 + (15)^2 = -125 + 225 = 100$$

$\therefore$ $v = 10$ km s^{-1}

9. **(3)** We know that, $g = \frac{GM}{R^2} = \frac{4}{3}\pi GM\rho$

$$\therefore \frac{g_p}{g_e} = \frac{R_p\rho_p}{R_e\rho_e} \text{ also } v_e = \sqrt{2gR}$$

$$\therefore \frac{(v_e)_p}{(v_e)_e} = \sqrt{\frac{g_pR_p}{g_eR_e}} = \sqrt{\frac{g_p}{g_e}\cdot\frac{g_p\rho_e}{g_e\rho_p}} = \frac{g_p}{g_e}\sqrt{\frac{\rho_e}{\rho_p}}$$

$$= \frac{\sqrt{6}}{11}\sqrt{\frac{3}{2}} = \frac{3}{11}$$

$$\therefore (v_e)_p = \frac{3}{11}(v_e)_e = \frac{3}{11}(11) = 3 \text{ km s}^{-1}$$

10. **(3200)** Let v be the speed of projectile at the highest point. By conservation of angular momentum and conservation of mechanical energy, we have

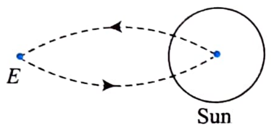
$$mu\,R\sin\alpha = mvr_{\max}$$

and $$\frac{1}{2}mu^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{r_{\max}}$$

Solving these two equations, we get $r_{\max} = \dfrac{3R}{2}$

$$\therefore \quad h_{\max} = r_{\max} - R = \frac{R}{2} = 3200 \text{ km}$$

11. **(65.00)** When the earth's motion is suddenly stopped, it would fall into the sun and (suppose) it comes back. If the effect of temperature of sun is ignored, we can say that the earth would continue to move along a strongly extended flat ellipse whose extreme points are located at the earth's orbit and at the centre of the sun.



The semi-major axis of such an ellipse is $R/2$.

$\dfrac{T'^2}{T^2} = \dfrac{(R/2)^3}{R^3}$, where T is the time period of normal orbit of the earth.

$$\therefore \quad T'^2 = \frac{T^2}{8} \Rightarrow T' = \frac{T}{2\sqrt{2}}$$

Now, required time is $t = \dfrac{T'}{2} = \dfrac{T}{4\sqrt{2}} = \dfrac{365}{4\sqrt{2}} = 65$ days

12. **(18.00)** When the particle is at the center, let their respective speeds are v_1 and v_2.

From conservation of linear momentum,

$$0 = 5.4\times10^9 v_1 - 6\times10^8 v_2$$

$$\therefore \quad v_2 = 9v_1$$

From conservation of energy, $(U+K)_i = (U+K)_f$

$$-\frac{Gm_1m_2}{\sqrt{4^2+3^2}} + 0 = -\frac{Gm_1m_2}{4} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$Gm_1m_2\left(\frac{1}{4}-\frac{1}{5}\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2(81v_1^2)$$

Now $m_1 = 9m_2$ (given)

$$G\frac{(9m_2^2)}{20} = \frac{1}{2}(90m_2)v_1^2$$

$$v_1 = \sqrt{\frac{Gm_2}{100}} = \sqrt{\frac{6.67\times10^{-11}\times6\times10^8}{100}} = 0.02 \text{ ms}^{-1}$$

$$\therefore \quad v_2 = 9v_1 = 0.18 \text{ ms}^{-1} = 18 \text{ cms}^{-1}$$

13. **(36.00)** Here, $m/g = 64$ N; $h = R/3$

The value of acceleration due to gravity at a height h (when h is not negligible as compared to R),

$$g' = g\frac{R^2}{(R+h)^2}$$

or $$mg' = mg\frac{R^2}{(R+h)^2} = 64\times\frac{R^2}{(R+R/3)^2} = \frac{9}{16}\times64 = 36 \text{ N}$$

14. **(0.25)** Here, $R = 6{,}400$ km; $x = 16$ km

The value of acceleration due to gravity at a depth x,

$$g' = g\left(1-\frac{x}{R}\right) = g\left(1-\frac{16}{6{,}400}\right) = \frac{399}{400}g$$

$$g - g' = g - \frac{399}{400}g = \frac{1}{400}g = 2.5\times10^{-3}g$$

If m is mass of the body, then mg and mg' will be respectively weight of the body on the surface of earth and at a depth of m below the surface of the earth. Then, % decrease in the weight of the body $= \dfrac{mg-mg'}{mg}\times100 = \dfrac{g-g'}{g}\times100$

$$= \frac{2\times5\times10^{-3}g}{g}\times100 = 0.25\%$$

15. **(32.00)** Let M and M' be masses and R and R' be the radii of the earth and the mars respectively. If g and g' are the values of acceleration due to gravity on the surface of the earth and the mars respectively, then

$$\frac{g'}{g} = \frac{GM'}{R'^2}\times\frac{R^2}{GM} = \frac{M'R^2}{MR'^2}$$

or $$g' = \frac{M'R^2}{MR'^2}\times g$$

If m is mass of the body, then its weight on the mars,

$$mg' = \frac{M'R^2}{MR'^2}\times mg$$

Here, $M = 10\,M'$, $R = 6{,}400$ km, $R' = 3{,}200$ km

and $mg = 80$ N

$$\Rightarrow \quad mg' = \frac{M'\times6{,}400^2}{10\,M'\times3{,}200^2}\times80 = 32 \text{ N}$$

16. **(130.0)** Let us write the expression for the acceleration due to gravity at the surface of the earth in terms of density of the earth,

We can write: $$mg = \frac{GMm}{R^2} \Rightarrow g = \frac{G\left(\frac{4}{3}\pi R^3\rho\right)}{R^2}$$

$$g = \frac{4}{3}\pi GR\rho \qquad \text{...(i)}$$

Suppose that the radius of the earth has to be increased to R' by putting up more soil around it, so that the value of acceleration due to gravity will become $g' = 10$ m s^{-2} exactly.

Then, $$g' = \frac{4}{3}\pi GR'\rho$$

From the Eqs. (i) and (ii), we get

$$\frac{g'}{g} = \frac{R'}{R}$$

or $$R' = \frac{g'}{g}R = \frac{10}{9.8}\times6.4\times10^6 = 6.53\times10^6 \text{ m}$$

Therefore, thickness of the required layer of the soil,

$$t = R' - R = 6.53\times10^6 - 6.4\times10^6$$

$$= 0.13\times10^6 \text{ m} = 130 \text{ km}$$

17. **(0.80)** Here, $M = 120$ kg; $m = 50$; $g = 0.05$ kg; and $R = 50$ cm $=$ 0.5 m and $G = 6.67\times10^{-11}$ Nm^{2}kg^{-2}

Now, initial potential energy of the system of sphere and particle,

$$U_i = -\frac{GMm}{R} = -\frac{6.67\times10^{-11}\times120\times0.05}{0.5}$$

$$= -8.0\times10^{-10}\text{ J}$$

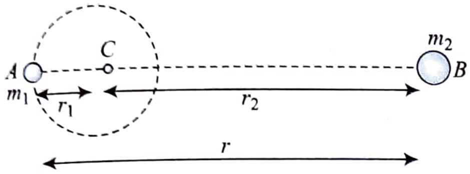
When the particle is far away from the sphere, the potential energy of the system will be zero i.e.

$$U_f = 0$$

Therefore, required work done,

$$W = U_f - U_i = 0-(-8.0\times10^{-10}) = 8.0\times10^{-10}\text{ J} = 0.8\text{ nJ}$$

18. (1) If r_1 and r_2 are distances of stars A and B from the centre of mass, then we have



$$m_1r_1 = m_2r_2 \text{ and } r_1+r_2 = r$$

This gives $r_1 = \dfrac{m_2r}{m_1+m_2}$

Given $m_1 = 2m_2$

Therefore, $r_1 = \dfrac{m_2r}{2m_2+m_2} = \dfrac{r}{3}$

Now from the condition of the circular path, if T_1 is period of revolution of star A about the centre of mass, then

$$m_1r_1\left(\frac{2\pi}{T_1}\right)^2 = \frac{Gm_1m_2}{r^2} \Rightarrow m_1\left(\frac{r}{3}\right)\left(\frac{2\pi}{T_1}\right)^2 = \frac{Gm_1m_2}{r^2}$$

$$\left(\frac{2\pi}{T_1}\right)^2 = \frac{3Gm_2}{r^3}$$

But $m_2 = \dfrac{M_s}{3}$; where M_s = mass of Sun

$$\left(\frac{2\pi}{T_1}\right)^2 = \frac{GM_s}{r^3} \qquad \text{...(i)}$$

Again period of revolution of earth (T_e) about the Sun is

$$m_eR\omega_e^2 = \frac{GM_sm_e}{R^2} \Rightarrow \left(\frac{2\pi}{T_e}\right)^2 = \frac{GM_s}{R^3}\quad\left(\because \omega_e = \frac{2\pi}{T_e}\right) \qquad \text{...(ii)}$$

As $T_1 = T_e = 1$ year $\Rightarrow r^3 = R^3 \Rightarrow r = R$

19. (7.10) Initial energy of the satellite,

$$E_1 = -\frac{GMm}{R} \quad \text{(since initial KE} = 0)$$

Final energy of the satellite, $E_2 = -\dfrac{GMm}{2(R+h)}$

Energy needed,

$$\Delta E = E_2 - E_1$$

$$= -\frac{GMm}{2(R+h)} - \left(-\frac{GMm}{R}\right) = GMm\left(\frac{1}{R} - \frac{1}{2(R+h)}\right)$$

But $GM = R^2g$

$$\Delta E = R^2gM\left(\frac{1}{R} - \frac{1}{2(R+h)}\right) = mgR\left[1 - \frac{1}{2\left(1+\dfrac{h}{R}\right)}\right]$$

$$\Rightarrow \quad \Delta E = 7.1\times10^{10}\text{ J}$$

20. (1.60) Total energy of sky laboratory, $E = -\dfrac{GMm}{2r}$

$$E_1 = -\frac{GMm}{2(2R)} \text{ and } E_2 = -\frac{GMm}{2(3R)}$$

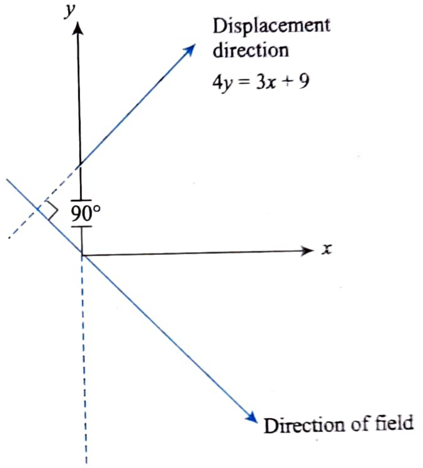
Therefore, the energy required,

$$\Delta E = E_2 - E_1 = GMm\left(\frac{1}{4R} - \frac{1}{6R}\right) = \frac{GMm}{12R}$$

$$= \frac{(R^2g)m}{12R} = \frac{mgR}{12} = 1.60\times10^{10}\text{ J}$$

21. (0) Slope of the displacement vector, $m_1 = \dfrac{3}{4}$

Slope of the force vector, $m_2 = -\dfrac{4}{3}$



$$m_1 = m_2 = \frac{3}{4}\times-\frac{4}{3} = -1$$

i.e., force and displacement directions are perpendicular. The work done is zero.

22. (04.00) Let the speed of the ball which can cross 10 m wide river be v.

Then $R = \dfrac{v^2\sin(2\times45^\circ)}{g} = 10, v = \sqrt{10g}$

Let the radius of planet be 'R_p', then mass of the planet

$$M_p = \frac{4}{3}\pi R_p^3\times2\sigma = \frac{4}{3}\pi R_p^3\times\frac{2\times M_e}{4/3\pi R_e^3} = \frac{2M_eR_p^3}{R_e^3}$$

Escape velocity of the planet,

$$v_e = \sqrt{\frac{2GM_p}{R_p}} = \frac{2G\times2\times M_eR_p^3}{R_p\times R_e^3} = \sqrt{\frac{10GM_e}{R_e^2}}$$

$$2R_p = \sqrt{10R_e} \Rightarrow 2R_p = \sqrt{10\times6.4\times10^6}$$

$$\Rightarrow \quad R_p = \frac{8\times10^3}{2} = 4\times10^3 = 4\text{ km}$$

23. (0.50) Here, escape velocity on the surface of the earth,

$$v_e = \sqrt{\frac{2GM}{R}}$$

Let v_e' be the escape velocity from the platform at a height h (= 3 R) above the surface of the earth. As the total energy of the body at infinity is zero,

$\frac{1}{2}mv_e'^2$+ P.E. of the body on the platform = 0

or $\frac{1}{2}mv_e'^2 + \left(-\frac{GMm}{R+h}\right) = 0$

or $\frac{1}{2}mv_e'^2 = \frac{GMm}{R+h} = \frac{GMm}{R+3R} = \frac{GMm}{4R}$

or $v_e' = \sqrt{\frac{GM}{2R}}$

But $v_e' = f\,v_e$

$$\sqrt{\frac{GM}{2R}} = f \times \sqrt{\frac{2GM}{R}}$$

or $f = 1/2 = 0.50$

24. (8000) (i) Let M and R be the mass and radius of the earth respectively. If m be the mass of satellite, then escape velocity from earth,

$$v_e = \sqrt{2gR_e}$$

Velocity of satellite, $v_s = \sqrt{\frac{gR_e}{2}}$ ...(i)

Further we know orbital speed of satellite at a height h is

$$v_s = \sqrt{\left(\frac{GM_e}{r}\right)} = \sqrt{\left(\frac{R_e^2 g}{R_e + h}\right)}$$

or $v_s^2 = \frac{R_e^2 g}{R_e + h}$ ...(ii)

From Eqs. (i) and (ii), we get

$h = R_e = 6400$ km

(ii) Now total energy at height h = total energy at earth's surface (principle of conservation of energy)

or $0 - GM_e\frac{m}{R_e + h} = \frac{1}{2}mv^2 - GM_e\frac{m}{R_e}$

or $\frac{1}{2}mv^2 = \frac{GM_e m}{R_e} - \frac{GM_e m}{2R_e}$ [As $h = R_e$]

Solving we get $v = \sqrt{gR_e}$

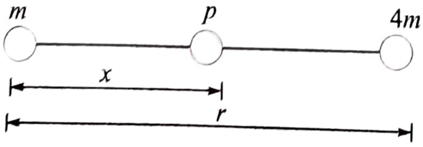
or $v = \sqrt{10 \times 6400 \times 10^3} = 8000$ m/s

Archives

JEE Main

Single Correct Answer Type

1. (4) Let x be the distance of the point P from the mass m where gravitational field is zero.



$$\therefore \quad \frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2} \quad \text{or} \quad \left(\frac{x}{(r-x)}\right)^2 = \frac{1}{4} \quad \text{or} \quad x = \frac{r}{3} \qquad \text{...(i)}$$

Gravitational potential at a point P is

$$V = -\frac{Gm}{x} - \frac{G(4m)}{(r-x)}$$

$$V = -\frac{Gm}{\left(\frac{r}{3}\right)} - \frac{G(4m)}{\left(r - \frac{r}{3}\right)} = -9\frac{Gm}{r} \qquad \text{(Using (i))}$$

2. (4) $W = 0 - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R} = gR^2 \times \frac{m}{R} = mgR$

$= 1000 \times 10 \times 6400 \times 10^3$

$= 64 \times 10^9\text{J} = 6.4 \times 10^{10}\text{J}$

3. (4) Energy of the satellite on the surface of the planet is

$$E_i = \text{KE} + \text{PE} = 0 + \left(-\frac{GMn}{R}\right) = -\frac{GMm}{R}$$

$$= \frac{1}{2}\frac{GMm}{3R} - \frac{GMm}{3R} = -\frac{GMm}{6R}$$

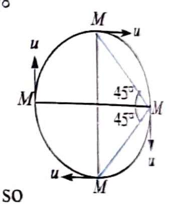
∴ Minimum energy required to launch the satellite is

$$\Delta E = E_f - E_i = -\frac{GMm}{6R} - \left(-\frac{GMm}{R}\right)$$

$$= -\frac{GMm}{6R} + \frac{GMm}{R} = \frac{5GMm}{6R}$$

4. (2) Net force on any one particle

$$= \frac{GM^2}{(2R)^2} + \frac{GM^2}{(R\sqrt{2})^2}\cos 45° + \frac{GM^2}{(R\sqrt{2})^2}\cos 45°$$



$$= \frac{GM^2}{R^2}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right]$$

This force will be equal to centripetal force so

$$\frac{Mu^2}{R} = \frac{GM^2}{R^2}\left[\frac{1 + 2\sqrt{2}}{4}\right]$$

$$u = \sqrt{\frac{GM}{R}\left[1 + 2\sqrt{2}\right]} = \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2} + 1)}$$

5. (2) $V = V_1 - V_2$

$$V_1 = -\frac{GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$V_2 = -\frac{3G\left(\frac{M}{8}\right)}{2\left(\frac{R}{2}\right)} \quad \Rightarrow \quad V = \frac{-GM}{R}$$

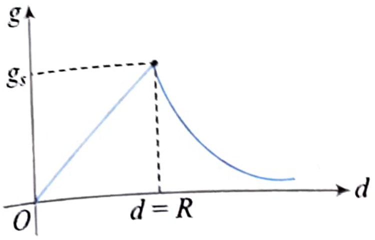
6. (4) Orbital velocity of a satellite revolving in a circular orbit at a height ($h << R$), $v_0 = \sqrt{Rg}$ and the velocity required to escape is $v_e = \sqrt{2Rg}$

So increase in velocity $\Delta v = v_e - v_0 = \left(\sqrt{2} - 1\right)\sqrt{Rg}$

7. (2) $g = \frac{GMx}{R^3}$ inside the Earth (straight line)

$g = \frac{GM}{r^2}$ outside the Earth

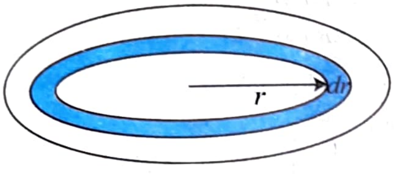
where M is mass of Earth.



dvanced

Correct Answer Type

(1) $V = -\int_{3R}^{4R} \frac{\sigma 2\pi r dr G}{\sqrt{r^2+16R^2}}$



(2) $V_c = \sqrt{2}v_0$

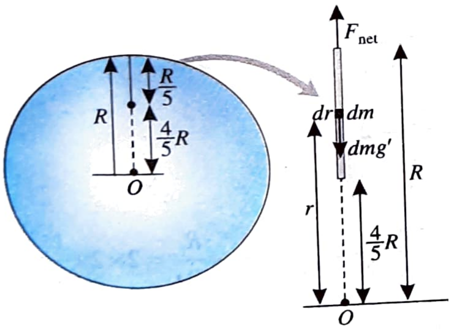
$$\text{KE} = \frac{1}{2}mv_e^2 = \frac{1}{2}m(\sqrt{2}v_0)^2 = mv_0^2$$

(2) The force applied at the top of the wire by a person holding it in place is $F_{\text{net}} = \int dm\, g'$

Inside planet $g' = g_0\left(\frac{r}{R}\right)$, g_0 = acceleration due to the gravity at the surface of planet.

$$= \int_{\frac{4R}{5}}^{R} (\lambda dr)\left(\frac{g_0 r}{R}\right)$$

$$= \frac{\lambda g_0}{R}\int_{\frac{4R}{5}}^{R} r dr = \frac{\lambda g_0}{R}\left(\frac{r^2}{2}\right)_{\frac{4R}{5}}^{R}$$



$$= \frac{\lambda g_0}{2R}\left(R^2 - \frac{16R^2}{25}\right) = \frac{\lambda g_0}{2R}\left(\frac{9R^2}{25}\right)$$

$$= \frac{9\lambda g_0 R}{50}$$

Acceleration due to the gravity at the surface of earth and the planet will be $g_e = \frac{GM_e}{Re^2}$ and $g_0 = \frac{GM}{R^2}$ respectively.

$$\frac{g_0}{g_e} = \left(\frac{M}{M_e}\right)\left(\frac{R_e}{R}\right)^2$$

$$\Rightarrow g_0 = g_e\left(\frac{M}{M_e}\right)\left(\frac{R_e}{R}\right)^2$$

$$= 10\left(\frac{R}{R_e}\right)^3\left(\frac{R_e}{R}\right)^2 = 10\left(\frac{R}{R_e}\right) = 1 \text{ m/s}^2$$

$$\Rightarrow F_{\text{net}} = \frac{9\times10^{-3}\times(1)6\times10^5}{50} = \frac{5400}{50} = 108 \text{ N}$$

4. **(3)** Given $v_e = 11.2 \text{ km/sec} = \sqrt{\frac{2GM_e}{R_e}}$

From energy conservation

$$\frac{1}{2}mv_s^2 - \frac{GM_s m}{r} - \frac{GM_e m}{R_e} = 0 - 0$$

where r = distance of rocket from the Sun

$$\Rightarrow v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2GM_s}{r}}$$

Given: $M_s = 3 \times 10^5 M_e$ and $r = 2.5 \times 10^4 R_e$

$$\Rightarrow v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2G3\times10^5 M_e}{2.5\times10^4 R_e}}$$

$$= \sqrt{\frac{2GM_e}{R_e}\left(1+\frac{3\times10^5}{2.5\times10^4}\right)} = \sqrt{\frac{2GM_e}{R_e}\times13}$$

$$\Rightarrow v_s = 42 \text{ km/s}$$

Multiple Correct Answers Type

1. **(2), (4)**

$$V_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\rho\cdot\frac{4}{3}\pi R^3}{R}} = \sqrt{\frac{8\pi G\rho}{3}}R$$

Surface area of $V_{es} \propto R$

$Q = 4A = 4\pi R_Q^2$

Surface area of $P = A = 4\pi R_P^2$

Mass R is $M_R = M_P + M_Q$

$\Rightarrow R_Q = 2R_P$

$$\rho\frac{4}{3}\pi R_R^3 = \rho\frac{4}{3}\pi R_P^3 + \rho\frac{4}{3}\pi R_Q^3$$

$$\Rightarrow R_R^3 = R_P^3 + R_Q^3 = 9R_P^3$$

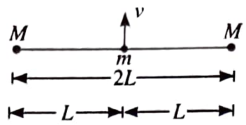
$$R_R = 9^{1/3}R_P \Rightarrow R_R > R_Q > R_P$$

Therefore $V_R > V_Q > V_P$

$$\frac{V_R}{V_P} = 9^{1/3} \text{ and } \frac{V_P}{V_Q} = \frac{1}{2}$$

2. (2), (4)

The situation is as shown in the figure



Applying the conservation of mechanical energy, we get

$$-\frac{GMm}{L}-\frac{GMm}{L}+\frac{1}{2}mv^2=0+0$$

$$\frac{1}{2}mv^2=\frac{2GMm}{L}\Rightarrow v=\sqrt{\frac{4GM}{L}}=2\sqrt{\frac{GM}{L}}$$

The energy of mass m will remain conserved hence option (d) is also correct.

3. (2), (3)

Gravitational field at a distance r from the centre of a uniform sphere is

$$E_r=\frac{GM}{R^3}r=\frac{G\left(\frac{4}{3}\pi R^3\rho\right)}{R^3}r$$

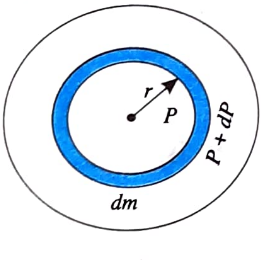
$$\Rightarrow E_r=\frac{4\pi G\rho}{3}r$$

Balancing the forces on a thin spherical shell at a distance r.

$$4\pi r^2\,dP=-\,g(r)dm$$

$$4\pi r^2dP=\frac{-4\pi G\rho}{3}r\,(4\pi r^2dr\rho)$$

$$\Rightarrow dP=\frac{-4\pi G\rho^2rdr}{3}$$



Integrating $\int dP=-\frac{4\pi G\rho^2}{3}\int r\,dr$

Hence, pressure at a radial distance r,

$$P(r)=\frac{-4\pi G\rho^2}{3}\cdot\frac{r^2}{2}+C$$

At surface $P=O$

So, $\frac{2\pi G\rho^2R^2}{3}=C$

$$\therefore\ P(r)=\frac{2\pi G\rho^2}{3}(R^2-r^2)$$

$$P\left(\frac{3R}{4}\right)=\frac{2\pi G\rho^2}{3}\left(1-\frac{9}{16}\right)R^2=\frac{2\pi G\rho^2}{3}\cdot\left(\frac{7}{16}\right)R^2$$

$$P\left(\frac{2R}{4}\right)=\frac{2\pi G\rho^2}{3}\left(1-\frac{4}{9}\right)R^2$$

$$=\frac{2\pi G\rho^2}{3}\cdot\left(\frac{5}{9}\right)R^2$$

Ratio $\frac{P(r=3R/4)}{P(r=2R/3)}=\frac{63}{80}$

Hence, (2).

$$P\left(\frac{3R}{5}\right)=\frac{2\pi G\rho^2}{3}\left(1-\frac{9}{25}\right)=\frac{16}{25}\text{ constant}$$

$$P\left(\frac{2R}{5}\right)=\frac{2\pi G\rho^2}{3}\left(1-\frac{4}{25}\right)=\frac{21}{25}\text{ constant}$$

Ratio $\frac{P(r=3R/5)}{P(r=2R/3)}=\frac{16}{21}$

Hence, (3).

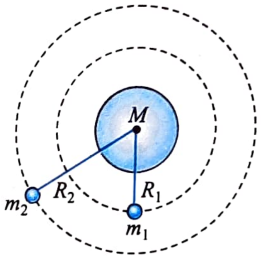
$$\frac{P\left(\frac{R}{2}\right)}{P\left(\frac{R}{3}\right)}=\frac{1-\frac{1}{4}}{1-\frac{1}{9}}=\frac{3}{4}\times\frac{9}{8}=\left(\frac{27}{32}\right)$$

Hence, (4) is not correct.

Matrix Match Type

1. (2) We are given $\frac{m_1}{m_2}=2$ and $\frac{R_1}{R_2}=\frac{1}{4}$

The orbital velocity is given by $v_0=\sqrt{\frac{GM}{R}}$



or $v_0\propto\frac{1}{\sqrt{R}}$

(P) $\frac{v_1}{v_2}=\frac{R_2}{R_1}=2$

(Q) $L=mvR$

$$\frac{L_1}{L_2}=\frac{m_1v_1R_1}{m_2v_2R_2}=\frac{m_1}{m_2}\times\frac{v_1}{v_2}\times\frac{R_1}{R_2}=2\times2\times\frac{1}{4}=1$$

(R) $K=\frac{1}{2}mv^2$

$$\frac{K_1}{K_2}=\frac{m_1v_1^2}{m_2v_2^2}=\frac{m_1}{m_2}\times\frac{v_1^2}{v_2^2}=2\times(2)^2=8$$

(S) $T=\frac{2\pi R}{v_0}$

$$\frac{T_1}{T_2}=\frac{R_1}{v_1}\times\frac{v_2}{R_2}=\frac{R_1}{R_2}\times\frac{v_2}{v_1}=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$$

merical Value Type

1. (3) $\frac{g'}{g} = \frac{\sqrt{6}}{11}; \frac{\rho'}{\rho} = \frac{2}{3} \Rightarrow \frac{R'}{R} = \frac{3\sqrt{6}}{22}$

$$\frac{v'_{esc}}{v_{esc}} \propto \sqrt{\frac{R'^2\rho'}{R^2\rho}} = \frac{3}{11} \Rightarrow v'_{esc} = 3 \text{ km/s}$$

2. (2) For a particle at a distance r from the center of Earth, force is given by, $F = \frac{Gm_1m_2}{r^2}$

Force becomes one-fourth, when $r = 2R$
(R = radius of Earth)

Escape velocity, $V_e = \sqrt{\frac{2GM}{R}}$

Using conservation of energy for the given particle,

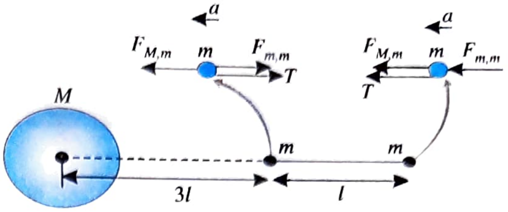
$$\frac{1}{2}mV^2 - \frac{GmM}{R} = -\frac{GmM}{2R}$$

This gives, $V = \sqrt{\frac{GM}{R}}$

$\Rightarrow V_e = V\sqrt{2}$

Hence, $n = 2$.

3. (7)



From FBD of middle mass: $\frac{GMm}{9l^2} - \frac{Gm^2}{l^2} - T = ma$

Since $T = 0$, $\frac{GMm}{9l^2} - \frac{Gm^2}{l^2} = ma$ ...(i)

Also, for the right most mass:

$$\frac{GMm}{16l^2} + \frac{Gm^2}{l^2} = ma \quad \text{...(ii)}$$

Equating (i) and (ii), $\frac{GMm}{9l^2} - \frac{GMm}{16l^2} = \frac{2Gm^2}{l^2}$

Solving: $m = \frac{7M}{288}$

Chapter 6

Concept Application Exercises

Exercise 6.1

1. (e) We have the relation, the pressure P_2 at the bottom of the container depends on the height h of the fluid above it. Since this height is the same for all three containers, the pressure at the bottom is the same for each container.

2. (b) The pressure at the top of each liquid is the same, since the U-tube is open at both ends. Also, the pressure at the location of the dashed line is the same in both the left and right sides of the U tube, since these two locations are at the same level. Thus, the pressure increment $\rho_1 gh_1$ for liquid 1 must be equal to the pressure increment $\rho_2 gh_2$ for liquid 2, where h_1 and h_2 are the heights of the liquids above the dashed line. Since h_1 is greater than h_2, ρ_1 must be less than ρ_2.

3. Here, $h = 3$ m $\therefore$ $P = h\rho g = 3\times10^3\times10$

   ($\because$ density of water, $\rho = 10^3\,\text{kg}\,\text{m}^{-3}$)

   Area of hole, $A = 5\text{ cm}^2 = 5\times10^{-4}\text{ m}^2$

   Therefore, force required to hold a patch covering the hole,

   $$F = P\times A = 3\times10^3\times10\times5\times10^{-4} = 1.5\times10\text{ N} = 1.5\text{ kg-f}$$

4. Let l be length of the mercury column in the barometer tube. As the barometer tube is inclined at an angle of 60° to the vertical, effective height (vertical) of the mercury column = l cos 60°. Since atmospheric pressure is 75 cm of mercury column, l cos 60° = 75

   or $$l = \frac{75}{\cos 60^\circ} = \frac{75}{1/2} = 150\text{ cm}$$

5. Here, $P = 270\text{ kPa} = 270\times10^3\,\text{Pa}$

   Let h be the height to which water rises in the pipes. Then, $F = h\rho g$

   or $$h = \frac{P}{\rho g} = \frac{270\times10^3}{10^3\times10} = 27\text{ m}$$

6. Given normal atmospheric pressure, $P = 1.013\times10^5\text{ N m}^{-2}$.

   Let h be the height of the alcohol column. Then, $P = \rho_{\text{alcohol}} gh_{\text{alcohol}}$

   or $$h = \frac{P}{\rho g} = \frac{1.013\times10^5}{984\times9.8} = 10.5\text{ m}$$

7. Let A be the area of contact of each tyre with the ground, when the person rides the bicycle.

   Then, pressure exerted by the person on the each tyre,

   $$P = \frac{\text{weight of the person and bicycle}}{\text{area of contact of the two tyres}} = \frac{Mg}{2\times A}$$

   $$= \frac{90\times10}{2\times A} = \frac{450}{A}$$

   Since the gauge pressure in both the tyres of the bicycle is 6.0×10^5 Pa, we have $\frac{450}{A} = 6.0\times10^5$

   or $$A = \frac{450}{6.0\times10^5} = 7.5\times10^{-4}\text{ m}^2$$

8. (a) Here, diameter of input piston, $d = 10$ cm $= 0.1$ m diameter of output piston, $D = 30$ cm $= 0.3$ m

   The weight lifted by the output piston, $F = 1.44\times10^4$ N.

   Let a and A be the areas of cross-section of the input and output pistons respectively. If f is the force applied on the input piston, then

   or $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ or $\frac{F_1}{\pi D^2/4} = \frac{F_2}{\pi d^2/4}$

   $$F_1 = F_2\left(\frac{A_1}{A_2}\right) = F_2\left(\frac{\pi r_1^2}{\pi r_2^2}\right) = 144\times10^4\left(\frac{0.1}{0.3}\right)^2 = 1.6\times10^3\text{ N}$$

   (b) Pressure on the input piston,

   $$P = \frac{F_1}{A_1} = \frac{F_1}{\pi d^2/4} = \frac{4F_1}{\pi d^2} = \frac{4\times1.6\times10^3}{\pi\times0.1^2} = 2.04\times10^5\text{ Nm}^{-2}$$

Exercise 6.2

1. Pressure at any depth h from the free surface of the water is given by $P = \rho gh + P_a = (1000)\times10\times10 + 1.0 + 10^5 = 2.00\times10^5\text{ N/m}^2$

2. The barometric height, $h = 75$ cm

   If l is the length of the mercury column in the tube, then $h/l = \sin 60^\circ$

   or $$l = \frac{h}{\sin 60^\circ} = \frac{75}{(\sqrt{3}/2)} = 86.6\text{ cm}$$

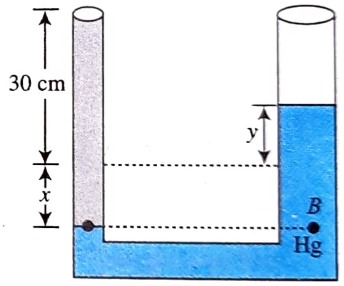
3. As the mercury column in the two arms of the U-tube is the same, so

   $$P_A = P_B \Rightarrow \rho_w gh_w = \rho_s gh_s$$

   $$\frac{\rho_s}{\rho_w} = \frac{h_w}{h_s} = \frac{10}{12.5} = 0.8$$

   Thus, specific gravity of the spirit is 0.8.

4. Suppose area of cross section of narrow limb be a, then area of cross section of right limb will be $3(a)$



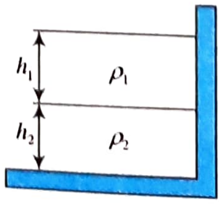
   Let the level of mercury in left limb fall be x and the rise of level in right limb be y, then

   $$ax = (3a)y$$
   $$x = 3y$$

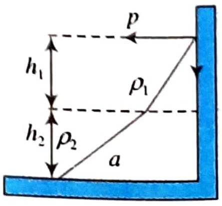
   According to Pascal's law, $P_A = P_B$

   $$(30 + x)\rho_w g = (x + y)\rho_{Hg} g$$
   $$(30 + 3y)\times1\times g = (3y + y)\times13.6\times g$$
   $$y = 0.58\text{ cm}$$

5. The inclination of the pressure diagram with the vertical wall is proportional with the density of the liquid. Since $\rho_2 > \rho_1$, therefore inclination of the lower diagram is always more than that of the upper one.



(a)



(b)

Figure (a) is correct.

Figures (b) and (c) are not possible.

6. As plug secures the pipe opening the force of friction between plug and pipe wall,

$$F = A\,(p_2 - p_1)$$

But $p_1 = p_0$ and $p_2 = p_0 + h\rho g$

$$F = Ah\rho g$$

$$F = \pi(2 \times 10^{-2})^2 \times 6 \times 10^3 \times 9.8 \Rightarrow F = 74\text{ N}$$

7. (a) The pressure intensity at the bottom of the container is

$$P = h\rho g = 0.20 \times 1000 \times 10 = 2 \times 10^3\text{ N/m}^2$$

Force at the bottom of container

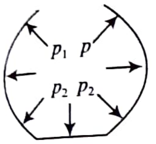
$= P \times$ base area $= 2 \times 10^3 \times (20 \times 10^{-4})$

$= 4$ N (Force exerted by water only)

(b) Weight of the water in the container

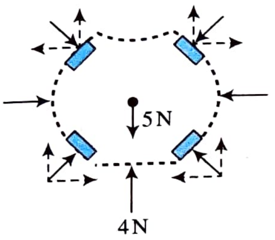
$=$ Volume × density of water × g $= 0.5 \times 10^3 \times 1000 \times 10 = 5$ N

$P_2 > P_1$



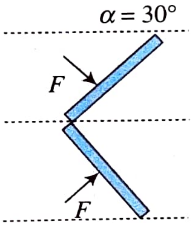
Force exerted by water on the container

(a)



Force exerted by container on water

(b)



(c)

From the diagram, the force exerted by the container is in upward direction because ($P_2 > P_1$). Let force exerted by sides of container be F, then

$$4 + F = 5\text{ N} \Rightarrow F = 1\text{ N}$$

8. Area of the plate = 1/2 × base × altitude

$= 1/2 \times 3 \times 3 = 4.5\text{ m}^2$

Density of oil $= 0.8 \times 1000 = 800\text{ kg m}^{-3}$

Depth of CG of the plate from the free surface

$= (1/3)\,3 = 1$ m

Now, total thrust on the plate

= (plate area) × (pressure at the CG of the plate)

$= (4.5)(800 \times 10 \times 1) = 36$ kN

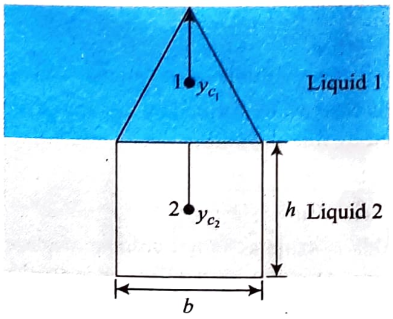
9. The force exerted by liquid on the gate is

$F_H = P_{av} \times$ vertical projection area of the gate

$$= \left(\frac{P_A + P_B}{2}\right) \times (2Rl)$$

$$= \left(\frac{R\rho g + 3R\rho g}{2}\right) \times 2Rl = 4\rho g R^2 l$$

10. (a) The hydrostatic force on the triangular area is $F_1 = \rho_1 g y_{c_1} A_1$



where $\rho_1 = \rho, y_{c_1} = 2h/3$ and $A_1 = bh/2$

This gives $F_1 = \dfrac{\rho g h^2 b}{3}$

(b) The hydrostatic force on the rectangular area is

$$F_2 = \left(\rho_1 g h_1 + \rho_2 g h y_{c_2}\right) A_2$$

Substituting $\rho_1 = \rho, h_1 = h, \rho_2 = 2\rho, y_{c_2} = \dfrac{h}{2}$ and $A_2 = bh$ we have $F_2 = 2\rho g h^2 b$

(c) The total hydrostatic force on the composite patch is

$$F = F_1 + F_2 = \frac{\rho}{3} g h^2 b + 2\rho g h^2 b = \frac{7}{3}\rho g h^2 b$$

11. Horizontal force on the cylinder due to two liquids must cancel out.

Force from left =force from right

$$(P_{\text{avg}})_{\text{left}}\, hL = (P_{\text{avg}})_{\text{right}}\, RL \qquad [L \text{ is length of the cylinder}]$$

$$\Rightarrow \frac{1}{2} 2\rho g h \times hL = \frac{1}{2} 3\rho g R \times RL \Rightarrow h = \sqrt{\frac{3}{2}}\, R$$

Exercise 6.3

1. At point A, $P_A = 0$

At point B, $P_B = P_A + \rho g h = \rho g h$

At point C, $P_C = P_B + \rho a L = \rho g h + \rho a L$

At point D, $P_D = P_C - \rho g h = \rho a L$

Or alternatively, $P_D = P_A + \rho a L = \rho a L$

2. Applying Pascal's law starting from point B, we get

$$P_B + \rho g H - \rho a L = P_A$$

Since $P_B = P_A = P_{\text{atm}}$, therefore $a = g\left(\dfrac{H}{L}\right)$

3. Let a be the acceleration of the elevator, then pressure inside the elevator is

$$P = \rho(g + a)h = \rho \times (g + a)\,0.76\text{ N/m}^2$$

Atmospheric pressure, $P_a = \rho g \times 0.76\text{ N/m}^2$

Clearly, the air pressure inside the elevator will be greater than P_a i.e., 76 cm of Hg.

4. Let a be the acceleration of the elevator and h be the barometer height, then

$$P_a = \rho(g + a)h \qquad \text{...(i)}$$

For the static elevator with barometric height h_0

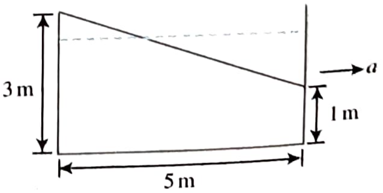
$$P_a = \rho g h_0 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$h = \frac{g h_0}{(g + a)}$$

Clearly, $h < h_0$ so barometric reading in an accelerating elevator will be less than 76 cm.

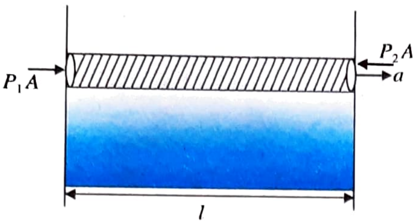
5. (a) The level of liquid rises by 1 m on the left side and falls by 1 m on the right side.



$$\frac{a}{g} = \frac{3 - 1}{5} = \frac{2}{5}$$

or $a = 0.4g = 4\text{ m/s}^2$

6. We consider a fluid body of cross-sectional area A (shown in the figure).



$$P_1A - P_2A = ma$$

or, $(P_1 - P_2)A = \rho Ala$

or, $P_1 - P_2 = \rho la = 0.92 \times 10 \times 3 = 27.6$ kPa

7. The acceleration of the trolley is $a = g\sin\theta$ down the inclined plane

$$a_x = a\cos\alpha = (g\sin\alpha)\cos\alpha$$

$$a_y = a\sin\alpha = (g\sin\alpha)\sin\alpha = g\sin^2\alpha$$

Let θ be the angle made by free surface of liquid, then

$$\tan\theta = \frac{a_x}{g - a_y} = \frac{g\sin\alpha\cos\alpha}{g - g\sin^2\alpha} = \tan\alpha$$

$$\theta = \alpha$$

8. (a) Since the original level of water before rotation is equal to the average of the minimum and maximum water level after rotation, therefore $h = \frac{+H}{2}$

(b) The equation of the parabolic surface is

$$y = \frac{\omega^2 r^2}{2g} \quad (\because y_{\min} = 0)$$

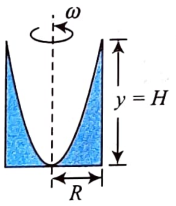
We know that for $r = R$, $y = H$, thus

$$H = \frac{\omega^2 R^2}{2g} \text{ or } \omega = \frac{\sqrt{2gH}}{R}$$

9. (a) We know that $y_{\max} = H = 3$ m,

$h = 2/3\ H = 2$ m,

$R = 1$ m, $g = 10$ m/s^2



$$\text{Thus, } \omega = \sqrt{\frac{4g(H-h)}{R^2}} = \sqrt{\frac{4(10)(3-2)}{1^2}}$$

$$= \sqrt{40} \text{ rad/s}$$

(b) We know that $y = \frac{\omega^2 r^2}{2g} - y_0$

Here, $y = H = 3$ m, $y_0 = 0$, $r = R = 1$ m, $g = 10$ m/s^2

$$\omega = \sqrt{\frac{2gH}{R^2}} = \sqrt{\frac{2(10)(3)}{1^2}} = \sqrt{60} \text{ rad/s}$$

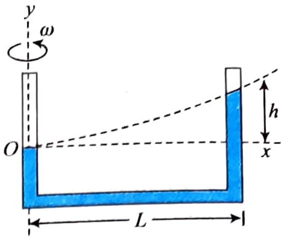
10. Let A and B be the two free liquid levels in the two limbs. If P_0 be the atmosphere pressure, then starting from point A and reaching point B through the tube, we have

$$P_0 + h_1\rho_1 g - l\rho_2 a - h_2\rho_2 g = P_0$$

$$\Rightarrow \quad h_1\rho_1 g = l\rho_2 a + h_2\rho_2 g = \rho_2(la + h_2 g)$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{la + h_2 g}{h_1 g}$$

11. Equation of parabola $y = \frac{\omega^2 x^2}{2g}$ ...(i)



At A, coordinate will be (L, h)

Substituting in Eq. (i) $h = \frac{\omega^2 L^2}{2g}$

Exercise 6.4

1. (a) If Mg of ice containing an air bubble is floating in water,

$$Mg = V_D\sigma_W g$$

i.e., $V_D = (M/\sigma_W)$

[as air bubble encloses air which is weightless]

Now when Mg ice melts, water formed will be Mg and it will occupy volume $V_F = (M/\sigma_W)$. As the volume of water formed by melting of ice is equal to the volume of water displaced by the floating ice, the level of water in the beaker will remain unchanged.

(b) Let the mass of ice be Mg and that of embedded piece m; then for floating

Weight = upthrust, i.e., $(M + m)g = V_D\sigma_W g$

So the volume of water displaced by the system when floating

$$V_D = \frac{M}{\sigma_W} + \frac{m}{\sigma_W} \quad \text{...(i)}$$

Now when ice melts, water formed will be Mg and so the volume of water formed by melting of ice

$$V_1 = (M/\sigma_W)$$

If the ice cube initially contains the lead piece, then as $\rho_{\text{Pb}} > \sigma_W$, it will sink when ice melts and so the water displaced by it will be equal to its own volume.

$$V_2 = (m/\rho_{\text{Pb}}) \qquad [\text{as } \rho = m/V]$$

So finally total volume occupied by water formed by melting of ice and lead piece

$$V_1 + V_2 = \frac{M}{\sigma_W} + \frac{m}{\rho_{\text{Pb}}} \quad \text{...(ii)}$$

Now as $\rho_{\text{Pb}} > \sigma_W$, this volume will be lesser than initial water displaced by the floating system Eq. (i); so the level of water in beaker will go down as the ice melts if the ice contains a sinking impurity.

(c) If the ice initially contains a cork piece, then as $\rho_{\text{cork}} < \sigma_W$, the cork will float on water when ice melts. So the water displaced by the floating cork piece will be

$$mg = V_2'\sigma_W g$$

$$\Rightarrow V_2' = m/\sigma_W$$

[and not its own volume (m/ρ_{cork})]

So in this case, water formed by melting of Mg of ice and water displaced by floating cork,

$$V_1 + V_2' = \frac{M}{\sigma_W} + \frac{m}{\sigma_W} \quad \text{...(iii)}$$

and as this is same as initial volume displaced by the floating system (Eq. (i)), the level of water in the beaker will remain unchanged.

For a floating body in two liquids,

Weight of the body = Buoyant force of liquid I + buoyant force of liquid II

$$V\rho g = V_1\rho_1 g + V_2\rho_2 g$$

$$V\rho = V_1\rho_1 + V_2\rho_2 \quad \text{...(i)}$$

Also, $V = V_1 + V_2$ ...(ii)

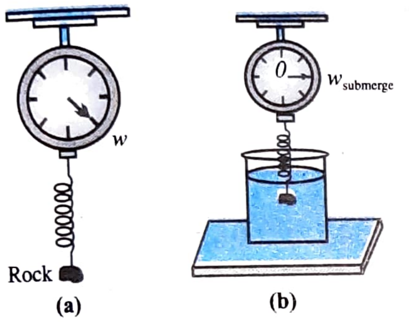
After solving Eqs. (i) and (ii), we get

$$V_1 = V\left(\frac{\rho-\rho_2}{\rho_1-\rho_2}\right) \text{ and } V_2 = V\left(\frac{\rho_1-\rho}{\rho_1-\rho_2}\right)$$

Let the volume of the rock be V, then magnitude of its weight.

$$w = \rho_{\text{rock}} Vg$$

In water, rock experiences an additional buoyant force with a magnitude equal to that of the weight of the water displaced by the system.



Buoyant force has magnitude, $F_{\text{buoy}} = \rho_{\text{water}} Vg$

The new scale reading is the total downward force on the system when it is in the water.

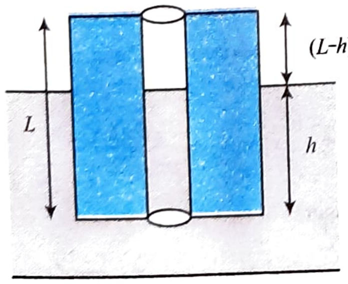
$$w_{\text{submerge}} = w - F_{\text{buoy}} = \rho_{\text{rock}} Vg - \rho_{\text{water}} Vg$$

$$\therefore \quad \frac{w_{\text{submerge}}}{w} = \frac{\rho_{\text{rock}} Vg - \rho_{\text{water}} Vg}{\rho_{\text{rock}} Vg} = \frac{\rho_{\text{rock}} - \rho_{\text{water}}}{\rho_{\text{rock}}}$$

Solving for the density of the system (i.e., the rock)

$$\rho_{\text{rock}} = \frac{w}{w - w_{\text{submerge}}} \rho_{\text{water}}$$

4. As ice is floating, $W = Th$, i.e., $V\rho g = V_{\text{in}}\sigma_w g$. Now if A is the cross section of the block, L is its thickness and out of L, h is inside water, $V = AL$ and $V_{\text{in}} = Ah$.



$$AL\rho_{\text{ice}} = hA\rho_{\text{water}}$$

$$h = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} L = 0.9 \times 5 = 4.5 \text{ m}$$

i.e., 4.5 m of ice will be submerged in water. So the level of water in the hole will be 5 – 4.5 = 0.5 m below the top of ice and hence the length of rope required to scoop up water is 0.5 m.

5. Volume of the cavities V_{cav} can be determined by taking difference between the volume V_{cast} of the casting as a whole and the volume of the iron in the casting.

$$V_{\text{cav}} = V_{\text{cast}} - V_{\text{iron}}$$

where, $V_{\text{iron}} = \dfrac{W}{\rho_{\text{iron}} g}$

W is the weight of casting;

$$W_{\text{eff}} = W - \rho_w g V_{\text{cast}}$$

$$\Rightarrow \quad V_{\text{cast}} = \frac{W - W_{\text{eff}}}{\rho_w g} \Rightarrow V_{\text{cav}} = \frac{W - W_{\text{eff}}}{\rho_w g} - \frac{W}{g\rho_{\text{iron}}}$$

$$= \frac{6000-4000}{(10^3)(10)} - \frac{6000}{(10)(8\times10^3)} = 0.125 \text{ m}^3$$

6. Let m be the mass of copper in neckless.

Mass of gold = $(50 - m)$

Volume of copper = $V_1 = m/10$

Volume of gold = $V_2 = \dfrac{50-m}{20}$

When immersed in water, $\rho_w = 1 \text{ g/cm}^3$

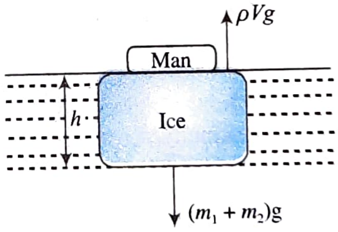
Decrease in weight = upthrust

$$(50 - 46)\, g = (V_1 + V_2)\, \rho_w\, g$$

$$4 = \frac{m}{10} + \frac{50-m}{20}$$

$$m = 30 \text{ g}$$

7. For equilibrium, $(m_1 + m_2)\, g = \rho Vg$



Here, m_1 = mass of man = 100 kg

m_2 = mass of ice = 0.917 × 1000 V = 917 V

$\rho = 1000 \text{ kg/m}^3$, $h = 0.5$ m

$\therefore \quad 100\, g + 917\, Vg = \rho Vg = 1000\, Vg$

$$V = \frac{100g}{1000g - 917g} \Rightarrow Ah = \frac{100}{83}$$

$$\therefore \quad A = \frac{100}{83 \times 0.5} = 2.41 \text{ m}^2$$

8. When a body is immersed in a liquid, it experiences an upthrust u, which makes spring balance S_1 to read

Apparent weight = true weight – upthrust = $W_A - u$ = 2.5 kg – wt

S_2 reads: Weight of B + weight of L + reaction (= upthrust of A)

$\therefore \quad W_B + W_L + u = 7.5$ kg weight

$W_B = 1$ kg weight, $W_L = 1.5$ kg weight

$1 + 1.5 + u = 7.5$ kg weight

$u = 5$ kg weight

$W_A = u + 2.5 = 7.5$ kg weight

S_1 will read 7.5 kg and S_2 will read 2.5 kg.

9. The gravity on moon is about one-sixth of that on the earth, i.e., $g_m = g/6$

On earth, water and air both exert buoyant force on the ball, but on moon as there is no air so weight of the ball is balanced only by buoyant force of the water. Thus

On earth, $mg = V_w \rho_w g + V_a \rho_a g$

or $\quad m = V_w \rho_w + V_a \rho_a \qquad$ ...(i)

On moon,

$$m\left(\frac{g}{6}\right) = V'_w \rho_w \left(\frac{g}{6}\right) \qquad \text{...(ii)}$$

or $\quad m = V'_w \rho_w$

From Eqs. (i) and (ii), we have $V'_w = V_w + \dfrac{V_a \rho_a}{\rho_w}$

Clearly $V'_w > V_w$. That is, the volume of the ball immersed in water on the moon will be greater than that on the earth. Hence ball will sink slightly more in water on the moon.

Exercise 6.5

1. Work done by per unit volume by force of gravity

$= -$ change in potential energy per unit volume

$\therefore \quad W = -\Delta U = -\rho g\,(h_2 - h_1) = -\rho g\,(6 - 3)$

$= -3 \times 500 \times 10 = -1.5 \times 10^4$ J/m³

2. Since the flow is horizontal, so the reduced form of Bernoulli's equation can be applied as

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Given $P_1 = 50 \times 13.6 \times 1000 = 6.8 \times 10^5$ dyn/cm²

and $\quad P_2 = 20 \times 13.6 \times 1000 = 2.72 \times 10^5$ dyn/cm²

and $\quad \rho = 1$ g/cc $\quad$ and $\quad v_1 = 5$ m/s $= 500$ cm/s

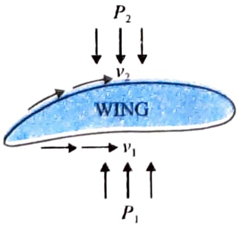
$\therefore \quad 6.8 \times 10^5 + 1/2 \times 1 \times (500)^2 = 2.72 \times 10^5 + 1/2 \times 1 \times v_2^2$

$\Rightarrow \quad \dfrac{1}{2}(v_2^2 - 250000) = 4.08 \times 10^5 \Rightarrow v_2 \approx 10.25 \text{ ms}^{-1}$

From the principle of continuity, $a_1 v_1 = a_2 v_2$

$$\frac{a_1}{a_2} = \frac{v_2}{v_1} = \frac{10.25}{5} = \frac{41}{20}$$

3. Applying Bernoulli's theorem to the lower and upper surface of a wing



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or, $\quad P_1 - P_2 = \dfrac{1}{2}\rho(v_2^2 - v_1^2)$

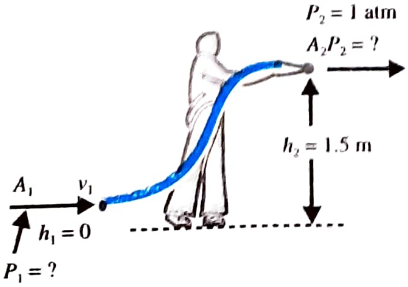
$[\because \text{lift} = P_1 - P_2]$

$\Rightarrow \quad 1000 = \dfrac{1}{2} \times 1.30(v_2^2 - 50^2)$

or $\quad v_2^2 - 2500 = 2000/1.3$

or, $\quad v_2 = 63.55$ m/s

4. (a) Figure illustrates the situation. From continuity equation, we can find the velocity of the fluid at the nozzle.



$$v_2 = \frac{A_1}{A_2} v_1 = \left(\frac{3.60}{0.250}\right)(50.0) = 720 \text{ cm/s} = 7.20 \text{ m/s}$$

(b) Thrust force experienced by gardener at hand due to flow of water

$$F_{\text{thrust}} = \rho A_2 v_2^2 = 1000 \times 0.25 \times 10^{-4} \times (7.2)^2 = 1.3 \text{ N}$$

(c) We will apply Bernoulli's equation to determine pressure P_1.

$h_1 = 0.00$ m, $\quad h_2 = 1.50$ m, $\quad P_2 = 1.01 \times 10^5$ Pa,

$v_1 = 0.50$ m/s, $\quad v_2 = 7.20$ m/s, $\rho = 1.00 \times 10^3$ kg/m³

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_1 = P_2 + \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$

$$= (1.01 \times 10^5) + \frac{1}{2}(1.00 \times 10^3)[(7.20)^2 - (0.50)^2]$$

$$+ (1.00 \times 10^3)(9.80)(1.50 - 0.00)$$

$$= 1.41 \times 10^5 \text{ Pa}$$

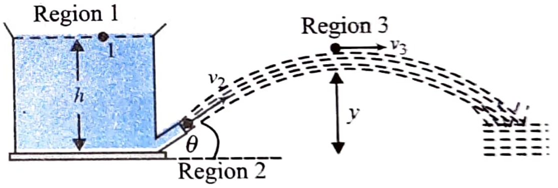
5. We apply Bernoulli's equation between points 2 and 3.

$$P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 = P_3 + \frac{1}{2}\rho v_3^2 + \rho g y_3$$

$$P_{\text{atm}} + \frac{1}{2}\rho v_2^2 + 0 = P_{\text{atm}} + \frac{1}{2}\rho v_3^2 + \rho g y \qquad \text{...(i)}$$

As both the points are open to atmosphere, $P_2 = P_3 = P_{\text{atm}}$.

Taking ground level as reference level, potential energy at point 2 is zero.



Since there is no horizontal acceleration, $v_3 = v_2 \cos\theta$.

From Eq. (i) we obtain

$$y = \frac{1}{2}\frac{(v_2^2 - v_2^2\cos^2\theta)}{g} = \frac{1}{2}\frac{v_2^2\sin^2\theta}{g} \qquad \text{...(ii)}$$

From Torricelli's theorem, $v_2^2 = 2gh$

$\Rightarrow \quad y = h\sin^2\theta$

$y = h$ when $\sin\theta = 1$, i.e., $\theta = 90°$, therefore y can never exceed h.

6. For horizontal pipe, gravitational head is same at two points, so using Bernoulli's theorem,

$$h_1 + \frac{P_1}{\rho g} + \frac{1}{2g}v_1^2 = h_2 + \frac{P_2}{\rho g} + \frac{1}{2}\frac{v_2^2}{g}$$

(P_1 and P_2: pressures at two points, v_1 and v_2: velocities of water at two points)

$$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2$$

$\Rightarrow \quad P_1 = 10$ cm of Hg $= h\rho g$

$= 10 \times 13.6 \times 980$ CGS units

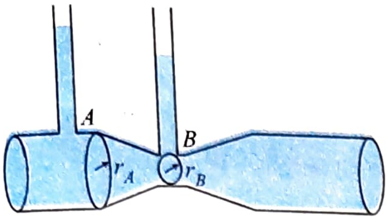
$v_1 = 40$ cm/s; $\quad v_2 = 50$ cm/s and $\rho = 1$ g/cc

$$\Rightarrow \quad \frac{10 \times 13.6 \times 980}{1} + \frac{1}{2}(40)^2 = \frac{P_2}{1} + \frac{1}{2}(50)^2$$

$$P_2 = 1.328 \times 10^5 \text{ dyn/cm}^2$$

$$P_2 = 9.97 \text{ cm of Hg}$$

Diameter of tube at $A = 60$ cm



Radius at $A = r_A = 30$ cm, radius at $B = r_B = 10$ cm,

area of cross section at $A = \pi(30)^2 \text{ cm}^2 = A_1$,

area of cross section at $B = \pi(10)^2 \text{ cm}^2 = A_2$

$P_A - P_B = 100$ cm of water $= 100 \times 1 \times 980$ dyn/cm²

Using the formula $Q = A_1 A_2 \sqrt{\dfrac{2(P_A - P_B)}{(A_1^2 - A_2^2)d}}$

substitute the values

$$Q = (900\pi)(100\pi)\sqrt{\frac{2 \times (100 \times 1 \times 980)}{(900^2\pi^2 - 100^2\pi^2)}}$$

$\Rightarrow \quad Q = 1.4 \times 10^5 \text{ cm}^3/\text{s} = 140$ l/s

8. In Pitot tube, velocity of liquids is given by

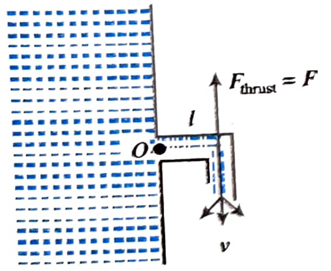
$$v = \sqrt{2 \times 1000 \times 20} = 200 \text{ m/s}$$

$$Q = Av = \pi r^2 v = \frac{22}{7} \times 7^2 \times 200 = 30800 \text{ cm}^3/\text{s} = 30.8 \text{ l/s}$$

9. For gases, velocity is given by

$$v = \sqrt{\frac{2h\rho g}{d}} = \sqrt{\frac{2 \times 0.1 \times 1000 \times 9.8}{1.3}} = 38.83 \text{ m/s}$$

10.



Velocity of flow $= v = \dfrac{Q}{\pi r^2}$

Reaction force $F = \rho A v^2$

$$\Rightarrow \quad F = \rho \pi r^2 \left(\frac{Q}{\pi r^2}\right)^2 = \frac{Q^2}{\pi r^2}\rho$$

Torque about $O = Fl = \dfrac{Q^2 \rho l}{\pi r^2}$

Exercises

Single Correct Answer Type

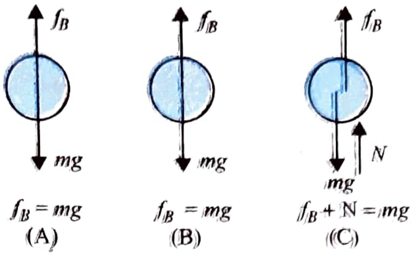
1. **(1)** W = weight of liquid

f_B = buoyant force on the ball

mg = weight of the ball

N = normal reaction between the ball and the surface

The free-body diagrams of the balls in each vessel are as follows.



At base, reaction force of buoyant force will act in downward direction.

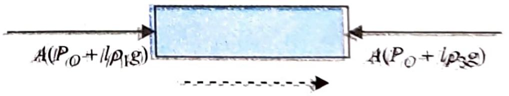
The forces acting at the base of each tank are

$$F_A = W + f_B = W + mg$$
$$F_B = W + f_B = W + mg$$
$$F_C = W + f_B + N = W + mg$$

Thus, $F_A = F_B = F_C$

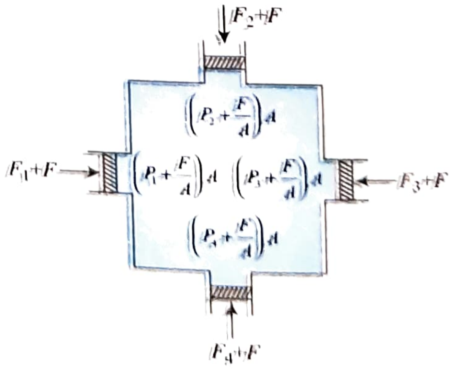
2. **(1)** The air through the horizontal tube will decrease the pressure and more liquid will be pushed into the capillary tube.

3. **(4)** Since both the vertical limbs of U-tube are open to atmosphere, at surface of liquids, the pressure is equal to the atmospheric pressure. Since heights of the liquids in the two vertical limbs are equal and the liquids have different densities, pressure exerted by them will be different from each other. The liquids can remain in static equilibrium relative to the tube only, when the system is accelerating down with acceleration g. In that case, liquids experience weightlessness. Hence, at every point of the tube, the pressure will become equal to atmospheric pressure. But if the tube is not accelerating down under gravity, then the heavier liquid will exert more pressure at the bottom. Hence, at the bottom of the left vertical limb, pressure will be greater than that at the bottom of the right vertical limb. It means on horizontal limb there will be a resultant horizontal force which will be towards the right as shown in the figure. Therefore, the liquid in the horizontal limb will have a rightward acceleration.



In fact, this whole system must have a horizontally rightward acceleration. Hence, only option (4) is correct.

4. **(1)** The four pistons are initially in equilibrium. As additional force F is applied to each piston, the pressure in the fluid at each point must be increased by F/A so that each piston retains state of equilibrium.

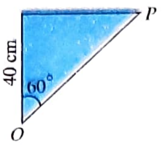

F_2+F
$\left(P_2 + \frac{F}{A}\right)A$
F_1+F
$\left(P_1 + \frac{F}{A}\right)A$
$\left(P_3 + \frac{F}{A}\right)A$
F_3+F
$\left(P_4 + \frac{F}{A}\right)A$
F_4+F

Thus, the increment in the pressure at each point is

$$\Delta P = \frac{F}{A} \quad \text{(by Pascal's law)}$$

5. (3) From Pascal's law, pressure is changing at every point by the same amount. Hence, buoyancy remains the same. So, the part of the block inside water remains the same.

6. (2) OP is the portion of the rod immersed in water.



$$\cos 60° = \frac{40}{OP}$$

or $$OP = \frac{40}{\cos 60°}\text{cm} = \frac{40}{1/2}\text{cm} = 80\,\text{cm}$$

The centre of buoyancy is at the centre of the immersed part of the rod. So, the required distance is 40 cm.

7. (2) $(V - \Delta V) \times 1.03 \times g = V \times 0.92 \times g$

or $$\frac{V - \Delta V}{V} = \frac{0.92}{1.03}$$

or $$\frac{\Delta V}{V} = 1 - \frac{0.92}{1.03} = \frac{0.11}{1.03} \quad \text{or} \quad \frac{\Delta V}{V} \times 100 = \frac{11}{1.03} \approx 11$$

8. (4) Let m be mass of the liquid. Let h_A, h_B and h_C be the heights of liquid level in three vessels. Then

$$m = \rho_A A \times h_A = \rho_B A \times h_B = \rho_C \times A \times h_C$$

$$\rho_A h_A = \rho_B h_B = \rho_C h_C = \frac{m}{A}$$

Pressure at the bottom in vessel A, $p_A = p_0 + \rho_A g h_A$

Similarly, at B and C, $p_B = p_0 + \rho_B g h_B$

$$p_C = p_0 + \rho_C g h_C$$

i.e., $p_A = p_B = p_C$

So, the same force will be exerted by the liquid in all the three vessels.

9. (3) $(2.4 \times 10^{-4}) \times 1.5 = 20 \times 2 \times 10^{-2} \times 10^{-4} \times v$

or $$v = \frac{2.4 \times 1.5 \times 10^2}{40}\text{m s}^{-1} = \frac{24 \times 15}{40}\text{m s}^{-1} = 9\,\text{m s}^{-1}$$

10. (1) Let ρ_S and ρ_L be the densities of silver and liquid, respectively, and m and V be the mass and volume, respectively, of the silver block. Therefore,

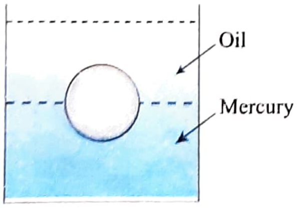
Tension in the string = mg – buoyant force

$$\Rightarrow T = \rho_S Vg - \rho_L Vg = (\rho_S - \rho_L) Vg$$

Also, $$V = \frac{m}{\rho_S}$$

$$\therefore \quad T = \left(\frac{\rho_S - \rho_L}{\rho_S}\right) mg = \frac{(10 - 0.72) \times 10^3}{10 \times 10^3} \times 4 \times 10 = 37.12\ \text{N}$$

11. (3)



$$\text{Weight} = \text{buoyant force} = \frac{V}{2}\rho_{\text{Hg}} g + \frac{V}{2}\rho_{\text{oil}} g$$

$$\therefore \quad \rho = \frac{\rho_{\text{Hg}} + \rho_{\text{oil}}}{2} = \frac{13.6 + 0.8}{2} = \frac{14.4}{2} = 7.2\ \text{g/cm}^3$$

12. (4) The points A and C are in same horizontal level hence $P_C - P_A = \rho a l$

(refer to figure in question to identify A, B, C)

Now, $P_B - P_C = \rho g h$

$$\Rightarrow P_B = (P_A + \rho a l) = \rho g h \Rightarrow P_B - P_A = h\rho g + l\rho a$$

13. (3) Apparent weight of the balloon = W_2

Apparent weight = real weight – upthrust

or W_2 = real weight – W

Now, real weight = $W_1 + W$

$\therefore \; W_2 = W_1 + W - W = W_1$

14. (3) When the vessel is stationary,

Weight = upthrust

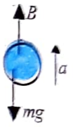
$$\Rightarrow V\rho_w g = V_i \rho_L g$$

ρ_w = density of wood

ρ_L = density of liquid

or $$\frac{V_i}{V} = \frac{\rho_w}{\rho_L} \qquad \text{...(i)}$$

When the vessel moves upwards.



Upthrust – weight = mass × acceleration

$$B - mg = ma$$

or $$V_i' \rho_L \left(g + \frac{g}{2}\right) - V\rho_w g = \frac{V\rho_w g}{2}$$

or $$\frac{V_i'}{V} = \frac{\rho_w}{\rho_L} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), $\dfrac{V_1'}{V} = \dfrac{V_i}{V}$

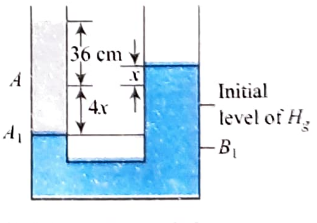
Hence, 50% will be immsersed.

15. (1) As the vessel is falling freely, the pressure at all the points in the liquids is same and equal to the atmospheric pressure and hence buoyancy becomes zero.

16. (3) Let the rise of level in the right limb be x cm, the fall of the level of mercury in the left limb be $4x$.

Therefore, the level of water in the left limb is $(36 + 4x)$ cm.

Equating pressure at the interface of mercury and water, we have (at A_1 and B_1)



$$(36 + 4x) \times 1\,g = 5x \times 13.6 \times g$$

Solving, we get $x = 0.56$ cm

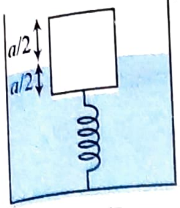
17. (3) $\rho_A = 0.75$, $\rho_P = 0.6$

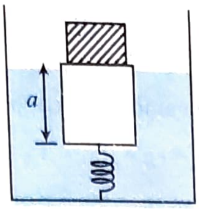
$\rho_B = 1.0$, $\rho_Q = 0.9$

As relative density of P is lesser than B, so it will float in liquid B and as relative density of Q is greater than liquid A so it will sink, because if density of the object is greater than that of the liquid in which it is immersed, then its weight is more than the upthrust and vice versa.

18. (4) Since density of block = 1/2(density of water), 50% of its volume is immersed in water.

In the second case, half of the volume of the block is further immersed in water.



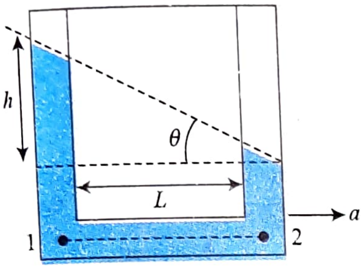


Therefore, W = Extra upthrust + Spring force

$$= a \times a \times \frac{a}{2} \times 2\rho \times g + K\left(\frac{a}{2}\right)$$

$$= a\left(a^2 \rho g + \frac{K}{2}\right)$$

(4) Let P_1 and P_2 be the pressures at the bottom of the left and right ends of the tube, respectively.



Then $F = (P_1 - P_2)A = \rho g h A$

where A is the cross section of the tube.

The mass of the liquid in the horizontal portion is

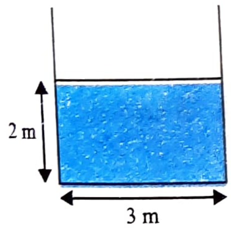
$m = \rho L A$

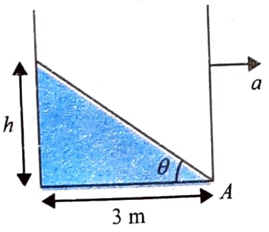
Now, $F = ma$

$\Rightarrow \rho g h A = \rho L A a$

$\therefore \; h = \dfrac{aL}{g}$

20. (2)





Volume equality gives

$$2 \times 3 = \frac{1}{2} \times h \times 3 \Rightarrow h = 4 \text{ m}$$

$$\therefore \tan\theta = \frac{4}{3} = \frac{a}{g} \Rightarrow a = \frac{4}{3}g$$

21. (3) Pressure at the bottom level of the block

$$P = \frac{mg}{a} + P_0 \qquad \text{...(i)}$$

The pressure at the same level remains the same.

Hence, $\dfrac{mg}{a} + P_0 = h\rho g + P_0 \Rightarrow h = \dfrac{m}{a\rho}$

22. (2) As block B falls, L will decrease.

Block B will displace volume of liquid (V_1)

Equal to its own volume when it is in the liquid.

$$V_1 = V_B = \frac{M_B}{\rho_B} \qquad \text{...(i)}$$

When block B is on block A, it will displace the volume of liquid (V_2) whose weight is equal to the weight of the block B.

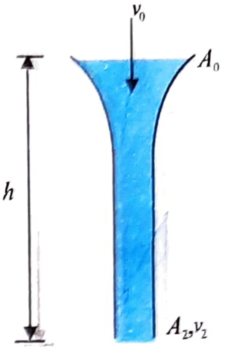
$V_2 \rho_L g = M_B g = (V_1 \rho_B) g$

or $V_2 \rho_L = V_1 \rho_B$

Since H increase, $V_1 > V_2$

So, $\rho_L > \rho_B$.

23. (2) $v_2^2 = v_0^2 + 2gh$ and $A_1 v_0 = A_2 v_2$ ...(i)



Solving, we get $\dfrac{A_2}{A_1} = \dfrac{v_0}{\sqrt{v_0^2 + 2gh}}$

$$\frac{A_2}{A_1} = \frac{1}{2} = \frac{v_0}{\sqrt{v_0^2 + 2gh}}$$

$$4v_0^2 = v_0^2 + 2gh$$

$$h = \frac{3v_0^2}{2g}$$

24. (1) Total pressure at the bottom = 3 atm

Pressure due to water in the tank = 3 atm − 1 atm = 2 atm = 20 m of water column

Height of water in the tank is $h = 20$ m

So, velocity of efflux = $\sqrt{2gh} = \sqrt{2 \times 10 \times 20} = \sqrt{400}$ m/s

25. (1) Let ρ be the density of the liquid. Then

$$F_1 = (\Delta P)A = \rho g h A \qquad \text{...(i)}$$

In the second case, F_2 = rate of change of momentum

$$= \rho A u^2 = \rho A(\sqrt{2gh})^2 = 2\rho g h A \qquad \text{...(ii)}$$

$$\therefore \frac{F_1}{F_2} = \frac{1}{2}$$

26. (3) Thrust force, $F = F_1 - F_2 = \rho a v_1^2 - \rho a v_2^2$

$$= \rho a(2gh_1) - \rho a(2gh_2)$$

$$= 2\rho a g(h_1 - h_2) = 2\rho a g h$$

27. (4) $v_1 = \sqrt{2g\left(\dfrac{h}{2}\right)} = \sqrt{gh}$ ...(i)

From Bernoulli's theorem, $\rho g h + 2\rho g\left(\dfrac{h}{2}\right) = \dfrac{1}{2}(2\rho)v_2^2$

$$\Rightarrow v_2 = \sqrt{2gh} \qquad \text{...(ii)}$$

$$\therefore \frac{v_1}{v_2} = \frac{1}{\sqrt{2}}$$

28. (1) From the equation of continuity,

$$\text{Rate} = R = \frac{A}{3}v_P + \left(A - \frac{A}{3}\right)v_Q$$

$$\therefore \; v_P + 2v_Q = \frac{3r}{A} = \frac{3 \times 0.1}{10^{-2}} = 30 \text{ m/s}$$

As $v_P = 20$ m/s, so $v_Q = 5$ m/s.

29. (2) Using equation of continuity, we have $v_2 = \dfrac{A_1}{A_2}v_1$

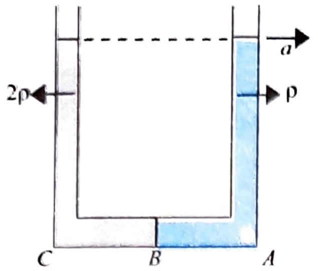
From Bernoulli's theorem,

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow g(h_1 - h_2) = \frac{1}{2}(v_2^2 - v_1^2) \qquad (\because p_1 = p_2)$$

$$\Rightarrow 60 = \left(\frac{A_1^2}{A_2^2} - 1\right)v_1^2 \Rightarrow \frac{A_1}{A_2} = \frac{4}{1}$$

30. (2)



For the given situation, liquid of density 2ρ should be behind that of ρ.

From the right limb, $P_A = P_{\text{atm}} + \rho gh$

$$P_B = P_A + \rho a\frac{l}{2} = P_{\text{atm}} + \rho gh + \rho a\frac{l}{2}$$

$$P_C = P_B + (2\rho)\, a\frac{l}{2} = P_{\text{atm}} + \rho gh + \frac{3}{2}\rho al \qquad \text{...(i)}$$

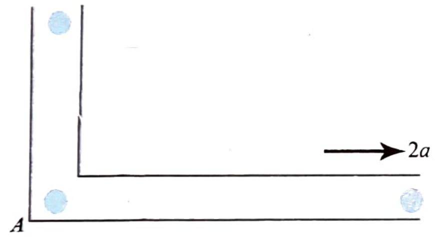
But from the left limb, $P_C = P_{\text{atm}} + (2\rho)\, gh$ ...(ii)

From Eqs. (i) and (ii),

$$P_{\text{atm}} + \rho gh + \frac{3}{2}\rho al = P_{\text{atm}} + 2\rho gh$$

$$\Rightarrow h = \frac{3al}{2g}$$

31. (1)



No sliding ⇒ pure rolling

Therefore, acceleration of the tube = $2a$ (since COM of the cylinders are moving at 'a')

$P_A = P_{\text{atm}} + \rho(2a)L$ (from the horizontal limb)

Also, $P_A = P_{\text{atm}} + \rho gH$ (from the vertical limb)

$$\therefore\ a = \frac{gH}{2L}$$

32. (4) Due to acceleration down the plane, both body B and liquid will receive the same component of acceleration down the plane.

33. (1) Given that $m_{\text{real}} = 36$ g, $m_{\text{app}} = 34$ g.

Density of gold $\rho_{\text{Au}} = 19.3$ g/cc

Density of copper $\rho_{\text{Cu}} = 8.9$ g/cc

We know that loss of weight = weight of displaced water $= 36 - 34 = 2$ g = Buoyant force = B

Here, $m_{\text{real}} = m_{\text{Au}} + m_{\text{Cu}} = 36g$ ...(i)

Let v be the volume of the ornament in centimetres. Then

$$B = v \times \rho_w \times g = 2 \times g$$

$$\Rightarrow \left(\frac{m_{\text{Au}}}{\rho_{\text{Au}}} + \frac{m_{\text{Cu}}}{\rho_{\text{Cu}}}\right)\rho_w \times g = 2 \times g$$

$$m_{\text{Au}}\rho_{\text{Cu}} + m_{\text{Cu}}\rho_{\text{Au}} = 2\rho_{\text{Au}}\rho_{\text{Cu}}$$

$$8.9m_{\text{Au}} + 19.3m_{\text{Cu}} = 2 \times 19.3 \times 8.9 = 343.54 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), $8.9m_{\text{Au}} + 19.3m_{\text{Cu}} = 343.54$

$$\Rightarrow 8.9(m_{\text{Au}} + m_{\text{Cu}}) + 10.4m_{\text{Cu}} = 343.54$$

$$\Rightarrow 8.9 \times 36 + 10.4m_{\text{Cu}} = 343.54$$

$$m_{\text{Cu}} = 2.225 \text{ g}$$

So the amount of copper in the ornament is 2.2 g.

34. (2) $v_A a_A = v_B \times a_B = \ v_A \times 4 = v_B \times 2 \quad v_B = 2v_A$ ...(i)

$$\text{Again, } \frac{1}{2}\rho v_A^2 + \rho g h_A + p_A = \frac{1}{2}\rho v_B^2 + \rho g h_B + p_B$$

$$\Rightarrow \frac{1}{2}\rho v_A^2 + p_A = \frac{1}{2}\rho v_B^2 + p_B \quad (\text{as } h_A = h_B)$$

$$\Rightarrow p_A - p_B = \frac{1}{2}\rho(v_B^2 - v_A^2) = \frac{1}{2} \times 1 \times (4v_A^2 - v_A^2)$$

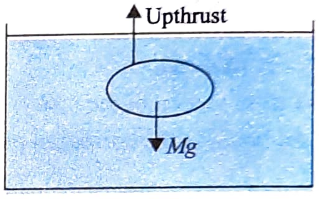
$$\Rightarrow 2 \times 1 \times 1000 = \frac{1}{2} \times 1 \times 3v_A^2$$

($p_A = p_B = 2$ cm of water column $= 2 \times 1 \times 1000$ dyn/cm^2)

$$\therefore\ v_A = \sqrt{\frac{4000}{3}} = 36.51 \text{ cm/s}$$

So the rate of flow $= v_A a_A = 36.51 \times 4 = 146$ cm^3/s

35. (1) As the water exerts upthrust on the stone, the stone also exerts the same force on water, according to Newton's third law.



Upthrust force acting on the stone is

$$U = V_{\text{stone}}\,\rho_\omega g = \left(\frac{m_{\text{stone}}}{\rho_{\text{stone}}}\right) = \frac{0.5}{10^4} \times 1000 \times 10 = 0.5 \text{ N}$$

Also, the weight is exerted on the beaker. Therefore the reading will be 1.5 + 0.5 = 2 kg.

36. (3) Let d_w and d_0 be the densities of water and oil, respectively. Then the pressure at the bottom of the tank is $h_w d_w g + h_0 d_0 g$.

Let this pressure be equivalent to pressure due to water of height h. Then,

$$hd_w g = h_w d_w g + h_0 d_0 g$$

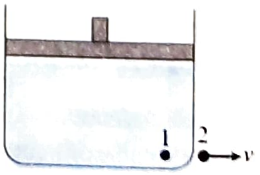
$$\therefore\ h = h_w + \frac{h_0 d_0}{d_w} = 100 + \frac{400 \times 0.9}{1}$$

$$= 100 + 360 = 460 = 4.6 \text{ m}$$

According to Toricelli's theorem,

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 4.6} \text{ m/s} = \sqrt{92} \text{ m/s}$$

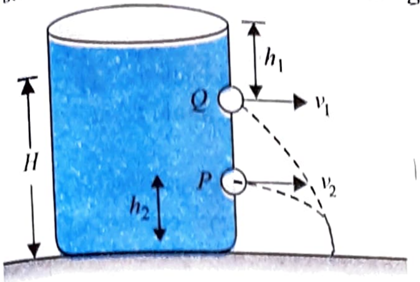
37. (2) Applying Bernoulli's theorem at points 1 and 2, difference in pressure energy between 1 and 2 = difference in kinetic energy between 1 and 2.



$$\text{Hence, } \rho hg + \frac{mg}{A} = \frac{1}{2}\rho v^2$$

$$\text{or } v = \sqrt{2gh + \frac{2mg}{\rho A}} = \sqrt{2\left(gh + \frac{mg}{\rho A}\right)}$$

(1) The two streams strike at the same point on the ground.



$R_1 = R_2 = R$

$v_1 t_1 = v_2 t_2$ ...(i)

where v_1 = velocity of efflux at $Q = \sqrt{2gh_1}$ and v_2 = velocity of efflux at $P = \sqrt{2g(H - h_2)}$

t_1 = time of fall of water stream through $Q = \sqrt{\dfrac{2(H - h_1)}{g}}$

t_2 = time of fall of the water stream through $P = \sqrt{\dfrac{2h_2}{g}}$

Putting these values is Eq. (i), we get

$(H - h_1)h_1 = (H - h_2)h_2$

or $[H - (h_1 + h_2)][h_1 - h_2] = 0$

$H = h_1 + h_2$ is irrelevant because the holes are at two different heights. Therefore, $h_1 = h_2$ or $h_1/h_2 = 1$

(2) Let v be the volume of the solid block of density ρ. Let ρ_1 be the density of water.

Weight of body = $v\rho g$.

When the body is immersed in water,

Tension in the string = Upward thrust – Weight of the body

$\Rightarrow T = v\rho_1 g - v\rho g = vg(\rho_1 - \rho_2)$

When the lift is moving upwards with acceleration a, the tension in the string is $T' = v(\rho_1 - \rho)(g + a)$

From Eqs. (i) and (ii), $T' = T(1 + a/g)$.

40. (1) Let ρ be the density of the material, ρ_0 be the density of water. When the sphere has just started sinking, the weight of the sphere = weight of water displaced (approx.)

$$\Rightarrow \frac{4}{3}\pi(R^3 - r^3)\rho g = \frac{4}{3}\pi R^3 \rho_0 g$$

$$\Rightarrow (R^3 - r^3)\,\rho = R^3\rho_0 \Rightarrow \frac{(R^3 - r^3)}{R^3} = \frac{\rho_0}{\rho}$$

$$\Rightarrow \frac{r}{R} = \frac{(7)^{1/3}}{2}$$

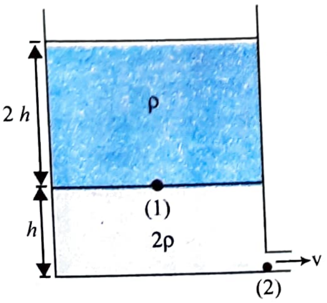
41. (1) Given $R = 5$ cm, $\omega = 2\pi$ (4) rad/s

Using $y = \dfrac{\omega^2 x^2}{2g}$,

the required level difference is y. When $x = R$,

$$y = \frac{(8\pi)^2(5 \times 10^{-2})}{2 \times 9.8} = 8 \text{ cm}$$

42. (2)



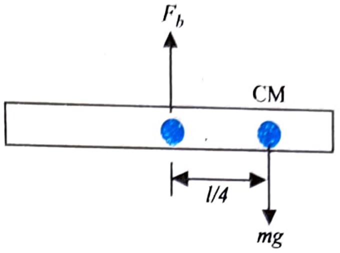
Pressure at (1), $P_1 = P_{atm} + \rho g(2h)$

Applying Bernoulli's theorem between points (1) and (2),

$$(P_{atm} + 2\rho gh) + \rho g(2h) = P_{atm} + \frac{1}{2}(2\rho)v^2$$

$$\Rightarrow v = 2\sqrt{gh}$$

43. (2) Torque about CM:



$$F_b \frac{l}{4} = I\alpha \Rightarrow \alpha = \frac{1}{I}(\pi r^2)(l)(\rho)(g)\frac{l}{4}$$

$$\Rightarrow \alpha = \frac{\pi r^2 l^2 g\rho}{4I}$$

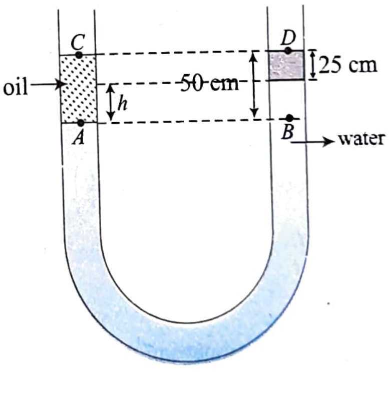
44. (3) $p_A + \rho aH - \rho gH = p_B$

Here, $p_A = \dfrac{p_0}{2}$ and $p_B = p_0$

$$\therefore -\frac{p_0}{2} = \rho gH - \rho aH$$

$$\Rightarrow \rho H(a - g) = \frac{p_0}{2} \Rightarrow a = \frac{p_0}{2\rho H} + g$$

45. (2) On pouring oil on the left side water rises by 25 cm from its previous level in the right limb of U-tube, creating a difference of levels of water by 50 cm. Let h cm be the height of oil above level A in the left limb of U-tube. Equating pressures at A and B, we get $P_A = P_B$.



$$\Rightarrow h \times \rho_{oil} \times g = 50 \times \rho_{water} \times g$$

$$\Rightarrow h = \frac{50 \times \rho_{water}}{\rho_{oil}} = \frac{50 \times 1}{0.8} = 62.5 \text{ cm}$$

Difference of levels of oil (C) and water (D) in the two limbs $= 62.5 - 50 = 12.5$ cm.

46. (2) Let V be the total volume of the ball and v be the volume of the ball in the upper liquid. Then $V - v$ is the volume of the lower liquid displaced.

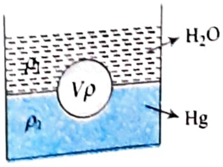
Using the law of floatation, we have

$$V\rho g = v\rho_1 g + (V - v)\rho_2 g$$

$$V\rho = v\rho_1 + V\rho_2 - v\rho_2$$

or $V(\rho - \rho_2) = v(\rho_1 - \rho_2)$

$$\frac{v}{V} = \frac{\rho - \rho_2}{\rho_1 - \rho_2} = \frac{\rho_2 - \rho}{\rho_2 - \rho_1}$$



47. **(3)** As $x = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2x}{r}}$

Velocity of efflux $v = \sqrt{2g(H-x)}$

Hence $R = vt = 2\sqrt{x(3H-x)}$

For range to be maximum $\frac{dR}{dx} = 0$

which gives $x = \frac{3}{2}H = 1.5\,H$

Alternatively:

Maximum range is also equal to average of two heights

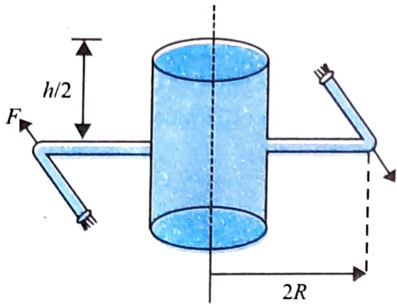
$$x = \frac{H+2H}{2} = \frac{3H}{2}$$

48. **(2)** $\frac{1}{3}\pi\left(\frac{H}{2}\right)^2 H\rho g = (P_0 + \rho g H)\pi\left(\frac{H}{2}\right)^2 - F_v$

$$F_v = P_0\pi\frac{H^2}{4} + \rho g\frac{H^3}{4}\pi - \frac{\pi H^3\rho g}{12}$$

$$= P_0\frac{\pi H^2}{4} + \frac{\rho g H^3\pi}{6} = \pi H^2\left(\frac{P_0}{4} + \frac{\rho g H}{6}\right)$$

49. **(1)**



Velocity of efflux of water $v = \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$

Force on ejected water = Rate of change of momentum of ejected water

$F = \rho\, av^2$

Torque of these forces about central line = $(\rho av^2) \times (4R)$

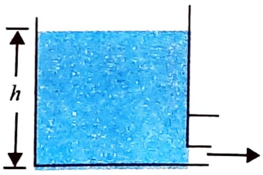
$= 4\rho av^2\, R = 4\rho agh\, R$

50. **(3)** Let d be the inner diameter of the hemispherical bowl. Then, $B = mg$

$$\frac{4}{3}\pi\times\left(\frac{1}{2}\right)^3\times\left(1.2\times10^3\right)g = \frac{4}{3}\pi\left[\left(\frac{1}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\times\left(2\times10^4\right)g$$

Solving, we get $d = 0.98$ m.

51. **(3)** The velocity with which the liquid comes out is $v = \sqrt{2gh}$.



Let cross-sectional area of the hole be A. Then the force exerted by the ejecting fluid due to change in momentum on the container is

$F = \rho Av^2 = ma$

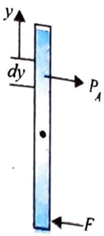
where m is the mass of the liquid inside the container.

$\rho Av^2 = \rho Ah \times a$

$\Rightarrow a = 2g$

Differentiating the above equation, we can have $v_0 = 2gt$ where t is the time in which the liquid comes out completely which is dependent on h.

52. **(3)** At a depth y from the surface of the fluid, the net force acting on the gate element of width dy is



$dF = (p_0 + \rho g y - p_0)\times 1dy = \rho g y dy$

Torque of this force about the hinge is

$$d\tau = \rho g y dy \times\left(\frac{1}{2} - y\right)$$

Net torque experienced by the gate is

$$\tau_{\text{net}} = \int d\tau + F\times\frac{1}{2}$$

$$= \int_0^1 \rho g y dy\left(\frac{1}{2} - y\right) + F\times\frac{1}{2} = 0$$

$$\Rightarrow F = \frac{\rho g}{6}$$

53. **(3)** When the levels equalize, the height of liquid in each arm is $(h_1 + h_2)/2$.

We may then visualize that a length $h_1 - (h_1 + h_2)/2 = (h_1 - h_2)/2$ of the liquid has been transferred from the left arm to the right arm.

Mass of liquid transferred = $(h_1 - h_2)A\rho/2$, where A is area of hole and ρ is density.

Vertical distance through which it moves $= (h_1 - h_2)/2$

Loss in gravitational potential energy $= \left[(h_1 - h_2)/2\right]^2 A\rho g$

Mass of the entire liquid = $(h_1 + h_2 + h)\,A\rho$

If this moves with velocity v,

kinetic energy $= \frac{1}{2}\;(h_1 + h_2 + h)A\rho v^2$

Equating energies, we get $v = (h_1 - h_2)\sqrt{\frac{g}{2(h_1 + h_2 + h)}}$

54. **(3)** Let M and m be the masses of the wooden piece and the man, V the volume of the cylinder and ρ the density of water. Then by law of floatation,

$$Mg = \frac{2V\rho}{3}g \Rightarrow M = \frac{2V\rho}{3} \quad \text{...(i)}$$

and $(M+m)g = \left(\frac{2}{3} + \frac{1}{6}\right)V\rho g = \frac{5V\rho}{6}g$

$$\Rightarrow (M+m) = \frac{5V\rho}{6} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get $\frac{m}{M} = \frac{1}{4}$

55. **(2)** Since two-third of the wooden block is immersed in water, its density $= 2\rho/3$, where ρ is the density of water.

Let the volume of the chip be V. Then its weight is $(2/3)V\rho g$. The upthrust acting on it inside the water is $V\rho g$. Hence, the unbalanced upward force is $V\rho g(1 - 2/3) = V\rho g/3$.

Hence, its acceleration a in the upward direction is

$$a = \frac{V\rho g/3}{(2/3)V\rho} = \frac{g}{2}$$

If t is the required time, then $1 = 0 + \frac{1}{2}at^2 = \frac{1}{2}\frac{g}{2}t^2$

Hence, $t^2 = \frac{4}{g} = \frac{4}{10} = 0.4 = 0.63$ s

(4) Weight of ornament in air = 0.25 kg

Weight of displaced water = $0.015 \times 10^{-3} \times \rho$ kg, where ρ is the density of water.

Hence, the relative density of the ornament is $0.25/(0.015 \times 10^{-3} \times \rho)$.

Let the percentage of copper in the ornament be n%. Then the relative density of ornament is

$$\frac{1}{100}[(100-n)19.3+8.9n] = \frac{1}{100}[1930-10.4n]$$

Hence,

$$\frac{1}{100}[1930-10.4n] = \frac{0.25}{0.015\times10^{-9}\times\rho} = \frac{0.25}{0.015} = \frac{50}{3}$$

$$\Rightarrow 3[1930-(10.4n)] = 5000$$

$$\therefore n = \frac{(5790-5000)}{3\times10.4} = \frac{790}{31.2} = 25\%$$

(1) When the oil is poured, the fraction of ice in the water decreases, i.e., volume of ice melted into water is greater than volume of water displaced by ice. So water level rises. Overall volume of ice will decrease as it melts. So the upper level of oil falls.

(3) Relative velocity of the water jet with respect to the moving plate = (18 + 3) = 21 m/s. The mass of water hitting the plate per second = $a\rho(21)$, where a is the cross-sectional area of the jet.

F = Force normal to the plate

= Rate of change of momentum along the direction of the water jet

= (Mass) × (Velocity) = $[a\rho\,(21)]\,21 = 441a\rho$

$$\therefore F = 441\times\rho\times\left(\frac{0.1}{18}\right) = \frac{441\times0.1\times1000}{18}\text{ N} = 2450\text{ N}$$

(3) Since the container alone floats to half its length in water, its relative density = 0.5. Since the levels of the liquid inside and water outside are the same, the volumes of the liquid and of the displaced water are the same. If this volume is V and the volume of the material of container is v, then the total mass of floating body is $M = [V\sigma + v(0.5)\rho]$, where ρ is the density of water. By law of floatation, $M = [V\sigma + v(0.5)\rho] = V\rho$ giving $V(1-\sigma) = v \times 0.5$.

Let the cross section of the container be A. Then the volume V' of water displaced V by the liquid is

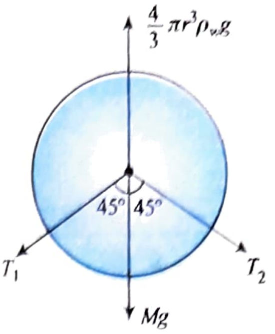
$$LA\left[\frac{2}{3}-\frac{1}{2}\right] = LA\left[\frac{1}{6}\right]$$

But $LA = \frac{3V}{2}$

Hence $V' = \frac{3V}{2}\times\frac{1}{6} = \frac{V}{4}$

Hence $V\sigma\rho = \left(\frac{V}{4}\right)\rho \Rightarrow \sigma = \frac{1}{4}$

(3) $T_1 = T_2$, $Mg + \sqrt{2}T_1 = \frac{4}{3}\pi r^3\rho_w g$, $T_1 = 250\sqrt{2}$ N



61. (1) According to Bernoulli's theorem,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$\Rightarrow \frac{P_1 - P_2}{\rho} = \frac{1}{2}(v_2^2 - v_1^2)$$

Change in kinetic energy per kilogram mass of oil is

$$\frac{P_1 - P_2}{\rho}$$

Substituting the numerical values, change in kinetic energy

$$= \frac{8}{800} = 10^{-2}\text{ J/kg}$$

62. (3) Velocity of the liquid just after coming out of section '3' is

$$v_3 = \sqrt{2gh} \quad \text{...(i)}$$

Using continuity equation at section '2' and section '3'

$$\frac{A}{2}v_2 = \frac{A}{4}v_3$$

$$v_2 = \frac{1}{2}v_3 = \sqrt{\frac{hg}{2}}$$

Now using Bernoulli's theorem at section 2 and at the opening end of pipe 3.

$$P_0 + \frac{1}{2}\rho v_3^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow P_2 = P_0 + \frac{1}{2}\rho(v_3^2 - v_2^2) = P_0 + \frac{1}{2}\rho\left(2gh - \frac{gh}{2}\right)$$

$$P_2 = P_0 + \frac{3\rho gh}{4}$$

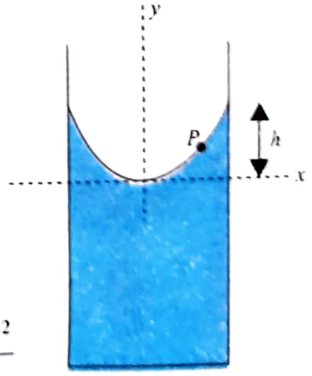
63. (1) When the cylindrical vessel rotates, the velocity at the sides is higher. Hence pressure at the sides is lower.

Equation of surface liquid is

$$y = \frac{\omega^2 x^2}{2g}$$

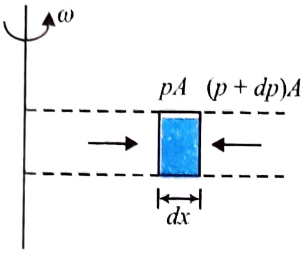
Here $\omega = 2\pi n$, $x = R$, $y = h$

$$\therefore h = \frac{(2\pi n)^2(R^2)}{2g} = \frac{2\pi^2 n^2 R^2}{g}$$



64. (1) Consider the condition of equilibrium for the mass of water contained between cross section separated by x and $x + dx$ from rotation axis.

$$dp = \rho\omega^2 x dx;\quad \int_{p_1}^{p_2} dp = \int_{r_1}^{r_2} \rho\omega^2 x dx$$



$$p_2 - p_1 = \rho\omega^2\left[\frac{x^2}{2}\right]_{r_1}^{r_2} = \rho\omega^2\left(\frac{r_2^2 - r_1^2}{2}\right)$$

$$\omega = \sqrt{\frac{2\,(p_2 - p_1)}{\rho\,(r_2^2 - r_1^2)}}$$

65. **(1)** Let a_1 and a_2 be cross sectional areas and v_1 and v_2 the velocities of liquid flow at A and B. If p_1 and p_2 are pressures of liquid recorded by manometer, then

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \Rightarrow p_1 - p_2 = \frac{1}{2}\rho v_1^2\left(\frac{v_2^2}{v_1^2} - 1\right)$$

By equation of continuity, $a_1 v_1 = a_2 v_2$

Also, $p_1 - p_2 = h\rho g$

Substituting these values, we have $v_1 = \sqrt{\dfrac{2gh}{\dfrac{a_1^2}{a_2^2} - 1}}$

Rate of flow of liquid is $a_1 v_1 = a_1 a_2\sqrt{\dfrac{2gh}{a_1^2 - a_2^2}}$

66. **(1)** Absolute pressure in bulb = Gauge pressure + Atmospheric pressure = $P + P_a$

Using Bernoulli's equation,

$$P_a + P = P_{BC} + \frac{\rho_a v^2}{2}$$

where ρ_a is the density of air.

$$\therefore\ P_{BC} = P_a + P - \left(\frac{1.3}{2}\right)v^2$$

$$P_{BC} = P_a - \rho gh$$

So equating these two values for P_{BC}, we get

$$V = \frac{P + \rho gh}{\sqrt{0.65}}$$

67. **(2)** Initially, $W - 2\,Kx = 0$ ...(i)

Finally, $W' - 2K\left(x + \dfrac{a}{2}\right) - a.a.\dfrac{a}{2}2\sigma.g = 0$ ...(ii)

$W' = W + W_0$

$W + W_0 - 2Kx - Ka - a_3\rho g = 0$ [From Eq. (ii)]

From Eq. (i), $W_0 = Ka + a^3\sigma g = a(K + a^2\sigma g)$.

68. **(4)** $(m_1 + m_2)g = \left[\dfrac{m_1}{\sigma_1} + \dfrac{m_2}{K\sigma_2}\right]\sigma g$

Divide by $m_2 g$, $\dfrac{m_1}{m_2} + \dfrac{1}{K} = \dfrac{m_1\sigma}{m_2\sigma_1} + \dfrac{\sigma}{K\sigma_2}$

Given: $\dfrac{m_1}{m_2} = \dfrac{1}{2}$, $\dfrac{\sigma}{\sigma_1} = 2$, $\dfrac{\sigma}{\sigma_2} = 0.5$

$$\frac{1}{2} + \frac{1}{K} = \frac{1}{2} \times 2 + \frac{0.5}{K}$$

$$\frac{1}{K} - \frac{0.5}{K} = 1 - \frac{1}{2} = \frac{1}{2}$$

$\Rightarrow K = 1$ (fully submerged)

69. **(4)** Acceleration when block B is in the liquid,

$$a_1 = \frac{m_A g - (m_B g - \text{upthrust})}{(m_A + 3m)}$$

$$= \frac{m_A g - \left(3mg - \dfrac{3m}{2\rho}\rho g\right)}{(m_A + 3m)}(\uparrow)$$

Acceleration when block B is outside of the liquid,

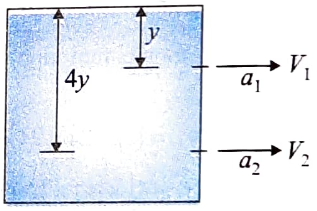
$$a_2 = \frac{3mg - m_A g}{(m_A + 3m)}(\downarrow)$$

Given $a_1 = a_2$, we get

$$m_A g - \frac{3}{2}mg = 3mg - m_A g$$

$$\Rightarrow 2m_A g - \frac{9}{2}mg \Rightarrow m_A = \left(\frac{9}{4}\right)m$$

70. **(1)** Velocity of efflux at a depth h is given by $V = \sqrt{2gh}$



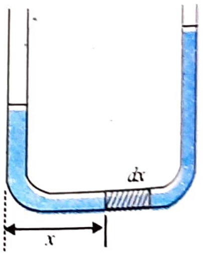
Given, $a_1 V_1 = a_2 V_2$

$$L^2\sqrt{2gy} = \pi R^2\sqrt{2g4y} \text{ or } R = L/\sqrt{2\pi}$$

71. **(2)** $dF = dmx\omega^2$

and $(dp)A = (\rho A dx)x\omega^2$

$$\int_{P_1}^{P_2} dp = \rho\omega^2\int_0^L x\,dx$$



$$\text{or}\quad P_2 - P_1 = \frac{\rho\omega^2 L^2}{2}$$

$$\Rightarrow \rho gH = \frac{\rho\omega^2 L^2}{2}$$

$$H = \frac{\omega^2 L^2}{2g}$$

72. **(2)** Upthrust = Net weight

$$\frac{3}{2}LAdg = (d_1 + d_2)LAg$$

$$\Rightarrow d_1 + d_2 = \frac{3}{2}d$$

and $d_2 - d_1 = d$

$$\Rightarrow d_2 = \frac{5}{4}d \text{ and } d_1 = \frac{d}{4}$$

(1) $v_A = \sqrt{2g \times \frac{h}{4}} = \sqrt{\frac{gh}{2}}$

$$(\text{Range})_A = v_A \times t = \sqrt{\frac{gh}{2}} \times \sqrt{2 \times \frac{3h}{4g}} \qquad \text{...(i)}$$

Bernoulli's theorem between surface and B

$$2\sigma g\frac{h}{2} + \sigma g\frac{h}{2} = \frac{1}{2}(2\sigma)v^2 + \left(2\sigma g\frac{h}{4}\right) \Rightarrow v = \sqrt{gh}$$

$$(\text{Range})_B = \sqrt{gh} \times \sqrt{\frac{2 \times h}{4g}}$$

$$\Rightarrow \frac{R_A}{R_B} = \sqrt{\frac{3}{2}}$$

(3) Applying continuity equation

$$v_A A_A = v_B A_B$$

$\Rightarrow A_A = A_B$ so $v_A = v_B$

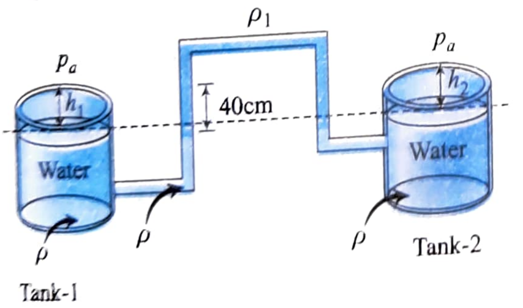
Applying Bernoulli's equation

$$P_A + \rho g h_A + \frac{1}{2}\rho v_A^2 = P_B + \rho g h_B + \frac{1}{2}\rho v_B^2$$

Since $v_A = v_B$; and $h_A = h_B$

$\Rightarrow P_A = P_B$

5. (2) Using Pascal's equation from first to second tank surface



$$P_a + h_1\rho g - 40\rho_1 g + 40\rho g = P_a + h_2\rho g$$

$$h_2\rho g - h_1\rho g = 40\rho g - 40\rho_1 g$$

as $\rho_1 = 0.9\rho$

$$(h_2 - h_1)\rho g = 40\rho g - 36\rho g$$

$$h_2 - h_1 = 4 \text{ cm}$$

76. (1) Buoyant force $= F_b = V_{sub} \cdot \rho_l \cdot g$

where, V_{sub}, ρ_l and g all are same w.r.t. O_1 and O_2.

77. (4) Acceleration of container is $a = a_0(\hat{i} - \hat{j} + \hat{k})$

As there is no gravity, the pressure difference will be only due to the acceleration. All points other than point 'H', are acted upon by a pseudo force. Hence, at point 'H' pressure developed is zero.

78. (2) Weight of sphere + chain = $(m + \lambda h)g$

$$\text{Buoyant force} = \left(3m + \frac{\lambda h}{7}\right)g$$

For equilibrium, weight = Buoyant force

$$\text{or} \quad m + \lambda h = 3m + \frac{\lambda h}{7}$$

$$\text{or} \quad h = \frac{7m}{3\lambda}$$

Multiple Correct Answers Type

1. **(1), (2), (4)**

Initially, the wooden block floats partially immersed in water. Initially, upthrust exerted by water is exactly equal to weight of the block. But when it is exactly pressed down, then the more water is displaced. Hence, upthrust exerted by water increases. The force required to press down the block is F = upthrust – mg. Hence, work done by the force F will be equal to work against (upthrust – mg). It means, work done by the external force = work done against upthrust – loss of gravitational potential energy of the block.

2. **(2), (4)**

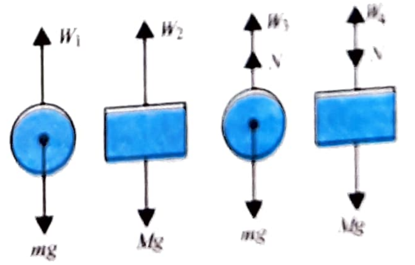
The option (1) is incorrect. A body will sink if its weight is greater than the upthrust. Option (2) is correct. Option (3) is incorrect. The density of hydrogen is less than that of air. Therefore, initially, the balloon rises because the upthrust due to air is greater than the weight of the hydrogen-filled balloon. The density of air decreases as we go up. Therefore, the upthrust on the balloon also decreases. It will stop rising when it attains a height at which the upthrust becomes equal to its own weight. Option (4) is correct.

3. **(1), (2), (3), (4)**

The system consisting of the body floating in water in the beaker is in a state of weightlessness when it is falling freely under gravity. If the body is initially floating (while starting to fall, say, from rest) it will continue to float during the free fall. The body falls along with the displaced liquid and the immersed portion of the body remains unchanged. However, the weight of the immersed portion of the body and the weight of the liquid displaced both becomes zero during the free fall. The upthrust on the body, therefore, becomes zero. The weight of the body in air also becomes zero because it becomes weightless. Hence, all the four options (1), (2), (3) and (4) are correct.

4. **(1), (2), (4)**

W_1, W_3 are the forces exerted by the ball and the spring balance on each other before and after immersion. W_2, W_4 are the forces exerted by the tank and the weighing machine on each other before and after immersion.



Let m and M be the masses of the ball and the tank, respectively, and let N be the force of interaction between the ball and the liquid in the tank.

$$W_1 = mg;\ W_2 = Mg;\ W_3 = N - mg;\ W_4 = N + Mg$$

5. **(2), (3)**

Height of liquid in vessel C is maximum. Therefore, force on the base of vessel C is maximum $[= (P_a + h\rho g)A]$. Net force on all the three vessels = weight of the liquid, which is the same for all the three vessels.

6. (1), (3), (4)

If the system is not in the state of free fall, pressure at any point in a liquid depends on its depth h w.r.t. its free surface.

$$P = P_0 + hdg$$

i.e., pressure is different at points lying in different depths in the liquid. In different liquids (i.e., density d is different) points at the same depth h will be at different pressures. But points lying at different depths in different liquids can be at the same pressure if

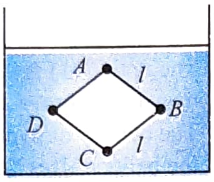
$$h_1 d_1 = h_2 d_2 = \cdots$$

So, options (1), (3) and (4) are correct.

7. (1), (4)

As per the continuity equation, Av = constant. $v_A = v_B$. As point B is at the lower level, hence $P_B > P_A$.

8. (1), (2), (4)



According to Pascal's law,

$$P_D = P_B$$

because B and D lie on the same horizontal plane.

Let l be the side of the square. Then the vertical distance between A and C is $l\sqrt{2}$.

$$P_B = P_D = P_C - \rho g \frac{L}{\sqrt{2}} \qquad \text{...(i)}$$

$$\text{Similarly, } P_B = P_D = P_A + \rho g \frac{L}{\sqrt{2}} \qquad \text{...(ii)}$$

$$P_B = P_D = \frac{P_A + P_C}{2}$$

9. (1), (3), (4)

$$\frac{P_1}{\rho} + \frac{v_1^2}{2} = \frac{P_2}{\rho} + \frac{v_2^2}{2} \Rightarrow P_1 - P_2 = \frac{\rho}{2}(v_2^2 - v_1^2)$$

$$\text{But, } \quad P_1 - P_2 = \rho g h = \frac{\rho}{2}(v_2^2 - v_1^2)$$

$$\text{or } \quad v_2^2 - v_1^2 = 2gh$$

10. (2), (3)

Since the liquids are in static equilibrium, pressure at R = pressure at S. Since liquid A is above liquid B, density of liquid A is less than that of B. Similarly, density of liquid C is less than that of B or liquid B has maximum density among these three liquids. Hence, option (3) is correct.

Since T is at lower level than Q, the pressure exerted by C is greater than that exerted by A. It is possible only when density of liquid A is less than that of C. Hence, option (2) is correct.

11. (1), (2), (3)

By the principle of conservation of energy, we have

$$mgh = \left(\frac{1}{2}\right) mv^2 \text{ or } v = \sqrt{2gh}$$

Since $\rho > \rho'$, the buoyant force of the lake is greater than the weight of the body. If a is the upward acceleration, then

$$ma = \rho g V - \rho' g V \quad \text{or} \quad (\rho' V)a = \rho g V - \rho' g V$$

Cancelling V on both the sides, we get

$$a = g\left(\frac{\rho - \rho'}{\rho}\right) = g\left(\frac{\rho}{\rho'} - 1\right)$$

Now, $v^2 = v_0^2 + 2ay$.

where v_0 is the velocity at $y = 0$. $v = 0$ is the velocity at the maximum depth and y is the negative displacement from the surface to the maximum depth. With these, we get $v_0^2 = -2ay$.

Hence, the depth is given by

$$-y = \frac{v_0^2}{2a} = \frac{2gh}{2g\frac{(\rho - \rho')}{\rho'}} = \frac{h\rho'}{\rho - \rho'}$$

12. (2), (3), (4)

$$\text{Density of the block} = \frac{1}{3 \times 10^{-4}} \text{ kg/m}^3 = \frac{10}{3} \times 10^3 \text{ kg/m}^3$$

Buoyant force $B = 3 \times 10^{-4} \times 10^3 \times g = 3$ N

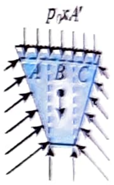
$\therefore \quad T_B = 10 - 3 = 7$ N

Reading of $B = 14 + 3 = 17$ N

Reading of $C = [14 + (10 - 3) + 3]$N $= 24$ N

13. (1), (2), (3), (4)

The free-body diagram of the entire liquid is as shown in the figure. m is the mass of the entire liquid in the vessel. As the liquid is in equilibrium, net force on the liquid in any direction has to be zero. From the above statement, we can say that option (1) is correct.



For liquid contained in section lettered A, vertical component of forces acting on the inclined face is equal to atmospheric pressure force and weight of liquid. Force on the base is atmospheric force plus weight in section B. From the above statement, we can say (directly or indirectly) that options (2), (3) and (4) are also correct.

14. (1), (2), (4)

Here, liquid is flowing from A to B. Hence, $P_A > P_B$

According to the equation of continuity. $a_1 v_1 = a_2 v_2$

As $a_2 > a_1$, hence $v_1 > v_2$. As the liquid is non-viscous, there is no loss of energy, i.e., energy per unit volume of the liquid is the same at every point.

So option (3) is incorrect.

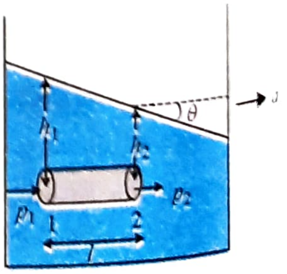
Here, $P_A > P_B$, so $v_A > v_B$ and energy/volume of liquid is same at every point. So, $(\text{PE})_A < (\text{PE})_B$.

Hence, section A must be at the lower level than section B, i.e., the pipe cannot be horizontal.

15. (1), (2)

If cross-sectional area of the tube is uniform everywhere speed will remain same and hence pressure will also be same. If tube's cross-sectional area is non-uniform then at different points of the cross section at different heights pressure may be same but not at points which are at same horizontal levels.

16. (1), (2), (3)



Pressure decreases in the direction of acceleration. If we take two points 1 and 2, two points in the horizontal line, than we can write $P_1 - P_2 = \rho a l$.

As the inclination of liquid surface with $\tan\theta = a/g$. The equipressure lines will be parallel to this line.

17. (3), (4)

$$R = 2\sqrt{hh'}$$

R would be maximum if hh' is maximum.

(1) $hh' = 70 \times 20 = 1400$

(2) $hh' = 60 \times 30 = 1800$

(3) $hh' = 50 \times 40 = 2000$

(4) $hh' = 40 \times 50 = 2000$

Clearly, the liquid coming out from holes 3 and 4 would fall at maximum horizontal distance.

(1), (3)

Let p_1, h_1, v_1 and p_2, h_2, v_2 be the pressures, heights and velocities of flow at the two points, respectively. Then Bernoulli's theorem gives

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \qquad \text{...(i)}$$

Putting $v_1 = 3.0$ m/s, $v_2 = 4.0$ m/s, $(h_2 - h_1) = 1$ m, $p_1 = 20$ kPa, we get

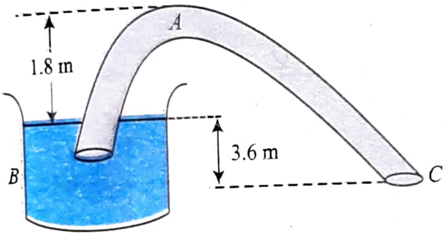
$$p_2 = 20 + \left[10^3 \times 10(-1) + \frac{10^3}{2}[9 - 16]\right] \times 10^{-3}$$

$$= 20 - 10 - 3.5 = 6.5 \text{ kPa}$$

Also when the flow stops, $v_1 = v_2 = 0$ and then from Eq. (i), we have $p_2 = 20 - 10 = 10$ kPa

(1), (2), (4)

Applying Bernoulli's equation at C and D, we have



$$P_0 + 0 + \rho g(3.6) = P_0 + \frac{1}{2}\rho v^2 + 0$$

$\Rightarrow \quad v = 6\sqrt{2}$ m/s

Volume blown per unit time

$$av = \pi r^2 v = 96\sqrt{2} \times 10^{-4} \text{ m}^3/\text{s}$$

Similarly, at A and C,

$$P_A + \frac{1}{2}\rho v^2 + \rho g(3.6 + 1.8) = P_0 + \frac{1}{2}\rho v^2 + 0$$

$\Rightarrow \quad P_A = 0.46 \times 10^5$ N/m^2

20. (1), (4)

According to Bernoulli's equation,

$$P_1 - P_2 = \frac{1}{2}\rho' v_1^2\left[\left(\frac{a_1}{a_2}\right)^2 - 1\right]$$

If h is the difference in level of mercury,

$$h\rho g = \frac{1}{2}\rho' v_1^2\left[\left(\frac{a_1}{a_2}\right)^2 - 1\right]$$

where ρ is density of mercury and ρ' is density of water. Hence,

$$1.4 \times 13.6 \times 980 = \frac{1}{2} \times v_1^2\left[\frac{v_2^2}{v_1^2} - 1\right] \qquad (\because a_1v_1 = a_2v_2, \rho' = 1)$$

$\therefore \quad 2.8 \times 13.6 \times 980 = v_2^2 - v_1^2$

$\Rightarrow \quad v_2^2 = 2.8 \times 13.6 \times 980 + v_1^2 = 2.8 \times 13.6 \times 980 + 60^2$

Solving, we get $v_2 = 202$ cm/s $\simeq 2$ m/s

$\therefore \quad v_2 > 60$ cm/s

21. (2), (4)

$$V_A = \left(2g\frac{H}{3}\right)^{1/2}$$

$$V_B = \left(\frac{4gH}{3}\right) = \sqrt{2}\left(\frac{2gH}{3}\right)^{1/2} = \sqrt{2}V_A$$

$$Q_A = V_A(2a)$$

and $\quad Q_B = V_B(a) = V_A\sqrt{2}(a)$

$$\Rightarrow \quad \frac{Q_A}{Q_B} = \sqrt{2}$$

22. (1), (3), (4)

The velocity with which the liquid comes out is given by the expression

$$P_0 + \rho_1 g \times 30 \text{ cm} + \rho_2 g \times 10 \text{ cm} = p_0 + \frac{\rho_2 v^2}{2}$$

$v = 2.24$ m/s

Due to transfer of liquid, change in momentum takes place and hence a force has been exerted on the liquid in backward direction. this force is given by $F = \rho_2 a v^2$, where a is the cross-sectional area of hole $= 1200 \times 5 \times 10^{-4} \times 5 = 3$ N.

Normal contact force between the vessel and the surface is

$N = mg$

$N = (\rho_1 A \times 0.3)g + (\rho_2 \times A \times 0.2)g = 420$

where A is the cross-sectional area of the vessel.

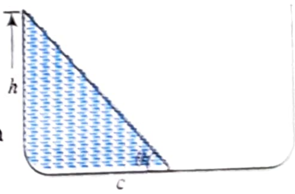
Limiting friction force, $f_L = \mu N = 0.04 \times 420 = 16.8$

As $f_L > F$, therefore the minimum force necessary to maintain the static equilibrium is 0. For the maximum value of the applied force.

$F_{\max} = f_L + F = 16.8 + 3 = 19.8$ N

23. (1), (3)

The maximum amount of water that can be retained is shown in the figure. If θ is the angle made by the water surface with the horizontal, then



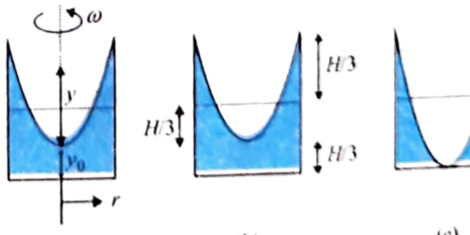
$$\tan\theta = \frac{h}{c} = \frac{a}{g} \quad \Rightarrow a = \frac{hg}{c}$$

So the maximum volume that can be retained is $(1/2) \times h \times c \times b$ and

$$F = \left[M + \frac{hcb\rho}{2}\right]\frac{hg}{c}$$

24. (1), (2), (3)

The equation of the free surface is $y = \dfrac{\omega^2 r^2}{2g} + y_0$

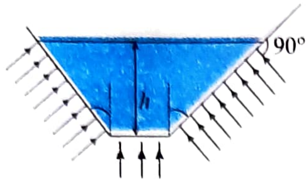


(a) (b) (c)

When the free surface touches the brim of the cylinder, the lowest point of the paraboloid is at a height $H/3$ above the base, as shown in Fig. (b). When the lowest point of the free surface touches the base of the cylinder, the height of water in the tank must be $H/2$. It means that water must spill out.

25. (1), (2), (3)

The forces on the walls of the vessel are shown. The liquid is in equilibrium, so net force on the liquid in any direction has to be zero (either $\alpha = \beta$ or $\alpha \neq \beta$).



26. (1), (3), (4)

At the equator due to rotation of earth the effective gravity decrease but at poles it remain same. In case of a spring balance the weight is measured by the gravity and the reading of equator will be less but for a beam balance the weights reading will remain same.

27. (1), (3), (4)

Fraction of volume immersed is given by $f = \frac{\rho_s}{\rho_l}$

Here, ρ_s and ρ_l are same. Base area in third case is uniform. Hence h_3 is minimum.

28. (1), (2), (3), (4)

Pressure at bottom of vessel is $P_B = 2h\rho g$

Force at bottom is $F_B = P_B A_2 = 2h\rho g A_2$

Pressure at level x is $P_x = h\rho g$

Force on horizontal walls of container is $P_x = h\rho g$

Force on horizontal walls of container is

$$F_x = h\rho g(A_2 - A_1) \text{ upward}$$

Hence walls will exert same downward force on liquid.

29. (1), (2), (4)

As ice displaces the water of same amount as that of its mass, water level will not change if it melts. If ice cube contains some lead pieces then on melting ice lead pieces will sink and will not be able to displace the water equal to their weights so water level will fall. If some cork pieces are inside the ice cube then after melting cork pieces will remain in floating state so water level will not change.

30. (1), (3), (4)

In case of liquid if $\rho_L > \rho$ the buoyancy force will be more than the weight of the ball so it will retard upto certain depth then start rising up and finally reaches the original position in a longer time t_2 and during motion the acceleration of ball outside and inside the liquid remain constant so the motion cannot be simple harmonic. If $\rho_L = \rho$ then inside the liquid net force on ball will be zero so it will continue to move with same constant velocity.

31. (1), (3)

Siphon will work only when the pressure inside the lower face of tube is more than atmospheric pressure for which $h_2 > h_1$. And to start the siphon the pressure due to column h_1 must be less than the atmospheric pressure otherwise it will stop hence option (3) is also correct.

32. (1), (3)

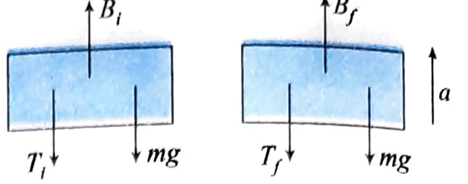
To get the maximum range on the level at a depth h below the base of tank we use

$$\text{Range } x = [2(2h - y)/g]^{1/2} x[2gy]^{1/2}$$

For maximum range we use $dx/dy = 0$ which gives $y = h$ and $x_{\max} = 2h$.

33. (1), (3)

Let σ is density of liquid and ρ is density at object



Initially Finally

Up thrust force on the block initially, $B_i = \sigma V g$

Upthrust force on the block when moving with acceleration

$$B_f = \sigma V(g + a) = B_i\left(\frac{g+a}{g}\right)$$

From F.B.D. of the block when it is accelerating

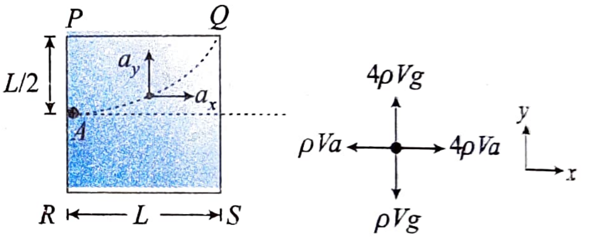
$$B_f - mg - T = ma$$

$$T = B_f - m(g + a) = \sigma V(g + a) - \rho V(g + a)$$

$$T_f = (\sigma - \rho)V(g + a) = T_i\left(\frac{g+a}{g}\right)$$

34. (2), (3)

FBD of ball in frame of container



Acceleration of the ball in y-direction

$$a_y = \frac{4\rho Vg - \rho Vg}{\rho V} = 3g$$

V is the volume of the ball

$$\frac{L}{2} = \frac{1}{2}3gt^2 \Rightarrow t = \sqrt{\frac{L}{3g}}$$

Acceleration of the ball w.r.t container in x-direction

$$a_x = \frac{4\rho Va - \rho Va}{\rho V} = 3a$$

$$L = \frac{1}{2}3at^2 \Rightarrow t = \sqrt{\frac{2L}{3a}}$$

Ball to collide at point Q, $t = \sqrt{\frac{2L}{3a}} = \sqrt{\frac{L}{3g}} \Rightarrow a = 2g$

Linked Comprehension Type

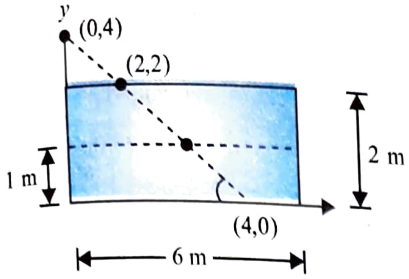
For Problems 1–5

1. (4) $\frac{dy}{dx} = \frac{a_x}{a_y + g} = \frac{g/2}{-g/2 + g} = 1$

(Effective acceleration due to gravity will be $g - a = g/2$)

$\therefore\ \theta = 45°$

2. (3) The slope of the free surface is 45°. Thus, the free surf passes through the centre of the box having coordinates (2,2) the top of the box.



Required length = 6 – 2 = 4 m

(2) $P = P_a + \rho g h = 10^5 + 1000 \times 10 \times 1$

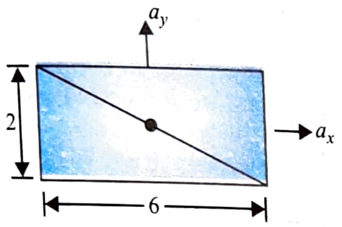
$= (10^5 + 10^4)\ \text{N/m}^2 = 0.11$ MPa

(2) $P = (10^5 + 10^3 \times 10 \times 4)\ \text{N/m}^2$

$= [0.1 + 0.04]$ MPa $= 0.14$ MPa

(1) As the maximum slope of the free surface is 1/3 for the condition of non-exposure of the bottom of the box, so

$$\frac{a_x}{a_y + g} = \frac{1}{3}$$



As $a_x = g/2$

$\therefore\ 3a_x = a_y + g$

$\Rightarrow a_y = g/2$, upwards

or Problems 6–7

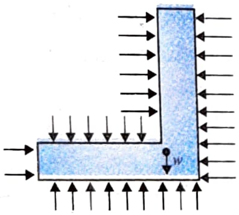
6. (3), 7. (4)

Pressure at bottom of vessel,

$P = \rho g \times 6$ cm

$= 1000 \times 10 \times \dfrac{6}{100}$

$= 600\ \text{N/m}^2$



Force acting at the base,

$F = pA_2 = 600 \times 50 \times 10^{-4} = 3$ N

Weight of liquid inside the vessel,

$W = \rho[A_1 \times 5\ \text{cm} + A_2 \times 1\ \text{cm}]g$

$= 1000[10 \times 5 + 50 \times 1] \times 10^{-6} \times 10 = 1$ N

So, $F > W$

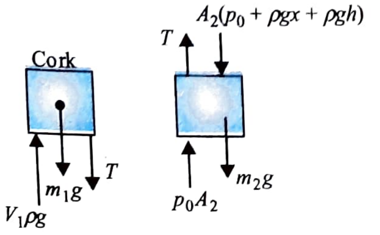
The free-body diagram of the liquid is as shown. The net force on the liquid is zero.

For Problems 8–9

8. (1), 9. (3)

Let x cm be the length of cork cylinder inside the water and T be the tension in the string.

The free-body diagrams of the two cylinders are as shown in the following figures.



Here, $m_1 = 15$ g, $m_2 = 25$ g, $h = 4$ cm, $v_{in} = A_1 x\ \text{cm}^3$

For cork cylinder, $T + m_1 g = V_{in} \rho g$

For aluminium cylinder,

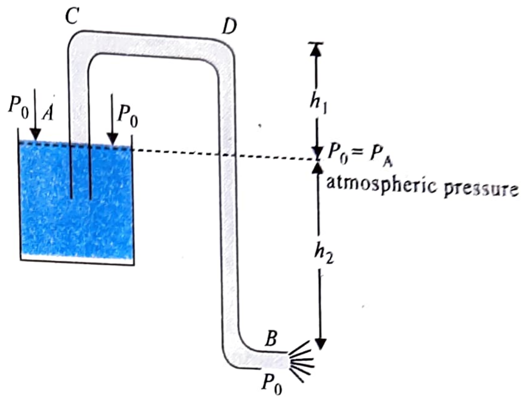
$T + p_0 A_2 = m_2 g + (p_0 + \rho g x + \rho g h)A_2$

$\Rightarrow\ x = 6$ cm

$T = 0.45$ N

For Problems 10–12

10. (1) As long as water fills the tube (as shown in the figure) and points A and B are open to the atmosphere, the velocity at B will be given by Torricelli's theorem.



Hence, $v_B = \sqrt{2gh_2}$ where h_2 is the difference in the levels A and B.

11. (3) We apply Bernoulli's theorem for level A and for the highest level, CD (labelled by subscript 1), to get

$$P_A = P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 \qquad \text{...(i)}$$

Since the tube has uniform cross section and water is incompressible,

$$v_1 = v_B = \sqrt{2gh_2} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$P_1 = P_A - \frac{1}{2}\rho\left[\sqrt{2gh_2}\right]^2 - \rho g h_1$$

$$= P_{\text{atm}} - \rho g(h_1 + h_2) \qquad \text{...(iii)}$$

The minimum value of $P_1 = 0$ (P_1 cannot be negative because then no water will reach the level). Hence, putting $P_1 = 0$,

$$(h_1)_{\max} = \frac{P_{\text{atm}}}{\rho g} - h_2 = \frac{1.0 \times 10^5}{1 \times 10^3 \times 9.8} - 3.0$$

$$= 10.2\ \text{m} - 3.0\ \text{m} = 7.2\ \text{m}$$

12. (2) Putting $h_1 = h_2 = 3.0$ m in Eq. (iii) above,

P_1 = pressure at level CD

$= P_{\text{atm}} - (\rho g)(3 + 3) = 1.0 \times 10^5 - 6\rho g$

The gauge pressure at level $CD = 6\rho g = 6 \times 10^3 \times 9.8\ \text{N m}^2$

$= 5.9 \times 10^4\ \text{N/m}^2$

For Problems 13–15

13. (4) The upward buoyant force $B = \rho g V$, where V is the volume of the body.

Hence, the equation of motion is

$B - mg = ma$, where a is the upward acceleration, i.e.,

$\rho g V - \sigma V g = \sigma V a$

Hence, $a = \dfrac{g(\rho - \sigma)}{\sigma} = g\left(\dfrac{\rho}{\sigma} - 1\right)$ upward

14. (1) If $v_0 = \sqrt{2gh}$ is the velocity downwards just when the body enters the liquid, the final velocity at the maximum depth it will sink is zero. Its acceleration is a.

Hence, $(-)\,y = \text{maximum depth} = \dfrac{v_0^2}{2a} = \dfrac{2gh}{2g\left(\dfrac{\rho}{\sigma} - 1\right)}$

$$\therefore \quad y = \frac{h\sigma}{\rho - \sigma}$$

15. (2) Time taken to reach the surface of liquid, $t_1 = \sqrt{\dfrac{2h}{g}}$

Time taken by object to travel from surface of liquid to maximum depth,

$$t_2 = \sqrt{\frac{2h\left(\dfrac{\sigma}{\rho - \sigma}\right)}{g\left(\dfrac{\rho - \sigma}{\sigma}\right)}}$$

As $\rho = 2\sigma$, $t_2 = \sqrt{\dfrac{2h}{g}}$

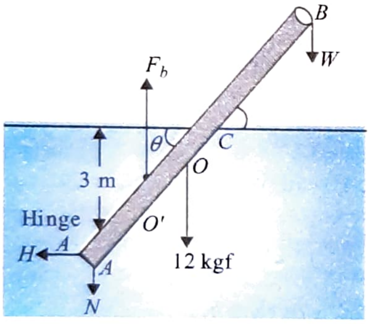
Total time taken to reach starting point is

$$t = 2t_1 + 2t_2 = 4\sqrt{\frac{2h}{g}}$$

For Problems 16–17

16. (2), 17. (1)

Mass per unit length of rod is 2 kg.



Mass of the submerged portion of the rod is 10 kg and its volume is $\dfrac{10}{500}\,\text{m}^3$.

Buoyant force, $F_B = \dfrac{10}{500} \times 1000 = 20\,\text{kgf}$

Let N and H be the vertical and horizontal reactions of the hinge on the rod, respectively. Considering the horizontal and vertical translational equilibrium of the rod,

$$N + 12 + W = 20$$

where W is the weight to be attached at the end.

$$H = 0$$

Considering the rotational equilibrium of the rod about A,

$$-20 \times g \times 2.5 \cos\theta = 12 \times g \times 3 \cos\theta + W \times 6 \cos\theta = 0$$

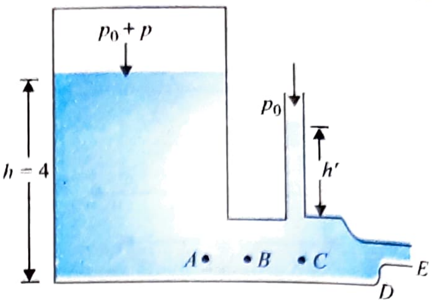
$$\Rightarrow 6W = 50g - 36g = 14g$$

$$\Rightarrow W = \frac{7}{3}g = \frac{7}{3}\,\text{kgf} \Rightarrow N = 8 - \frac{7}{3} = \frac{17}{3}\,\text{kgf}$$

For Problems 18–20

18. (1), 19. (3), 20. (2)

Let atmospheric pressure be p_0. Then absolute pressure of air is $p + p_0 = 4$ atm



Pressure at A, $p_A = p + p_0 + \rho g h$

From equation of continuity and Bernoulli's theorem,

$$p_B = p_C$$

Applying Bernoulli's theorem at A and E, we have

$$p + p_0 + \rho g h = p_C + \frac{\rho v_C^2}{2} = p_0 + \frac{\rho v_E^2}{2}$$

where v_C and v_E are the velocities of flow of liquid at C and E, respectively.

From hydrostatics, $p_C = p_0 + \rho g h'$

From continuity equation, $A_C v_C = A_E v_E$.

where $A_C = 6\text{ cm}^2$ and $A_E = 3\text{ cm}^2$

Solving above equations, we get $v_E = 13.4$ m/s

$v_C = 6.7$ m/s

$h' = 4.5$ m

Discharge rate, $Q = A_E v_E = 3 \times 10^{-4} \times 13.4\ \text{m}^3/\text{s}$

When a hole has been created at the top of the vessel, gauge pressure drops to zero, i.e., in above equation, put $p = 0$ and get h'.

$\therefore \ h' = 3$ m

For Problems 21–22

21. (3), 22. (1)

Weight of the rod, $w = LA\rho g$

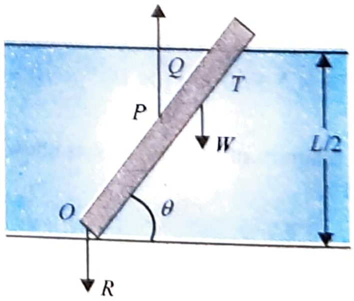
(A = area of cross section, ρ = density of material)

Submerged length of the rod is given by

$$OT = \frac{L}{2\sin\theta}$$

Buoyant force $B = \left(\dfrac{L}{2\sin\theta}\right) A\rho_w g$

from Archimedes' principle (ρ_w = density of water).



This force acts at P, where $OP = \dfrac{OT}{2} = \dfrac{l}{4\sin\theta}$

Balancing torque about O is

$$WOQ\cos\theta = BOP\cos\theta$$

$$(LA\rho g)\left(\frac{L}{2}\right) = \left(\frac{L}{2\sin\theta}\right)A\rho_w g\left(\frac{L}{4\sin\theta}\right)$$

$$\sin^2\theta = \frac{\rho_w}{4\rho} \Rightarrow \sin^2\theta = \left(\frac{1}{2}\right) \Rightarrow \theta = 45°$$

Also, balancing forces $R + W = B$

$$R = B - W = \frac{LA\rho_w g}{2\sin\theta} - LA\rho g$$

$$= LA\rho g\left[\frac{1}{\sin\theta} - 1\right] = \left(\sqrt{2} - 1\right)mg \qquad (\because \rho_w = 2\rho)$$

For Problems 23–25

23. (3) Since area of a hole is very small in comparison to base area A of the cylinder, velocity of liquid inside the cylinder is negligible. Let velocity of efflux be v and atmospheric pressure be P_0. Consider two points A (inside the cylinder) and B (just outside the hole) in the same horizontal line as shown in the figure in question.

Pressure at A, $P_A = P_0 + h\rho_2 g + (h - y)\rho_1 g$

Pressure at B, $P_B = P_0$

According to Bernoulli's theorem,

Pressure energy at A = Pressure energy at B + Kinetic energy at B

$$\therefore \; P_A = P_B + \frac{1}{2}\rho_1 v^2$$

$$\therefore \; v = 4 \text{ m s}^{-1}$$

24. (1) When the cylinder is on the smooth horizontal plane, force F required to keep the cylinder stationary equals horizontal thrust exerted by fluid jet. But that is equal to mass flowing per second × change in velocity of this mass.

$$\therefore \; F = (av\rho)(v - 0) = a\rho v^2 \text{ or } F = 7.2 \text{ N}$$

25. (4) Total mass of the liquid in the cylinder is

$$m = Ah\rho_1 + Ah\rho_2 = 450 \text{ kg}$$

Limiting friction = μmg = 45 N

Since $F <$ limiting friction, therefore minimum force required is zero. Considering the free-body diagram for maximum vertical forces,

$$N = mg$$

Now considering the horizontal forces,

$$F_{\text{max}} = F + \mu N = 52.2 \text{ N}$$

For Problems 26–28

26. (1), 27. (2), 28. (1)

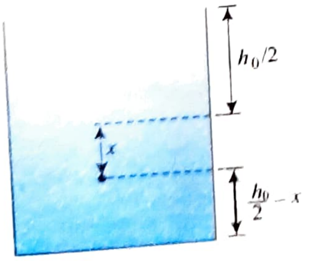
Let the net force on this particle be zero at height h. Then

weight = upthrust (at h)

$$\Rightarrow \frac{4}{3}\pi r^3 g \frac{5}{2}\rho_0 = \frac{4}{3}\pi r^3 g\rho_0\left[4 - \frac{3h}{h_0}\right]$$

$$\Rightarrow \frac{3h}{h_0} = 4 - \frac{5}{2} = \frac{3}{2} \quad \Rightarrow h = \frac{h_0}{2} \qquad \text{...(i)}$$

Therefore, at $h = h_0/2$, the equilibrium position, the particle has a speed in downward direction due to which it will move downward. Let at a further depth x from $h_0/2$,



F_{net} = upthrust − weight

$$= \frac{4}{3}\pi r^3 g\rho_0\left[4 - \frac{3\left(\frac{h_0}{2} - x\right)}{h_0}\right] - \frac{4}{3}\pi r^3 g\frac{5}{2}\rho_0$$

$$= \frac{4}{3}\pi r^3\rho_0 g\left[4 - \frac{3}{2} + \frac{3x}{h_0} - \frac{5}{2}\right] = \frac{4\pi r^3\rho_0 g}{h_0}x$$

$$\therefore \; F = \frac{4\pi r^3\rho_0 g}{h_0}x$$

Since the net force on the particle and displacement of ball from its mean position are oppositely directed, the particle executes SHM about its mean position, i.e., $h = h_0/2$.

$$\therefore \; F = ma = -\frac{4\pi r^3\rho_0 g}{h_0}x$$

$$\Rightarrow \frac{4}{3}\pi r^3\frac{5}{2}\rho_0 a = -\frac{4\pi r^3\rho_0 g}{h_0}x$$

$$\Rightarrow a = -\frac{6g}{5h_0}x = -\frac{12}{h_0}x = -\omega^2 x$$

$$\text{where } \omega = \sqrt{\frac{12}{h_0}} = \sqrt{\frac{12}{\frac{12}{\pi^2}}} = \pi$$

Hence, time taken by the particle to reach the bottom is

$$t = \frac{T}{2} = \frac{1}{2}\frac{2\pi}{\omega} = \frac{\pi}{\omega} = \frac{\pi}{\pi} = 1 \text{ s}$$

For Problems 29–30

29. (2) Equating the pressures at the same level of third liquid at the boundary of first and third liquids on left hand side.

Pressure on left hand side = pressure on right hand side

$$\therefore \; P_0 + \rho(20)g = P_0 + 10(1.5)g + h(2\rho)g.$$

Solving this equation, we get h = 2.5 cm

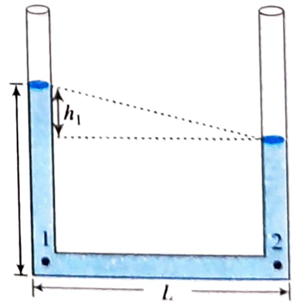
30. (3) Rewriting the equation as

$$P_0 + \rho(20)g = P_0 + 10(1.5)g + h(2\rho)g$$

From here we can see that h will decrease.

For Problems 31–32

31. (3)

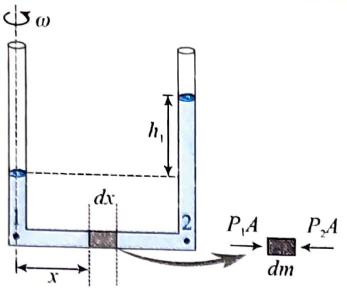


$$P_1 - P_2 = \rho g h_1$$

for horizontal section $F = (P_1 - P_2)A = \rho g h_1 = (\rho LA)a$

$$\Rightarrow \; h_1 = \frac{La}{g}$$

32. (1) $dF = (dm)\omega^2 x \Rightarrow (dP)A = (\rho A dx)\omega^2 x$



$$\Rightarrow \int_{P_1}^{P_2} dP = \rho\omega^2 \int_0^L x\,dx$$

$$\Rightarrow P_2 - P_1 = \rho g h_1 = \rho\omega^2 L^2/2$$

$$\Rightarrow h_1 = \frac{\omega^2 L^2}{2g}$$

For Problems 33–34

33. (4) Pressure on the water surface

$$P_s = \frac{Mg}{A} = \frac{3\times 10}{\pi[16\times10^{-4} - 1\times10^{-4}]}$$

$$= \frac{30\times10^4}{\pi\times15} = \frac{2}{\pi}\times10^4\,\text{Pa}$$

According to Pascal law $P_s = h\rho g$

$$\Rightarrow h = \frac{P_s}{\rho g} = \frac{\frac{2}{\pi}\times10^4}{10^3\times10} = \frac{2}{\pi}\,\text{m}$$

34. (3) Mass of water in the pipe

$$= (\pi r^2 h)\rho = \pi\times10^{-4}\times\frac{2}{\pi}\times10^{-3} = 0.2\text{ kg}$$

mass of water in cylinder = 750 – 200 = 550 g = 0.55 kg

$$\Rightarrow 0.55 = (\pi R^2 H)\rho$$

$$\Rightarrow H = \frac{0.55}{\pi\times16\times10^{-4}\times10^3} = \frac{11}{32\pi}\,\text{m}$$

Matrix Match Type

1. i. → b.; ii. → c.; iii. → a.

Kinetic energy is 1/2 mv^2. Hence,

$\frac{v^2}{2g} = \frac{\left(\frac{1}{2}mv^2\right)}{mg}$ is energy per unit weight.

$\frac{\rho v^2}{2} = \frac{\left(\frac{1}{2}mv^2\right)}{m\rho}$ is energy per unit volume and

$\frac{v^2}{2} = \frac{\frac{1}{2}mv^2}{m}$ is energy per unit mass.

2. i. → c., d.; ii. → e.; iii. → b.; iv. → c., a.

Bernoulli's theorem is applicable for incompressible, non-viscous liquids with streamlined flow. So, i → c. It is based on the principal of conservation of energy. So, i → d.

A ball moving with a spin will have higher velocity of air above the ball than below. So the pressure below will be more than that above, causing a lift. So, ii → e.

Based on the principle of Bernoulli's theorem, artificial high pressure created pushes the paint through the gun. So, iii → b.

Streamlined flow is possible in only non-viscous medium. So, iv → c.

As the stream passes through smaller cross section or narrower pipes, the velocity increases causing the pressure to fall. So, iv → a.

3. i. → d.; ii. → c.; iii. → a.; iv. → a.

Magnus effect talks of difference in pressure caused due to difference in velocity to lift a structure. Lifting of asbestos roofs during cyclonic storms is based on this phenomenon. So, i → d.

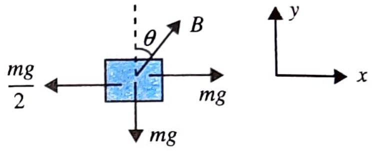
Viscous force is wet friction and brings loss in internal energy. So, ii → c.

At the same level in the same liquid in a container, the pressure is the same according to Pascal's law. So, iii → a.

Hydraulic machines use the idea of Pascal's law to transfer equal pressure in all directions from the free surface where it is applied. So, iv → a.

4. i. → d.; ii. → c.; iii. → a.; iv. → b.

i. FBD of the cube with respect to the container



$$B = \left(\frac{m}{\sigma}\right)\rho g_{\text{eff}} = m\left(\frac{\rho}{\sigma}\right)\sqrt{a^2+g^2}$$

$$\tan\theta = \frac{a}{g}$$

$$\sin\theta = \frac{a}{\sqrt{a^2+g^2}}; \ \cos\theta = \frac{g}{\sqrt{a^2+g^2}}$$

Let acceleration of the cube w.r.t. container in horizontal direction in a'_x

$$mg + B\sin\theta - \frac{mg}{2} = ma'_x$$

$$a'_x = \frac{g}{2} + \frac{\rho}{\sigma}a = a'_x \Rightarrow \frac{g}{2} + \frac{\rho}{\sigma}\cdot\frac{g}{2} = a'_x$$

Hence, $a_x = a'_x + \frac{g}{2} = g + \frac{g}{2}\frac{\rho}{\sigma}$

$$a_x = g\left[\frac{2\sigma+\rho}{2\sigma}\right] \quad \text{...(i)}$$

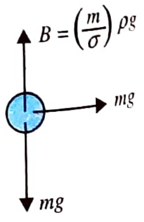
In vertical direction: $B\cos\theta - mg = ma_y$

$$m\left(\frac{\rho}{\sigma}\right)\sqrt{a^2+g^2}\cdot\frac{g}{\sqrt{a^2+g^2}} - mg = ma_y$$

$$a_y = g\left[\frac{\rho}{\sigma} - 1\right] = g\left[\frac{\rho-\sigma}{\sigma}\right] \quad \text{...(ii)}$$

$$\frac{a_x}{a_y} = \frac{(2\sigma+\rho)}{2(\rho-\sigma)}$$

ii. FBD of the sphere:

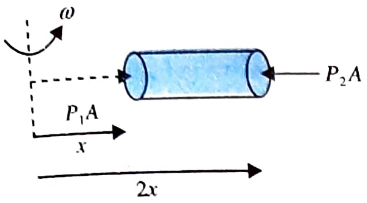


$$a_x = g$$

$$a_y = \frac{-mg + \frac{m\rho}{\sigma}g}{m} = \left(\frac{\rho-\sigma}{\sigma}\right)g$$

Hence, $\dfrac{a_x}{a_y} = \dfrac{\sigma}{\rho - \sigma}$

FBD of the cylinder



Hence, $(P_2 - P_1)A = ma_x$

$$\frac{\rho\omega^2}{2}\left[(2x)^2 - x^2\right]A = ma_x$$

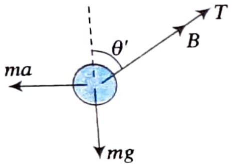
$$\frac{\rho}{2}\left(\frac{2g}{3x}\right)3x^2 A = (Ax)\sigma a_x$$

This gives $\dfrac{a_x}{g} = \dfrac{\rho}{\sigma}$

iv. When the cart is not filled with liquid, $\tan\theta = \dfrac{a}{g}$

When the car is filled with liquid

FBD of the pendulum,



Cart: $(T + B)\sin\theta' = mg$ ...(i)

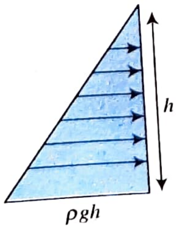
$(T + B)\cos\theta' = mg$ ...(ii)

$\tan\theta' = \dfrac{a}{g}$ ...(iii)

Hence, $\dfrac{\tan\theta'}{\tan\theta} = \dfrac{1}{1} = \dfrac{\theta'}{\theta} = 1:1$

5. **i. → d.; ii. → c.; iii. → a.; iv. → b.**

i. Hydrostatic force on horizontal direction



$$\frac{F_H}{h} = \frac{1}{2}\rho gh^2 = \frac{1}{2}\rho gA$$

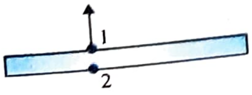
$$F_H = \frac{1}{2}\rho gAh$$

ii. Buoyant force on the cube

$$F_B = (Ah)\rho g = \rho ghA$$

iii. The thrust force (impact) due to liquid coming out of nozzle = ρAv^2

iv. Applying Bernoulli's equation just above and just below the roof,



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$(P_2 - P_1) = \frac{1}{2}\rho v^2 = \Delta P \qquad (\because v_2 = 0)$$

Hence, lift force $F_{\text{lift}} = \Delta PA$

or, $F_{\text{lift}} = \dfrac{1}{2}\rho Av^2$

6. **(2)** $F_2 - F_1$ = upthrust
= weight of cylinder in equilibrium or floating condition.
From liquid-1, horizontal pair of forces cancel out.
So, net force = 0

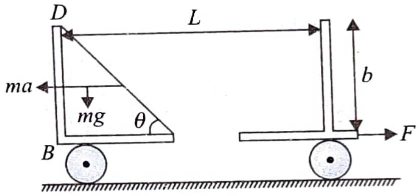
Numerical Value Type

1. **(1)** As the cart is drawn by a force F, the water in the vessel takes up a slant position rising upward at the back wall of the vessel. To prevent water flowing out of the hole H, the acceleration of the vessel should have such a value that it occupies a face area DBH and a width of vessel given by A/L.

Area of $\Delta DBH = \dfrac{1}{2}bc$

Volume of liquid retained $= \dfrac{1}{2}bc \times \dfrac{A}{L}$

Mass of cart and water $= M + \dfrac{bcA\rho}{2L}$



$$\tan\theta = \frac{ma}{mg}$$

$$a = g\tan\theta = g \times \frac{b}{c}$$

$$\text{Required force} = \left(M + \frac{bcA\rho}{2L}\right)\frac{gb}{c} = [1.5 + 0.5] \times 50$$

$$= 2.0 \times 50 = 100\ \text{N}$$

2. **(5)** Let the volume of the cylinder be V. When the cylinder is floating, upthrust = weight. Hence,

$$\rho\left(\frac{3}{4}V\right)g = mg \Rightarrow V = \frac{4m}{3\rho}$$

Let the acceleration of the particle vessel be A (upwards). In the reference frame of the vessel, the acceleration of the cylinder is $A/3$.

$$\therefore \quad mg + mA - \text{uptrust} = m\left(\frac{A}{3}\right)$$

$$mg + mA - \rho Vg' = m\left(\frac{A}{3}\right)$$

where $g' = g + A$ = effective value of g for upthrust.

$$\therefore \quad mg + mA - \rho V(g + A) = m\left(\frac{A}{3}\right)$$

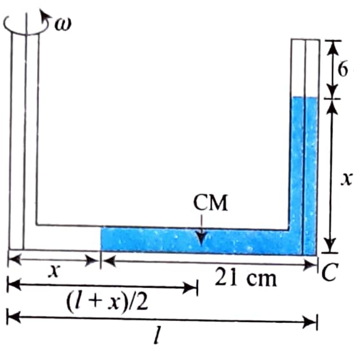
$$\Rightarrow \quad mg - \frac{4}{3}m(g + A) = -m\frac{2}{3}A$$

$$A = \left(-\frac{g}{2}\right)\text{upwards}$$

The acceleration of the vessel should be $\frac{g}{2} = \frac{10}{2} = 5\ \text{ms}^{-2}$ (downwards).

3. (5) Due to rotation, let the shift of the liquid be x cm.

Let the cross-sectional area of the tube be A.



In the right limb for compressed air,

$$p_1 v_1 = p_2 v_2 \Rightarrow p_0 A \times 6 = p_2 A\,(6 - x)$$

or $p_2 = \dfrac{6p_o}{6 - x}$ ...(i)

Force at the corner C of right limb climb due to liquid above,

$$p_1 = \left[\frac{6p_o}{6 - x} + x\rho g\right]A.$$

Mass of the liquid in the horizontal arm, $M = \rho(l - x)A$

It is rotated about left limb. Then the centripetal force is

$$F_2 = m\omega_0^2 r = \rho(l - x)A\omega_0^2\frac{l + x}{2} = \frac{\rho A\omega_0^2}{2}(l^2 - x^2)$$

But $F_1 = F_2$

$$\Rightarrow \frac{\rho A\omega_0^2(l^2 - x^2)}{2} = \left[\frac{6p_0}{6 - x} + x\rho g\right]A$$

$$= \frac{10^5 \times 100 \times (21^2 - x^2) \times 10^{-4}}{2}$$

$$= \left[\frac{6 \times 10500}{(6 - x)} + x \times 10^5 \times 10 \times 10^{-2}\right]$$

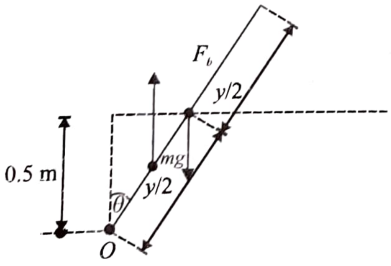
On solving, we get $x = 0.01$ m = 1 cm

The length of the air column = 6 – 1 = 5 cm.

4. (2) Let y be the length of the plank inside water.

$$y = \frac{0.5}{\cos\theta}$$

Let A be the cross-sectional area of the plank. Then the buoyant force on it is $F_b = V\rho_\omega g = (Ay)\rho_\omega g$



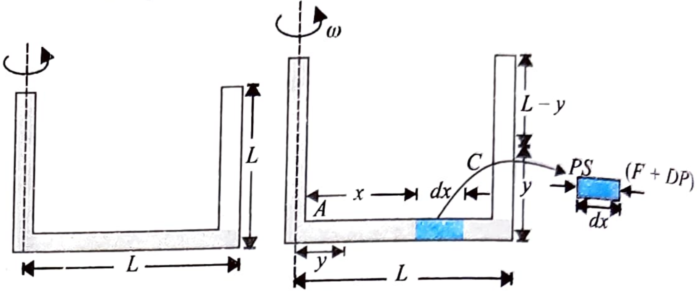
Since the plank is in rotational equilibrium, so $\sum \vec{\tau}_0 = 0$

or $mg \times \frac{l}{2}\sin\theta \quad F_b \times \frac{y}{2}\sin\theta = 0$ or $mgl \quad F_b \times y = 0$

or $(Al \times 0.5)gl \quad (Ay)d_v y = 0$ or $0.5l^2 = y^2$

or $0.5 \times (1)^2 = \left(\frac{0.5}{\cos\theta}\right)^2$ or $\cos^2\theta = \frac{1}{2}$

5. (1) When the tube is rotated, the liquid starts to flow radially outwards and air in sealed arm is compressed. Let the shift of the liquid be y as shown in figure.



Let the cross-sectional area of the tube be S. Here element of length dx is rotating with the tube. From the free-body diagram of the element, we can write

$$(P + dP)S - PS = dm\,\omega^2 x = (\rho S dx)\,\omega^2 x$$

$$dPS = \rho\omega^2 Sxdx$$

Here the pressure difference between points A and B can be given by integrating the pressure difference across an element of width dx, which is given as

$$dP = \rho\omega^2 xdx$$

Now integrating from A to B, we get

$$P_B - P_A = \int_y^L \rho\omega^2 xdx = \frac{\rho\omega^2}{2}(L^2 - y^2)$$

$$\Rightarrow P_B = P_0 + \frac{\rho\omega^2}{2}(L^2 - y^2)$$

The pressure at point C can be given as

$$P_C = P_A - \rho gy$$

At point A, pressure is atmospheric. Thus, we have

$$P_C = \frac{\rho\omega^2}{2}(L^2 - y^2) + P_0 - \rho gy = 10^5\ \text{N/m}^2$$

6. (12.40)

$$\frac{1}{2}\rho_{air}v^2 = \rho_w g h_w$$

$$v = \sqrt{\frac{2\rho_w g h_w}{\rho_{\text{air}}}} = \sqrt{\frac{2 \times 10^3 \times 10 \times 10^{-2}}{1.3}}$$

$$= 12.4\ \text{m/s}$$

7. (32) The speed of water at section X and Y should be equal.

Applying Bernoulli equation, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$$\Rightarrow \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = P_1 - P_2 = h\rho g$$

$$v_2^2 = v_1^2 + 2gh$$

$$v_2 = \sqrt{(2)^2 + 2 \times 1000 \times 0.51} = 32\ \text{cm/s}$$

8. (2.00) Torque of hydrostatic force about centre of sphere is already zero as hydrostatic force passes through the centre

For translational equilibrium, the horizontal thrust provided by both the liquids should be equal and opposite

$\therefore\ F_{\text{LHS}} = F_{\text{RHS}}$

$$\frac{1}{2}(2\rho)(g)h^2 = \frac{1}{2}(3\rho)(g)R^2$$

$$h = \sqrt{\frac{3}{2}}R = \sqrt{\frac{3}{2}} \times \sqrt{\frac{8}{3}}\text{ m} = 2\text{ m}$$

(37.50) In equilibrium, $\dfrac{600 \times 10}{800 \times 10^{-4}} = \dfrac{F}{25 \times 10^{-4}} + h\rho g$

$$\Rightarrow \frac{F}{25 \times 10^{-4}} = \frac{60}{8} \times 10^4 - 8 \times (0.75 \times 10^3) \times 10$$

$$\frac{F}{25 \times 10^{-4}} = 1.5 \times 10^4$$

$$\Rightarrow F = 37.5\text{ N}$$

(46.00) Velocity of flow $= \sqrt{2gh} = \sqrt{2 \times 10 \times 3.6} = 6\sqrt{2}$ m/s

Bernoulli's theorem between surface and A

$$P_{\text{atm}} = P + \frac{1}{2}\rho v^2 + \rho g h$$

$$P = P_{\text{atm}} - \frac{1}{2}\rho v^2 - \rho g h$$

$$= 10^5 - \frac{1}{2} \times 10^3 (6\sqrt{2})^2 - 10^3 \times 10 \times 1.8$$

$$= 4.6 \times 10^4\text{ N/m}^2 = 46\text{ kPa}$$

(10.00) Using Bernoulli's theorem, we have

$$P_B = P_{\text{atm}} + \frac{1}{2}\rho v^2 \qquad \text{...(i)}$$

and $P_B - P_{\text{atm}} = \displaystyle\int_{l-h}^{l} dr \cdot \rho \cdot \omega^2 x = \frac{\rho\omega^2}{2}[l^2 - (l-h)^2]$

$$P_B - P_{\text{atm}} = \frac{1}{2}\rho\omega^2(2l - h)h \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\frac{1}{2}\rho\omega^2(2l-h)h = \frac{1}{2}\rho v^2$$

$$\Rightarrow v = \omega h\sqrt{\left(\frac{2l}{h} - 1\right)}$$

$$= 5 \times 1.0\sqrt{\left(\frac{2 \times 2.5}{1} - 1\right)} = 10\text{ m/s}$$

2. **(6.00)**

$$h_w + 8\text{ cm} + h_0 = 22\text{ cm} + 22\text{ cm}$$

$$\Rightarrow h_w + h_0 = 36\text{ cm}$$

$$P_C = P_B \Rightarrow h_0\rho_0 g = h_w\rho_0 g \Rightarrow h_0 \times 0.8 = h_w \times 1$$

$$\Rightarrow h_0 = \frac{h_w}{0.8} = 1.25h_w \Rightarrow h_w + 1.25\,h_w = 36$$

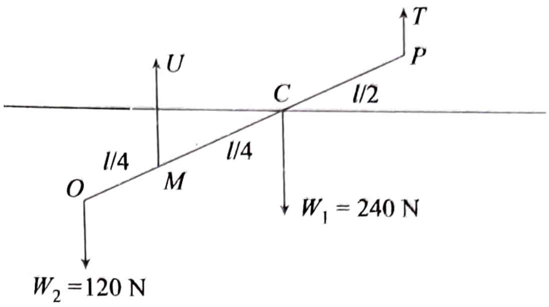
$$\Rightarrow h_w = \frac{36}{2.25} = 16\text{ cm} \Rightarrow BE = 22 - 16 = 6\text{ cm}$$

13. **(64.00)**

$$T + U = W_1 + W_2 = 360\text{ N} \qquad \text{...(i)}$$

$$U = \frac{V}{2}\rho_w g = (V/2)(10^3)(10)$$

or $U = 0.5 \times 10^4\,V$ ...(ii)



Taking torque about $M = 0$

$$\therefore\ 120\left(\frac{l}{4}\right) + T\left(\frac{3l}{4}\right) = 240\left(\frac{l}{4}\right)$$

or $T = \dfrac{240 - 120}{3} = 40\text{ N} = 4g$

Now from Eqs. (i) and (ii), we get

$$V = 6.4 \times 10^{-2}\text{ m}^3 = 64 \times 10^{-3}\text{ m}^3$$

14. (30.00) Taking torque about hinge $mg\dfrac{b}{2}\sin\theta = f_B\left(b - \dfrac{x}{2}\right)\sin\theta$

$$(Ab)d_m g\frac{b}{2} = (Ax)d_L g\left(b - \frac{x}{2}\right);\ \frac{5}{9}\frac{b^2}{2} = bx - \frac{x^2}{2}$$

By solving it $x = \dfrac{b}{3} = \dfrac{90}{3} = 30$ cm

15. (6.00) Upthrust on the block,

$$U = \frac{2}{5}V \times 1500\left(g + \frac{g}{2}\right) + \frac{3}{5}V \times 1000 \times \left(g + \frac{g}{2}\right)$$

$$= 1800 \times 10^{-3} \times 10 = 18\text{ N}$$

Weight of the block $= 10^{-3} \times 800 \times \left(g + \dfrac{g}{2}\right) = 12$ N

So tension in the string = $U - mg$ = 18 – 12 = 6 N

Archives

JEE Main

Single Correct Answer Type

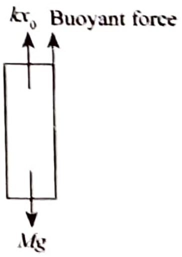
1. (3) $\rho_{\text{oil}} < \rho < \rho_{\text{water}}$

Oil is the least dense of them, so it will settle at the top with water at the base. Now the ball is denser than oil but less denser than water. So it will sink in oil but not in water. It will stay at the oil-water interface.

2. (2) At equilibrium, $\sum F = 0$

$$kx_0 + \left(\frac{AL}{2}\sigma g\right) - Mg = 0$$

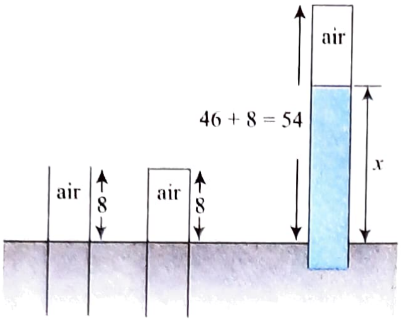
$$x_0 = \frac{Mg}{k}\left[1 - \frac{LA\sigma}{2M}\right]$$



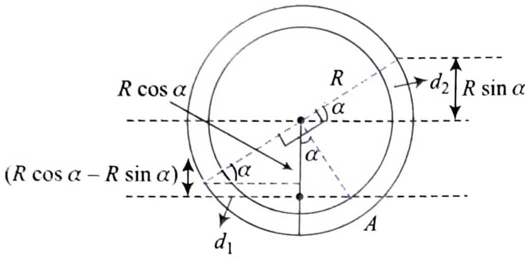
3. (3) (76) (8) = (54 – x) (76 – x)

x = 38 cm

Length of air column = 54 – 38 = 16 cm



4. **(1)** Equating pressure at A



$$(R\cos\alpha + R\sin\alpha)\,d_2 g = (R\cos\alpha - R\sin\alpha)\,d_1 g$$

$$\Rightarrow \frac{d_1}{d_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1+\tan\alpha}{1-\tan\alpha}$$

JEE Advanced

Multiple Correct Answers Type

1. **(1), (2), (4)**

Let V be the volume of each sphere and T be the tension in the string.

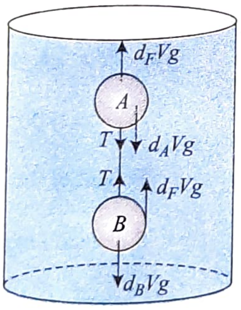
For the string to be taut,

$$d_F Vg > d_A Vg$$

$$\Rightarrow d_F > d_A$$

and $d_B Vg > d_F Vg$

$$\Rightarrow d_B > d_F$$

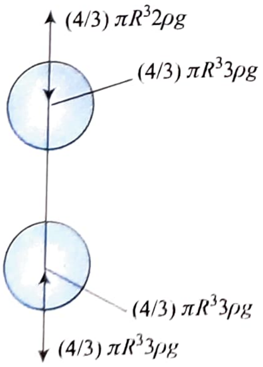


For equilibrium,

$$d_F Vg + d_F Vg + T = T + d_A Vg + d_B Vg$$

or $d_A + d_B = 2d_F$

2. **(1), (4)**



At equilibrium,

$$\frac{4}{3}\pi R^3 2\rho g = \frac{4}{3}\pi R^3 \rho g + T$$

$$T = \frac{4}{3}\pi R^3 \rho g$$

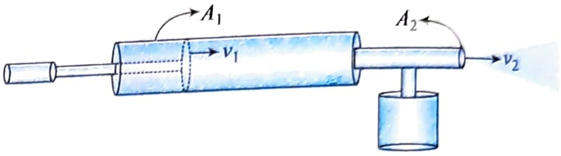
$$\therefore\ \Delta l = \frac{4}{3k}\pi R^3 \rho g$$

For equilibrium of the complete system, net force of buoyancy must be equal to the total weight of the sphere which holds true in the given problem. So, both the spheres are completely submerged.

Linked Comprehension Type

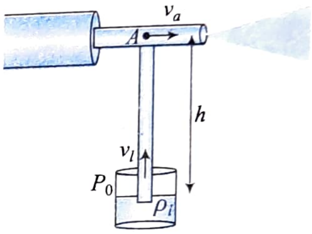
For Problems 1–2

1. **(3)** By continuity equation $A_1 v_1 = A_2 v_2$



$$\Rightarrow \pi(20)^2 \times 5 = \pi(1)^2 v_2 \quad \Rightarrow \quad v_2 = 2\text{ m/s}$$

2. **(1)** Pressure at the point A in the nozzle is P.



Now, $P = P_0 - \frac{1}{2}\rho_a v_a^2$ ...(i)

and also $P = P_0 - \frac{1}{2}\rho_l v_l^2 - \rho_l gh$ ...(ii)

From Eqs. (i) and (ii)

$$\frac{1}{2}\rho_a v_a^2 = \frac{1}{2}\rho_l v_l^2 + \rho_l gh$$

$$\Rightarrow v_l = \sqrt{\frac{\rho_a}{\rho_l}v_a^2 - 2gh}$$

So for the given v_a and h, $v_l \propto \sqrt{\frac{\rho_a}{\rho_l}}$

If a is area of cross-section of thin tube, then rate of flow of liquid will be equal to av_l.

Hence, the rate (volume per unit time) at which the liquid is sprayed $(av_l) \propto \sqrt{\frac{\rho_a}{\rho_l}}$

Numerical Value Type

1. **(6)** $P = P_0 - \rho gh = 98 \times 10^3\ \text{N/m}^2$

$$P_0 V_0 = PV$$

$$10^5[A(500 - H)] = 98 \times 10^3[A(500 - 200)]$$

$$H = 206\text{ mm}$$

Level fall = 206 − 200 = 6 mm

Chapter 7

Concept Application Exercises

Exercise 7.1

1. (b) Stress $= \frac{\text{Force}}{\text{Area}} = \frac{F}{\pi r^2}$

$\therefore$ Stress $\propto \frac{1}{r^2}$

$$\Rightarrow \frac{(\text{Stress})_B}{(\text{Stress})_A} = \left(\frac{r_A}{r_B}\right)^2 = (2)^2$$

$\Rightarrow (\text{Stress})_B = 4 \times (\text{stress})_A$ [As F = constant]

2. (b) Shearing strain, $\phi = \frac{x}{L} = \frac{0.02\,\text{cm}}{0.1\,\text{m}} = 0.002$

3. (c) Breaking force = Breaking stress × Area of cross section

For a given material of the wire, breaking stress is constant.

$$\therefore \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = F_1\left(\frac{A_2}{A_1}\right) = F_1\left(\frac{A_1/4}{A_1}\right) = \frac{F_1}{4} = \frac{W}{4}$$

4. (d) Breaking force = Breaking stress × Area of cross-section. When the wire is cut to half, there is no change in cross-sectional area of the wire. Therefore, there is in no change in the maximum load (i.e. breaking force) the wire can support.

5. (d) If same force is applied on four wires of same material then elongation in each wire depends on the length and diameter of the wire and given by $l \propto L/d^2$ and the ratio of L/d^2 is maximum for option (d).

6. We will use the impulse momentum theorem to find the force on the hand, which is equal in magnitude to the force on the board. Then the definition of shear stress will let us compute that quantity to see if it is large enough to break the board or not.

The impulse momentum theorem describes the force of the board on his hand:

$$Ft = mv_f - mv_i$$

$$F(0.002\,\text{s}) = (1.00\,\text{kg})\,(1.00\,\text{ms}^{-1} - 10.0\,\text{ms}^{-1})$$

and $F = -4500$ N

(i) Therefore, the force of his hand on board is 4500 N.

(ii) That force produces a shear stress on the area that is exposed when the board snaps:

$$\text{Stress} = \frac{F}{A} = \frac{4500\text{ N}}{(10^{-1}\text{ m})\,(10^{-2}\text{ m})} = 4.50 \times 10^6\ \text{Ns}^{-1}\text{m}^{-2}$$

(iii) Yes, this is larger than 3.60 MN m^{-2} and suffices to break the board.

We do not have the right combination of information to use the work–kinetic energy theorem to find the force. We could compute the acceleration of the hand and use Newton's second law to find the same 4.50 kN force but writing the impulse–momentum theorem is more direct. The area in stress = force/area is always the area of the layers of molecules between which the stress produces distortion (strain).

7. $\frac{F}{A} = Ye$

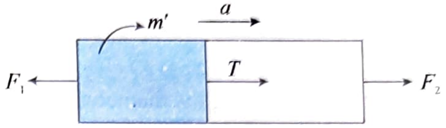
or $F = YeA = Ye\pi r^2$

$$= (9\times10^9)\left(\frac{0.1}{100}\right)\times3.14\times(2\times10^{-3})^2$$

$$= 9\times3.14\times4\times10^9\times10^{-3}\times10^{-6}$$

$$\simeq 1.13\times10^2\text{ N}$$

8. (i) Let T be the tension at a distance x as shown.

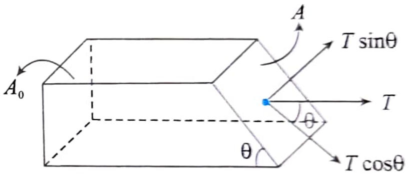


$$T - F_1 = m'a = \frac{m'(F_2 - F_1)}{m}$$

or $T - F_1 = (F_2 - F_1)\frac{x}{l}$

or $T = (F_2 - F_1)\frac{x}{l} + F_1$

(ii) At cross-section PQ,



$$p_{sh} = \frac{T\cos\theta}{A} = \frac{T\cos\theta}{A_0/\sin\theta} = \frac{T}{A_0}\sin\theta\cos\theta = \frac{T}{2A_0}\sin 2\theta$$

$$p_{\text{long}} = \frac{T\sin\theta}{A} = \frac{T\sin\theta}{\frac{A_0}{\sin\theta}} = \frac{T}{A_0}\sin^2\theta$$

(iii) p_{long} is maximum, when $\theta = 90°$

p_{sh} is maximum, when $\theta = 45°$

Exercise 7.2

1. (c) According to equation $\left[F = Y\left(\frac{\Delta L}{L_0}\right)A\right]$, the force F required to stretch a piece of material is proportional to Young's modulus Y, the amount of stretch ΔL, and the cross-sectional area A of the material, but is inversely proportional to the initial length L_0 of the material. Solving this equation for the amount of stretch gives $\Delta L = \frac{FL_0}{YA}$. Thus, the greater the cross-sectional area, the smaller is the amount of stretch, for given values of Young's modulus, the initial length, and the stretching force. Thus, B stretches more than A, because B has the smaller cross-sectional area of solid material.

2. (b) The Young's modulus depends upon the nature of the material and not on geometrical dimensions.

3. (a) $l = \frac{FL}{\pi r^2 Y}$

$$\Rightarrow \frac{l_2}{l_1}=\frac{F_2}{F_1}\frac{L_2}{L_1}\left(\frac{r_1}{r_2}\right)^2=2\times2\times\left(\frac{1}{2}\right)^2=1$$

$\therefore$ $l_2=l_1$ i.e. the increment in length will be same.

4. (d) $Y=\dfrac{FL}{\pi r^2 l}$ $\therefore$ $F=Y\pi r^2\dfrac{l}{L}$

$$\frac{F_A}{F_B}=\frac{Y_A}{Y_B}\left(\frac{r_A}{r_B}\right)^2\left(\frac{l_A}{l_B}\right)\left(\frac{L_B}{L_A}\right)=1\times\left(\frac{2}{1}\right)^2\times(1)\times\left(\frac{2}{1}\right)=8$$

5. (i) As $Y=\dfrac{FL}{A\Delta L}$,

$$\Delta L=\frac{FL}{AY}=\frac{FL}{(\pi D^2/4)Y}$$

where $A=(\pi D^2/4)$, D being the diameter of the cable,

or $\Delta L=\dfrac{4FL}{\pi D^2 Y}$

As $\Delta L\propto\dfrac{1}{D^2}$, if the diameter is doubled, the elongation becomes (1/4)th.

(ii) Further, as $F=\dfrac{\pi D^2 Y\Delta L}{4L}$, $F\propto D^2$.

Therefore, when the diameter is doubled, F becomes four times, i.e., the wire can support a load which is four times the previous value.

6. As $Y=\dfrac{FL}{A\Delta L}$, $\Delta L=\dfrac{FL}{YA}$

or $\Delta L\propto\dfrac{1}{Y}$ (as F, L and A are constants in this case)

Thus, $\dfrac{\Delta L_s}{\Delta L_c}=\dfrac{Y_c}{Y_s}=\dfrac{12\times10^{10}}{20\times10^{10}}=\dfrac{3}{5}$ ...(i)

Also, $\Delta L_s+\Delta L_c=1\text{ cm}$ ...(ii)

From Eqs. (i) and (ii),

$\Delta L_s=0.375\text{ cm}$, $\Delta L_c=0.625\text{ cm}$

7. Because the legs are vertical and share the load, the additional force on each femur is half of 600 N, i.e., 300 N.

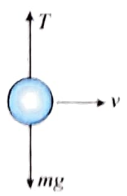
Here, $F=300\text{ N}$, $L=0.52\text{ m}$, $Y=7\times10^9\text{ N/m}^2$,

As $Y=\dfrac{FL}{A\Delta L}$

$$\Delta L=\frac{FL}{YA}=\frac{(300\text{ N})(0.52\text{ m})}{(7\times10^9\text{ N/m}^2)(7.5\times10^{-4}\text{ m}^2)}$$

$$=3\times10^{-5}\text{ m}=0.03\text{ mm}$$

8. The tension in wire when the mass is at the bottom of the circle,



$$T-mg=\frac{mv^2}{R}\Rightarrow T=mg+\frac{mv^2}{R}$$

$$\Rightarrow T=100\times10+\frac{100+5^2}{1}=3500\text{ N}$$

As $Y=\dfrac{FL}{A\Delta L}$

Here, $\Delta L=\dfrac{FL}{AY}=\dfrac{3500\times1}{(0.070\times10^{-4})(2\times10^{11})}$ $(\because F=T)$

$$=2.5\times10^{-3}\text{ m}=2.5\text{ mm}$$

9. Let L be the greatest length of Al wire that can hang without breaking.

Mass of the wire, $M=AL\rho$

$$\text{Stress}=\frac{Mg}{A}=\frac{(AL\rho)}{A}=L\rho g$$

As breaking stress $=7.5\times10^7\text{ N/m}^2$, $L\rho g=7.5\times10^7$

or $L=\dfrac{7.5\times10^7}{\rho g}=\dfrac{7.5\times10^7}{(2.7\times10^3)9.8}=2.8\times10^5\text{ cm}\approx28\text{m}$

Exercise 7.3

1. (d) Fluids can develop volumetric strain only.

2. (b) As $\eta=\dfrac{\text{shearing stress}}{\text{shearing strain}}$

and since liquids cannot sustain shearing stress

$\therefore$ $\eta=0$ in their case.

3. Here, $\Delta P=1.1\times10^8\text{ Pa}$, $V=0.32\text{ m}^3$,

$B=160\text{ GPa}=160\times10^9\text{ Pa}$

As $B=\dfrac{\Delta P\times V}{\Delta V}$, $\Delta V=\dfrac{\Delta P\times V}{B}$

$$\Delta V=\frac{(1.1\times10^8\text{ Pa})(0.32\text{ m}^3)}{160\times10^9\text{ Pa}}=2.24\times10^{-4}\text{ m}^3$$

4. Diameter of a rivet, $D=6\text{ mm}=6\times10^{-3}\text{ m}$

Cross sectional area of the rivet,

$$A=\frac{\pi D^2}{4}=\frac{3.142\times(6\times10^{-3}\text{ m})^2}{4}=28.26\times10^{-6}\text{ m}^2$$

Maximum shearing stress on the rivet,

$$\frac{F}{A}=6.9\times10^7\text{ Pa}=6.9\times10^7\text{ N/m}^2$$

Maximum force on the rivet, $F=\left(\dfrac{F}{A}\right)\times A$

$$=(6.9\times10^7\text{N/m}^2)(28.26\times10^{-6}\text{ m}^2)=1950\text{ N}$$

Since there are four rivets, maximum tension that can be exerted by the riveted strip = 4 × 1950 = 7800 N

5. As $\eta=\dfrac{F/A}{\Delta x/L}$

Shear strain, $\dfrac{\Delta x}{L}=\dfrac{F}{\eta A}=\dfrac{200\times2.8}{(2\times10^9)(49\times10^{-4})}=0.2$

$$\Delta x=0.2\times L=0.2\times7=1.4\text{ cm}$$

Exercise 7.4

1. Energy stored $=\dfrac{1}{2}$ (stretching force) (deformation)

$=\dfrac{1}{2}(10\times9.8)$ (compression)

Compression $=\Delta l=\dfrac{Mgl}{AY}=\dfrac{10\times9.8\times0.1}{10^{-4}\times7\times10^{10}}=10^{-6}\times1.4$

Energy stored $=\dfrac{1}{2}(10\times9.8)(1.4\times10^{-6})$

$=6.86\times10^{-5}$ joule

2. Work done, $W=\dfrac{1}{2}Fx$ (F is force, x is elongation)

Here elongation is same and $W\propto F$

But elongation is given by $x = \frac{Fl}{AY}$

As x and Y are constants $F \propto \frac{A}{l}$

If F_1 and F_2 are the forces in the first and second cases

$$\frac{F_1}{F_2} = \frac{A_1}{A_2} \times \frac{l_2}{l_1} = \frac{r_1^2}{r_2^2} \times \frac{l_2}{l_1}$$

$$= \left(\frac{r_1}{2r_1}\right)^2 \times \left(\frac{l_1/2}{l_1}\right) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

Then $\frac{W_1}{W_2} = \frac{F_1}{F_2} = \frac{1}{8} \Rightarrow W_2 = 8W_1 = 8 \times 4 = 32\text{ J}$

$U = \frac{1}{2}\text{stress} \times \text{strain} \times \text{volume}$

$$U = \frac{1}{2} \cdot \text{stress} \cdot \frac{\text{stress}}{Y} \cdot A \cdot L$$

$$U = \frac{1}{2}\frac{Mg}{A} \cdot \frac{Mg}{AY} \cdot A \cdot L$$

$$U = \frac{1}{2}\frac{M^2g^2L}{AY}$$

(i) Breaking stress, $\frac{F}{A} = 0.9 \times 10^3$,

(here, $A = 4\text{ cm}^2 = 4 \times 10^{-4}\text{ m}^2$)

Therefore, $F = (0.9 \times 10^8) \times (4 \times 10^{-4}) = 3.6 \times 10^4\text{ N}$

We have, $U = mgh = \frac{1}{2}F\Delta L$

or $\Delta L = \frac{2mgh}{F}$

$$\Rightarrow Y = \frac{FL}{A\Delta L} = \frac{F^2L}{A(2mgh)}$$

$$= \frac{(3.6 \times 10^4)^2 \times 1}{(4 \times 10^{-4})(2 \times 40 \times 9.8 \times 2)} = 2.1 \times 10^9\text{ N/m}^2$$

(L = length of two bones in a leg = 2 × 50 cm = 1 m)

(ii) As $F = k\Delta L$, $k = \frac{F}{\Delta L} = \frac{YA\Delta L/L}{\Delta L} = \frac{YA}{L}$

For a single bone, $L = 50\text{ cm} = 0.5\text{ m}$

Thus, $k = \frac{YA}{L} = \frac{(0.21 \times 10^{10})(4 \times 10^{-4})}{0.5} = 1.7 \times 10^6\text{ N/m}$

As $k = \frac{YA}{L} \Rightarrow \frac{k_s}{k_c} = \frac{Y_s}{Y_c}$ ...(i)

(as A and L are constants)

Also, $u = \frac{1}{2}k(\Delta L)^2$, $\frac{u_s}{u_c} = \frac{\frac{1}{2}k_s(\Delta L)^2}{\frac{1}{2}k_c(\Delta L)^2} = \frac{k_s}{k_c}$ ...(ii)

From Eqs. (i) and (ii): $\frac{u_s}{u_c} = \frac{Y_s}{Y_c}$

As $Y_s > Y_c, u_s > u_c$. Hence work done in steel spring will be more.

As $u = \frac{1}{2}k(\Delta L)^2$ and $F = k\,\Delta L$,

$$u = \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{F^2}{2l}$$

Clearly, $\frac{u_s}{u_c} = \frac{k_c}{k_s}$ (as F is constant)

But as $\frac{k_s}{k_c} = \frac{Y_s}{Y_c}$, $\frac{u_s}{u_c} = \frac{Y_c}{Y_s}$

As $Y_c < Y_s, u_s < u_c$ or $u_c > u_s$, hence work done in copper spring will be more.

Exercise 7.5

1. (b) In ductile materials, yield point exists while in brittle material, failure would occur without yielding.
2. (d) On the graph stress is represented on x-axis and strain on y-axis.

So, from the graph $Y = \cot\theta = \frac{1}{\tan\theta} \propto \frac{1}{\theta}$ [where θ is the angle from stress axis]

$\therefore Y_P < Y_Q < Y_R$ [As $\theta_P > \theta_Q > \theta_R$]

We can say that elasticity of wire P is minimum, and R is maximum.

3. (a) Young's modulus $Y = \frac{FL}{Al}$ $\therefore l \propto \frac{1}{A}$

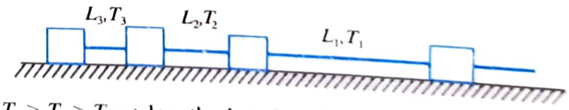
(As Y, L and F are constant)

From the graph, for same load elongation is minimum for curve OD. As elongation (l) is minimum, therefore area of cross-section (A) is maximum. So thickest wire is represented by OD.

4. (b) At point b, yielding of material starts.
5. (a) Area of hysteresis loop gives the energy loss in the process of stretching and unstretching of rubber band and this loss will appear in the form of heat.

Single Correct Answer Type

1. (2) Tension in any of the wire is same at all points as cross-sectional area are the same and tension also. So stress (= F/A) in both the wires would be the same but from Y = Stress/Strain as Y is different for both, so strain is different in both.
2. (2)



$T_1 > T_2 > T_3 \Rightarrow$ lengths $L_1 > L_2 > L_3$

3. (3) As already discussed in key concepts, if the cut is not very near to end of the bar, the internal developed force is distributed uniformly over the cross section.
4. (1) $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{x}{\frac{2l - l}{l}} = \frac{x}{\frac{l}{l}} = x$

Actually, the above expression is not exact for this much elongation.

5. (1) $Y = \frac{F/a}{\Delta l/l} = \frac{Fl}{a\Delta l}$

or $Y = \frac{Fl \times 4}{\pi D^2 \times \Delta l}$ or $\Delta l \propto \frac{1}{D^2}$ or $\frac{\Delta l_2}{\Delta l_1} = \frac{D_1^2}{D_2^2} = \frac{n^2}{1}$

6. (1) Breaking strength = Breaking stress × $\frac{\pi D^2}{4}$

Breaking stress is unchanged.

D is halved. So, breaking strength becomes one-fourth, i.e.,

$$\frac{1}{4}\times 1.5\times 10^5\ \text{N} \quad \text{or} \quad 0.375\times 10^5\ \text{N}$$

7. (4) $Y=\frac{F/a}{\Delta l/l}=\frac{Fl}{a\,\Delta l}$ or $\Delta l \propto \frac{l}{D^2}$

(1) $\frac{100}{1^2}=100$ (2) $\frac{200}{4}=50$

(3) $\frac{300}{9}=33.33$ (4) $\frac{50}{(1/2)^2}=200$

8. (1) Energy density $=\frac{1}{2}\times\text{Stress}\times\text{Strain}$

$$=\frac{1}{2}\times\text{Stress}\times\frac{\text{Stress}}{Y}=\frac{(\text{Stress})^2}{2Y}\propto\frac{1}{D^4}$$

Now, $\frac{u_A}{u_B}=\frac{D_B{}^4}{D_A{}^4}=[2]^4=16$

9. (3) $Y=\frac{Fl}{a\Delta l}$ or $\Delta l\propto\frac{1}{a}$

Again, $m=al\rho$ or $m\propto a$

$\therefore\ \Delta l\propto\frac{1}{m}$

$$\frac{\Delta l_1}{\Delta l_2}=\frac{m_2}{m_1}=\frac{2}{3}$$

10. (4) $Y=\frac{Fl}{a\,\Delta l}$

In the given problem, Y, l and Δl are constants.

$\therefore\ F\propto a$

or $F\propto\pi r^2$ or $F\propto r^2$ or $\frac{F_1}{F_2}=\frac{r_1{}^2}{r_2{}^2}=\frac{1}{4}$

11. (2) $Y=\frac{Mg\times 4\times l}{\pi D^2\times\Delta l}$ or $\Delta l\propto\frac{1}{D^2}$

When D is doubled, Δl becomes one-fourth, i.e.,

$\frac{1}{4}\times 2.4$ cm, i.e., 0.6 cm

12. (2) $Y=\frac{F/a}{\Delta l/l}$ or $Y=\frac{Fl}{a\Delta l}$

or $\Delta l=\frac{Fl}{aY}=\frac{Fl}{\pi r^2 Y}$

In the given problem, $\Delta l\propto\frac{l}{r^2}$

When both l and r are doubled, Δl is halved.

13. (4) 10 m column of water exerts nearly 1 atmosphere pressure. So, 100 m column of water exerts nearly 10 atmosphere pressure, i.e., 10×10^5 Pa or 10^6 Pa.

Now, $K=\frac{(\Delta P)V}{\Delta V}=\frac{10^6\times 100}{0.1}\ \text{Pa}=10^9\ \text{Pa}$

14. (2) Energy per unit volume $=\frac{1}{2}\times\text{Stress}\times\text{Strain}$

$$=\frac{1}{2}\times\text{Stress}\times\frac{\text{Stress}}{Y}=\frac{S^2}{2Y}\qquad\left[\because Y=\frac{\text{Stress}}{\text{Strain}}\right]$$

15. (3) Work done $=\frac{1}{2}F\times\text{Extension}$

$$=\frac{1}{2}\times\frac{YA}{L}\times 1 \qquad Y=\frac{F\times L}{A\times 1}$$

$$=\frac{YA}{2L} \qquad F=\frac{YA}{L}$$

16. (3) $\Delta l=\frac{Fl}{aY}$

$$\Delta l=\frac{2\times 10^5\times 10^{-5}\times l}{2\times 10^{-4}\times 10^4}\quad\text{or}\quad \Delta l=l$$

So, new length is $2l$.

17. (3) $Y=\frac{Fl}{a\Delta l}$ or $\Delta l\propto\frac{1}{a};\Delta l\propto\frac{1}{D^2}$

$$\frac{\Delta l_2}{\Delta l_1}=\frac{D_1{}^2}{D_2{}^2}=4 \quad\text{or}\quad \Delta l_2=4\Delta l_1=4\ \text{cm}$$

18. (3) $Y=\frac{Fl}{\pi r^2\Delta l}$ or $\Delta l=\frac{Fl}{\pi r^2 Y}$

$$\Delta l\propto\frac{l}{r^2},\quad \Delta l'\propto\frac{2l}{(\sqrt{2}r)^2}\quad\text{or}\quad \Delta l'\propto\frac{l}{r^2}$$

$\therefore\ \frac{\Delta l}{\Delta l'}=1$

19. (2) $\Delta l=\frac{Fl}{\pi r^2 Y}$ or $\frac{\Delta l}{l}\propto\frac{1}{r^2 Y}$

$$\frac{\left(\frac{\Delta l}{l}\right)_s}{\left(\frac{\Delta l}{l}\right)_{\text{Cu}}}=\frac{r_{\text{Cu}}{}^{2}{}_{\text{Cu}}}{r_s{}^2 Y_s}=\frac{4r_s{}^2\frac{Y_s}{2}}{r_s{}^2 Y_s}=2$$

$$\therefore\ \left(\frac{\Delta l}{l}\right)_s=2\left(\frac{\Delta l}{l}\right)_{\text{Cu}}=2\%$$

20. (3) Breaking stress $=\frac{\text{Maximum weight}}{\text{Area of cross section}}$

$$10^6=\frac{al\rho g}{a}=l\rho g$$

or $l=\frac{10^6}{\rho g}=\frac{10^6}{4\times 10^3\times 10}\ \text{m}=25\ \text{m}$

21. (2) Work done $=\frac{1}{2}\times$ stretching force × extension

Therefore, net work done in increasing the length from 0.164 mm to 1.02 mm is

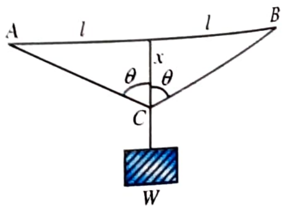
$$W_2-W_1=\frac{1}{2}\times 5\times 9.8\times 1.02\times 10^{-3}-\frac{1}{2}\times 3\times 9.8\times 0.61\times 10^{-3}$$

$$=0.016\ \text{J}$$

22. (3) $Y\propto$ weight applied

$$\therefore\ \frac{Y_1}{Y_2}=\frac{W_1}{W_2}\ \Rightarrow\frac{W_1}{W_2}=\frac{3}{1}$$

23. (3) Here change in length is



$$\Delta l=(AC+BC)-2l$$

$$=2(l^2+x^2)^{1/2}-2l$$

$$=2l\left(1+\frac{x^2}{l^2}\right)^{1/2}$$

$$=2l\left(1+\frac{1}{2}\frac{x^2}{l^2}\right)-2l=\frac{x^2}{l}$$

$$\therefore\ \text{Strain}=\frac{\Delta l}{2l}=\frac{x^2}{2l^2}$$

(4) We know $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{Wl}{2Ax} \times \frac{2l^2}{x^2}$

$$Y = \frac{Wl^3}{Ax^3} \text{ or } x = \left(\frac{W}{AY}\right)^{1/3} \times l$$

(1) $W = \frac{1}{2}\tau\phi = \frac{1}{2} \times 8 \times \frac{\pi}{4} = \pi$ J

(3) The density would increase by 0.1% if the volume decreases by 0.1%.

$$K = \frac{\Delta P}{\Delta V/V}$$

$$\Delta P = K\frac{\Delta V}{V} = 2 \times 10^9 \times \frac{0.1}{100} = 2 \times 10^6 \text{ N/m}^2$$

(2) $Y = \frac{F/A}{\Delta l/l}$

For the first wire, $Y = \frac{F \times L}{\pi r^2 \times l}$

For the second wire, $Y = \frac{2F \times 2L}{\pi (2r)^2 \times l'}$

From the above two equations, $l' = l$

(3) Force = Weight suspended + Weight of $3L/4$ of wire

$$= W_1 + \frac{3W}{4}$$

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \left(W_1 + \frac{3W}{4}\right)/s$$

(4) $Y = \frac{Fl}{a\Delta l}$

Y, l and F are constants.

$$\therefore \quad \Delta l \propto \frac{1}{D^2}$$

$$\frac{\Delta l_2}{\Delta l_1} = \frac{D_1^2}{D_2^2} = \frac{1}{16}$$

$$\therefore \quad \Delta l_2 = \frac{1}{16} \text{ mm}$$

(2) $Y = \frac{F/A}{\Delta l/l} = \frac{Fl}{\Delta l A} = \frac{Fl}{\Delta l\left(\pi R^2\right)}$

Hence $Y \propto \frac{1}{R^2}$

$$\frac{Y_B}{Y_S} = \frac{R_S^2}{R_B^2}$$

$$\frac{10 \times 10^{10}}{20 \times 10^{10}} = \frac{R_S^2}{R_B^2}$$

$$R_B^2 = 2R_S^2$$

$$R_B = \sqrt{2}R_S$$

$$\Rightarrow \quad R_S = \frac{R_B}{\sqrt{2}}$$

31. (3) $Y = \frac{Fl}{a\Delta l}$

Y, l and a are constants

$$\therefore \quad \frac{Fl}{\Delta l} = \text{constant or } \Delta l \propto F$$

Now, $l_1 - l = T_1$ and $l_2 - l = T_2$

Dividing, $\frac{l_1 - l}{l_2 - l} = \frac{T_1}{T_2}$

or $l_1T_2 - lT_2 = l_2T_1 - lT_1$ or $l(T_1 - T_2) = l_2T_1 - l_1T_2$

or $l = \frac{l_2T_1 - l_1T_2}{T_1 - T_2}$ or $l = \frac{l_1T_2 - l_2T_1}{T_2 - T_1}$

32. (2) $T = \frac{2m_1m_2}{m_1 + m_2}g = \frac{2 \times 1 \times 2}{1 + 2} \times 10 \text{ N} = \frac{40}{3}$ N

If r is the minimum radius, then

$$\text{Breaking stress} = \frac{\frac{40}{3}}{\pi r^2} \text{ or } \frac{40}{3\pi} \times 10^6 = \frac{40}{3\pi r^2}$$

or $r^2 = \frac{1}{10^6}$ or $r = \frac{1}{10^3}$ m

or $r = \frac{1}{10^3} \times 10^3$ mm = 1 mm

33. (2) The elastic potential is

$$\text{Energy} = \frac{1}{2}\text{ Stress} \times \text{Strain} = \frac{1}{2}Y(\text{strain})^2 = \frac{1}{2}Y\left(\frac{\Delta l}{L}\right)^2$$

$$\therefore \quad \frac{U_2}{U_1} \propto \left(\frac{\Delta l_2}{\Delta l_1}\right)^2 = \left(\frac{10}{2}\right)^2$$

$$\frac{U_2}{U_1} = 25 \Rightarrow U_2 = 25\,U_1 = 25\,U$$

34. (2) $\frac{1}{K} = \frac{\Delta V/V}{\Delta P}$ or $\frac{\Delta V}{V} = \Delta P\left(\frac{1}{K}\right)$

or $\frac{\Delta V}{V} \times 100 = 10^5 \times 8 \times 10^{-12} \times 100 = 8 \times 10^{-5}$ %

35. (1) $mr\omega^2$ = Breaking stress × Cross-sectional area

$$10 \times 0.3\omega^2 = 4.8 \times 10^7 \times 10^{-6} = 48$$

or $\omega^2 = \frac{48}{3} = 16$ or $\omega = 4$ rad/s

36. (2) $\Delta P = h\rho g = 200 \times 10^3 \times 10 \text{ N/m}^2 = 2 \times 10^6 \text{ N/m}^2$

$$K = \frac{\Delta P}{\frac{\Delta V}{V}} = \frac{2 \times 10^6}{\frac{0.1}{100}} = \frac{2 \times 10^8}{0.1} \text{ N/m}^2 = 2 \times 10^9 \text{ N/m}^2$$

37. (4) $\frac{20a \times 2.5 \times 10^3 \times 10 + 5 \times 10^5}{a} = 1.6 \times 10^6$

or $500a + 500 = 1600a$

or $1100a = 500$ or $a = \frac{5}{11} \text{ m}^2 = 0.45 \text{ m}^2$

38. (4) $W = \frac{1}{2}F\Delta l$

$$W = \frac{1}{2} \times \frac{Y\pi r^2\Delta l}{l}\Delta l \qquad \left(\because F = \frac{Y\pi r^2\Delta l}{l}\right)$$

or $W \propto \frac{r^2}{l}$, $W' \propto \frac{(2r)^2 2}{l}$

$$\frac{W'}{W} = 8 \quad \text{or} \quad W' = 8 \times 2\text{J} = 16\text{J}$$

39. **(3)** $m = al\rho$, m and ρ are constants.

$$\therefore \quad \frac{a_1}{a_2} = \frac{l_2}{l_1} = \frac{3}{2}$$

$$\text{Now, } Y = \frac{Fl}{a\,\Delta l} \quad \text{or} \quad \Delta l \propto \frac{l}{a}$$

$$\text{or} \quad \frac{\Delta l_1}{\Delta l_2} = \frac{l_1}{l_2} \times \frac{a_2}{a_1} = \frac{2}{3} \times \frac{2}{3} \quad \text{or} \quad \frac{\Delta l_1}{\Delta l_2} = \frac{4}{9}$$

40. **(3)** $Y = \dfrac{Fl}{a\,\Delta l}$; Y, l and a are constants

$\therefore \quad F = \Delta l$

In the first case, $V\rho g = 16$

In the second case, $(V\rho g - V \times 1 \times g) = 14$

$$\text{Dividing, } \frac{Vg(\rho - 1)}{Vg\rho} = \frac{14}{16} = \frac{7}{8}$$

$$\text{or} \quad \frac{\rho - 1}{\rho} = \frac{7}{8} \quad \text{or} \quad 8\rho - 8 = 7\rho \quad \text{or} \quad \rho = 8\ \text{g/cm}^3$$

41. **(1)** $Y = \dfrac{Fl}{a^2\Delta l}$

$$\Delta l \propto \frac{1}{a^2} \quad \text{or} \quad \Delta l \propto \frac{1}{D^4}$$

$$\frac{\Delta l_A}{\Delta l_B} = \frac{{D_B}^4}{{D_A}^4} = \frac{1^4}{\left(\frac{1}{2}\right)^4} = 16$$

42. **(1)** $Y = \dfrac{\text{stress}}{\Delta L / L}$ or $\dfrac{\Delta L}{L} = \dfrac{\text{stress}}{Y} = \dfrac{5 \times 10^7}{2 \times 10^{11}} = 2.5 \times 10^{-4}$

Now, $V = \pi r^2 L$

$$\frac{\Delta V}{V} = \frac{\pi \Delta(r^2 L)}{\pi r^2 L}$$

$$\text{or} \quad \frac{\Delta V}{V} = \frac{r^2 \Delta L + L \times 2r\Delta r}{r^2 L} \quad \text{or} \quad \frac{\Delta V}{V} = \frac{\Delta L}{L} + 2\frac{\Delta r}{r}$$

$$\text{or} \quad 2\frac{\Delta r}{r} = \frac{\Delta V}{V} - \frac{\Delta L}{L} \quad \text{or} \quad 2\frac{\Delta r}{r} = \frac{0.02}{100} - 2.5 \times 10^{-4}$$

$$\text{or} \quad \frac{\Delta r}{r} = 1 \times 10^{-4} - \frac{2.5}{2} \times 100^{-4} = -0.25 \times 10^{-4}$$

43. **(3)** $\dfrac{\rho'}{\rho} = \dfrac{\frac{M}{V - \Delta V}}{M / V} = \dfrac{V}{V - \Delta V}$

$$\text{or} \quad \frac{\rho'}{\rho} = V(V - \Delta V)^{-1} \quad \text{or} \quad \frac{\rho'}{\rho} = 1 + \frac{\Delta V}{V}$$

(using binomial theorem)

$$\text{or} \quad \frac{\Delta V}{V} = \frac{\rho'}{\rho} - 1 \quad \text{or} \quad \frac{\Delta V}{V} = \frac{\rho' - \rho}{\rho}$$

$$\text{Again, } K = \frac{\Delta P}{\frac{\Delta V}{V}} = \frac{\rho \Delta P}{\rho' - \rho} \quad \text{or} \quad \rho' - \rho = \frac{\rho \Delta P}{K}$$

But, $\Delta P = P$ (given)

$$\therefore \quad \rho' - \rho = \frac{\rho P}{K}$$

44. **(4)** $Y = \dfrac{Mg}{A} \times \dfrac{L/2}{\Delta L}$

(Length is taken as $L/2$ because weight acts at C.G.)

Now, $M = AL\rho$

(For the purpose of calculation of mass, the whole of geometrical length L is to be considered.)

$$\therefore \quad Y = \frac{AL\rho g L}{2A\Delta L}$$

$$\text{or} \quad \Delta L = \frac{\rho g L^2}{2Y} = \frac{1.5 \times 10^3 \times 10 \times 8 \times 8}{2 \times 5 \times 10^6}\ \text{m}$$

$$= 9.6 \times 10^{-2}\ \text{m} = 9.6 \times 10^{-2} \times 10^3\ \text{mm} = 96\ \text{mm}$$

45. **(2)** As the weight of wire acts at centre of gravity. Therefore, only half the length of wire gets extended.

$$\text{Now } Y = \frac{F}{A} \cdot \frac{(L/2)}{\Delta l} = \frac{Mg(L/2)}{A\,\Delta l}$$

$$\Rightarrow \quad \Delta l = \frac{MgL}{2AY} \quad \Rightarrow \Delta l = \frac{AL\rho g L}{2AY}$$

$$\therefore \quad \Delta l = \frac{\rho L^2 g}{2Y}$$

46. **(4)** All of the statements are factual statements. Stress is defined as internal force developed in the system per unit cross-sectional area and is not defined as force applied per unit cross-section area, although in equilibrium, stress is numerically equal to applied force per unit area.

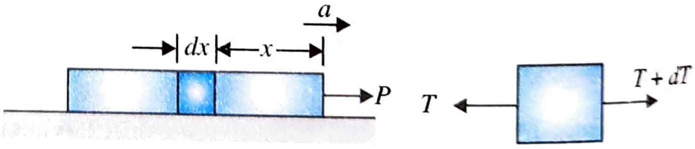
47. **(2)** If the rod is allowed to expand, then it will expand by $\Delta l = l\,\alpha\,\Delta T$ due to increase in temperature by ΔT. But as the length of the cylinder is kept constant by applying pressure, a stress is developed in the cylinder. The decrease in length of cylinder due to elasticity is $\Delta l = l\ \alpha\ \Delta T$ and a compressive stress will develop in it.

Stress = Excess pressure applied at ends

$$= Y \times \frac{\Delta l}{l} = Y\alpha\,\Delta T$$

$$= 2 \times 10^{11} \times 11 \times 10^{-6} \times 100 = 2.2 \times 10^3\ \text{atm}$$

48. **(3)** Let the force applied be P, then $P = ma$



$$\Rightarrow \quad P = 5 \times 2 = 10\ \text{N}$$

For the element shown in the figure, $T = \dfrac{m}{l}(l - x) \times a$

$$\Rightarrow \quad T = \frac{5}{1}(1 - x) \times 2 = 10(1 - x)$$

$$\text{Elongation in } dx \text{ is } \frac{\Delta(dx)}{dx} = \frac{T/A}{Y}$$

$$\Delta(dx) = \frac{10(1 - x)}{(5 \times 10^{-2})^2} \times \frac{1}{5 \times 10^9}\,dx$$

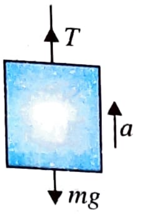
$$\text{Total elongation, } \Delta l = \int_0^1 \frac{10(1 - x)}{25 \times 10^{-4}} \times \frac{1}{5 \times 10^9}\,dx$$

$$= 0.4 \times 10^{-6}\ \text{m} = 0.4\ \mu\text{m}$$

(3) From the free-body diagram of the elevator,

$$T - mg = ma$$

$$T = m(g+a)$$



Stress in cable is $\alpha = \frac{T}{A} = \frac{m(g+a)}{A}$

From the given condition, $\alpha \le \frac{\alpha_{max}}{2}$

$$\frac{m(g+a)}{A} \le \frac{\sigma_{max}}{2}$$

$$g + a \le \frac{2\times10^9}{2} \times \frac{10^{-4}}{2\times10^3} = 50$$

$$a = 40 \text{ ms}^{-2}$$

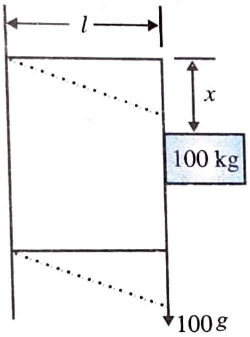
(2) The maximum load which a wire can sustain is a property of the material of wire and it depends on the cross-sectional area. $\sigma_{breaking}$ is a characteristic property of material of wire and is independent of length or breadth (dimensions) of the wire.

1. (1) $Y = \frac{F/A}{\Delta l/l}$

So $\frac{Y_s}{Y_c} = \frac{F\times l_s}{A_s\times\Delta l} \div \frac{F}{A_c}\times\frac{l_c}{\Delta l}$

$$\frac{A_c l_s}{A_s l_c} = \frac{4\times10^{-6}\times4.7}{3\times10^{-6}\times3.5} = 1.8$$

2. (2) The situation is as shown in the figure. A force of 100*g* acts on farther face of cube as shown, due to which shear will develop.



$$\eta = \frac{F/A}{x/l}$$

$$x = \frac{Fl}{A\eta} = \frac{100\times10\times l}{l^2\times25\times10^9}$$

$$x = 4\times10^{-7}\text{ m}$$

53. (2) Change in pressure due to placing of mass on prism is $\Delta p = \frac{Mg}{A}$.

From bulk modulus definition $K = \frac{-dp}{dV/V}$

$$\left|\frac{dV}{V}\right| = \frac{\Delta p}{K} = \frac{Mg}{AK}$$

From $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{V} = \frac{3dR}{R} \Rightarrow \frac{dR}{R} = \frac{1}{3}\frac{dV}{V} = \frac{Mg}{3AK}$$

54. (1) From the definition of bulk modulus $B = -\frac{dp}{dV/V}$

As we move from surface to place where pressure changes to αp_0, let us assume volume changes by ΔV, then

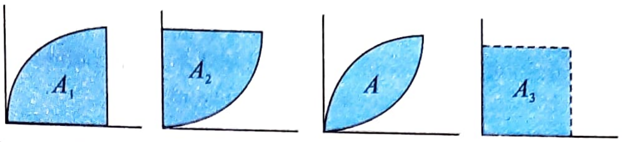
$$B = \frac{V\Delta p}{\Delta V} = \frac{V(\alpha-1)p_0}{\Delta V}$$

New volume, $V' = V - \Delta V = V\left[1 - \frac{(\alpha-1)p_0}{B}\right]$

Density at the given depth, $\rho' = \rho V/V'$, where ρ is density at surface

$$\rho' = \frac{\rho\times B}{B-(\alpha-1)p_0}$$

55. (4) Hystersis loss corresponding to elasticity per unit volume of a substance is given by the area of hysteresis loop, i.e., stress-strain curve corresponding to one complete loading and deloading.



Area of an ellipse = π × semi-major axis × semi-minor axis

$$A_1 = \frac{1}{4}(\pi\times8\times4\times10^2) \text{ and } A_2 = \frac{1}{4}(\pi\times8\times4\times10^2)$$

Also, $A_3 = 8\times4\times10^2$

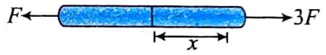
Area of hysteresis loop is $A = A_1 + A_2 - A_3$

$$A = 2\left[\frac{\pi}{4}\times8\times4\times10^2\right] - [8\times4\times10^2] = \left(\frac{\pi}{2}-1\right)\times32\times10^2 J$$

= work done per cycle

= energy lost per cycle per unit volume

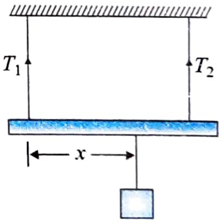
56. (1) Tension in rod at a distance *x* from right edge is



$$T = F\left(3 - 2\frac{x}{K}\right)$$

∴ Net extension in rod $= \int_0^L \frac{T}{4A}dx = \frac{2F}{YA}L$

57. (3) $\frac{T_1}{0.1} = \frac{T_2}{0.2}$ or $\frac{T_1}{T_2} = \frac{1}{2} = 0.5$



Further, $T_1x = T_2(2-x)$

or $\frac{T_1}{T_2} = \frac{2-x}{x}$

Hence, $\frac{2-x}{x} = 0.5 = \frac{1}{2}$ or $4 - 2x = x$

or, $x = \frac{4}{3}\text{m} = \frac{400}{3}\text{cm}$

58. (3)

(1) Bulk modulus is reciprocal of compressibility.

(2) Breaking force ∝ D^2.

Multiple Correct Answers Type

1. (1), (3)

The slope of the linear portion of the curve gives the Young's modulus of the material. The slope of the linear portion *OP* for material *A* is greater than that of the linear portion *OR* for material *B*. Hence, statement (1) is correct. The plastic region for material

A (from P to Q) is greater than that (from R to S) for material B, which indicates that material A is more ductile. Hence, statement (2) is incorrect. The breaking stress for material B (i.e., stress corresponding to point S) is less than that for material A (i.e., stress corresponding to point Q), which implies that material B can break more easily than material A. Thus, material B is more brittle. Material A is stronger than material B because it can withstand a greater stress before it breaks. The breaking stress is the stress corresponding to point Q for material A and to point S for material B. Hence, the correct choices are (1) and (3).

2. (1), (4)

We know that $Y = \dfrac{FL}{Al}$

$$\therefore \quad l = \frac{FL}{AY}$$

Since, the two wires are made of the same material, Young's modulus Y is same for both. Since, load F and the cross-sectional area A are the same,

$$\therefore \quad l \propto L$$

i.e., extension is proportional to the length of the wire. Hence, option (1) is correct. The strain in a wire is given by

$$\frac{l}{L} = \frac{4F}{AY}$$

Since F, A and Y are the same, the strains in wires A and B will be equal. Thus, the correct options are (1) and (4)

3. (1), (3)

Area of cross section, $A = \dfrac{\pi d^2}{4}$

where d is the diameter of the wire. Therefore, $l = \dfrac{4FL}{\pi d^2 Y}$

Since F, L and Y are the same for wires A and B

$$\therefore \quad l \propto \frac{1}{d^2}$$

i.e., the extension is inversely proportional to the square of the diameter. Hence, choice (1) is correct. The strain is

$$\frac{l}{L} = \frac{4F}{\pi d^2 Y}$$

Thus, strain $\propto \dfrac{1}{d^2}$

Hence, the correct choices are (1) and (3).

4. (2), (3)

Since, wires A and B are made of the same material they have the same Young's modulus. Now, breaking load = breaking stress × cross-sectional area.

The wire having a greater area of cross section can withstand a greater load. Hence, the correct options are (2) and (3)

5. (1), (4)

Statement (1) is correct because the Young's modulus of steel is greater than that of rubber.

Statement (2) is incorrect. If a spring is stretched, both the total length of the wire in the coil and the volume of the wire do not change. Only the shape of the coils of the wire undergoes a change. Hence, it is the shear modulus that determines the stretching of the coil.

Statement (3) is also incorrect. The bending moment of the prongs of a tuning fork is determined by the Young's modulus of the material. Hence, the restoring force on the prongs depends on Young's modulus which determines the frequency of the fork.

Statement (4) is correct. When the material is not subjected to any stress, its atoms are in their normal (equilibrium) positions. When a tensile stress is applied, the separation R between the atoms becomes greater than the equilibrium separation R_0. For $R > R_0$, the interatomic forces are attractive.

6. (1), (2)

(1) Within elastic limit, Stress = strain

(2) Beyond elastic limit, a small increase in stress produces large deformation, i.e., large strain. So,

$$\frac{\text{Stress}}{\text{Strain}} \text{ decreases.}$$

(3) Both the wires will have the same stress

$$\text{Again } Y = \frac{\text{Stress}}{\text{Strain}} \quad \text{or} \quad \text{Strain} = \frac{\text{Stress}}{Y}$$

Since Y is different, strain will be different. So, option (3) is not correct.

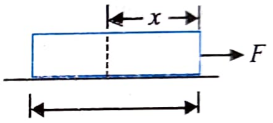
7. (1), (2), (3)

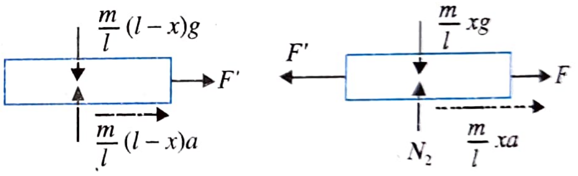
Even a small stress causes large strain.

In the case of a perfectly rigid body, strain is zero.

8. (2), (3)

Since, force is applied at one end only, the plank starts to accelerate along the direction of this force and a stress is developed in its material. Hence, option (3) is incorrect.





To calculate stress in the material of the plank, at a distance x from the end at which force is applied, the free-body diagrams are considered as shown in the figure.

$$\therefore \quad F' = \frac{m}{l}(l-x)a \qquad \text{...(i)}$$

$$F - F' = \frac{m}{l}xa \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$F' = \frac{F(l-x)}{l}$$

$$\therefore \quad \text{Stress} = \frac{F'}{A} = \frac{F(l-x)}{Al}$$

where A is cross-sectional area of the plank.

It shows that stress varies linearly with x. It is maximum at $x = 0$ and zero at $x = l$.

Hence, option (1) is correct.

9. (1), (3)

The compressive force in the rod will be same at left end therefore, compressive stress will be maximum at this end. Hence, option (2) alone is correct.

10. (1), (2)

Loss in gravitational potential energy of M is Mgl as M falls down by l.

Elastic potential energy stored in wire is

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$= \frac{1}{2} \times \frac{Mg}{A} \times \frac{l}{L} \times AL = \frac{Mgl}{2}$$

Now, work done by gravity force is not equal to elastic potential energy stored in wire. This is due to the fact that some work has been done against air friction etc., which increases the internal energy of wire.

(1), (3), (4)

In this question, the applied force is not constant. Let us consider at any instant the wire elongates by x. Then work done by F to elongate it more by dx is

$$dW = Fdx = \frac{YAx}{L}dx \qquad [x \ll L]$$

$$W = \int_0^l \frac{YAxdx}{L} = \frac{YAl^2}{2L}$$

As internal restoring force and applied force are almost same (equal and opposite), numerical value of work done by these two forces would be equal.

(1), (2), (4)

Here, for all three wires, length l, elongation Δl and cross-sectional area A are the same. From

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{T/A}{\Delta l/l}$$

$\Rightarrow \quad T \propto Y$

Let T_1, T_2 and T_3 be tension in three wires I, II and III, respectively.

For vertical equilibrium, $T_1 + T_2 + T_3 = 150$ g

For rotation equilibrium, $T_1 \times x - T_3 x = 0$

$\Rightarrow \quad T_1 = T_3 \quad$ so $\quad Y_1 = Y_3$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{T_2}{Y_1} = 2$$

$\Rightarrow \quad T_2 = 2T_1$

So, $\quad 2T_1 + T_2 = 150$ g

$\Rightarrow \quad T_2 = 75$ g

3. **(1), (2), (3)**

$$(\text{stress})_S = \frac{F}{2A}, \ (\text{stress})_{\text{Cu}} = \frac{F}{A}$$

Given that $\frac{Y_S}{Y_{\text{Cu}}} = \frac{2}{1}$

$$\frac{(\text{strain})_S}{(\text{strain})_{\text{Cu}}} = \frac{(\text{stress})_S/Y_S}{(\text{stress})_{\text{Cu}}/Y_{\text{Cu}}}$$

$$= \frac{(\text{stress})_S}{(\text{stress})_{\text{Cu}}} \times \frac{Y_{\text{Cu}}}{Y_S} = \frac{(F/2A)}{(F/A)} \times \frac{1}{2} = \frac{1}{4}$$

or $\quad \frac{L_S/\Delta L_S}{L_{\text{Cu}}/\Delta L_{\text{Cu}}} = \frac{1}{4} \quad$ or $\quad \frac{L_S \Delta L_{\text{Cu}}}{L_{\text{Cu}} \Delta L_S} = \frac{1}{4}$

or $\quad \frac{25}{50} \times \frac{\Delta L_{\text{Cu}}}{\Delta L_S} = \frac{1}{4} \quad$ or $\quad \frac{\Delta L_{\text{Cu}}}{\Delta L_S} = \frac{1}{2}$

So, options (1), (2) and (3) are correct.

14. (2), (3)

Thermal force $= YA\alpha d\theta = Y\pi r^2 \alpha d\theta$

$$r_1 = r, \quad r_2 = r\sqrt{2}, \quad r_3 = r\sqrt{3}, \quad r_4 = 2r$$

$$F_1 : F_2 : F_3 : F_4 = 1:2:3:4$$

Thermal stress $= Y\alpha d\theta$

As Y and α are same for all the rods, hence stress developed in each rod will be the same. As strain $= \alpha d\theta$, so strain will also be the same.

$$E = \text{Energy stored} = \frac{1}{2}Y(\text{strain})^2 \times A \times L$$

$\therefore \quad E_1 : E_2 : E_3 : E_4 = 1:2:3:4$

So, options (2) and (3) are correct.

15. (1), (2), (3)

Tension in $B = T_B = \frac{mg}{3}$

Tension in $A = T_A = T_B + mg = \frac{4mg}{3}$

$\therefore \quad T_A = 4T_B$

Stress, $S = \frac{T}{\pi r^2}$

Wire breaks when S is equal to breaking stress

For $\quad r_A = r_B, \quad S_A = 4S_B$

Therefore, A breaks before B

For $\quad r_A = 2r_B$

$$S_B = \frac{T_B}{\pi r_A^2}, \ S_A = \frac{T_A}{\pi r_A^2} = \frac{4T_B}{\pi(2r_B)^2} = \frac{T_B}{\pi r_B^2} = S_B$$

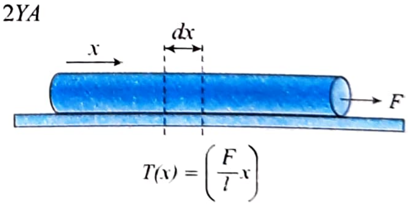
As the stresses are equal, either of the wires may break.

16. (2), (3)

Suppose $d\delta$ is increment in dx.

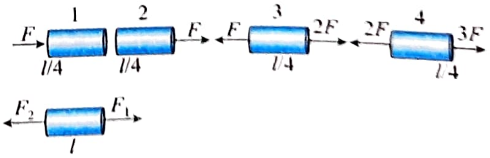
Then $\int d\delta = \frac{F}{YAl}\int_0^L xdx$

$$\delta = \frac{Fl}{2YA}$$



17. (1), (2), (3), (4)

In steady state, the FBD of the rod can be drawn by dividing it into four parts as shown



$$\gamma = \frac{\Delta l}{l} = (\text{stress})_{\text{average}} \Rightarrow \Delta l = \frac{l}{\gamma} \cdot \frac{(F_1 + F_2)}{2A}$$

Now, $\Delta l_{1(\text{compression})} = \frac{F\frac{l}{4}}{2A\gamma} = \frac{Fl}{8A\gamma}$;

$$\Delta l_{2(\text{elongation})} = \frac{Fl}{8A\gamma}$$

$$\Delta l_{3(\text{elongation})} = \frac{3Fl}{8A\gamma},\ \Delta l_{4(\text{elongation})} = \frac{5Fl}{8A\gamma}$$

$$\Delta l = \frac{8Fl}{8A\gamma} = \frac{Fl}{A\gamma}$$

Linked Comprehension Type

For Problems 1–3

1. (2)

2. (3) TY and NX are showing the plastic regions in A and B.

So $L_A > L_B$

Since A withstands more stress before yielding than B, $S_A > S_B$

So (3) is correct and the rest are incorrect.

3. (1) Time to regain original status in B is greater than the time to regain original status in A. So relatively B is more fatigued than A. So (1) is correct and the rest are incorrect. However, this answer can only be relative and never absolute.

For Problems 4–6

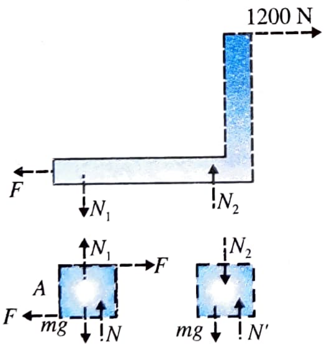
4. (2), 5. (1), 6. (3)

Here, first of all, draw the free-body diagram of the steel rods and the L-shaped structure as shown in the figure.

The direction of N_1 and N_2 can be found by using the equilibrium condition.

For L-shaped structure,

For vertical equilibrium, $N_1 = N_2$,



For horizontal equilibrium, $F = 1200$ N

For rotational equilibrium, torque about E (or any other point) = 0

$\Rightarrow\ N_2 \times 4 - 1200 \times 3 = 0$

$\Rightarrow\ N_2 = 900$ N

$$\text{Shearing stress in } A = \frac{F}{A} = \frac{1200}{5}\ \text{N/cm}^2 = 240\ \text{N/cm}^2$$

Shearing stress in $B = 0$

$$\text{Longitudinal stress in } A = \frac{N_1}{A} = 180\ \text{N/cm}^2$$

$$\text{Longitudinal stress in } B = \frac{N_2}{A} = 180\ \text{N/cm}^2$$

(Compressive in nature)

For Problems 7–9

7. (4) Young's modulus is a material constant. So, length and radius do not affect its value. So, ratio has to be 1:1.

8. (1), (3) On crossing the yield region, the material will experience the breaking stress and further elongation causes reduction in stress and breaking of the wire. So, options (1) and (3) are correct.

9. (2) Stress–strain curve flattens on crossing the elastic limiting region. So, (2) is correct.

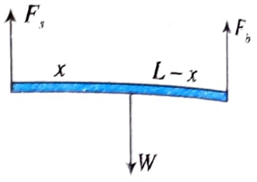
For Problems 10–11

10. (1) Let F_s and F_b be the forces in the wires A and B, respectively. The free-body diagram of the rod is shown in the figure.

Applying the condition of rotational equilibrium about the point of application of load, we get,

$$F_s x = F_b (L - x)$$

$$\frac{F_s}{F_b} = \frac{L - x}{x} \quad \text{...(i)}$$



Free body diagram of the rod

If the stresses are equal in two wires, we have

$$\frac{F_s}{F_b} = \frac{A_s}{A_b}$$

Here $A_s = 1 \times 10^{-5}$ m² and $A_b = 2 \times 10^{-5}$ m²

$$\frac{F_s}{F_b} = \frac{1}{2} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{L - x}{x} = \frac{1}{2} \quad \text{or} \quad x = \frac{2L}{3}$$

Since $L = 2$ m, $x = (4/3) = 1.33$ m

11. (1) For equal strains,

$$\in = \frac{\sigma}{Y} = \frac{F}{AY};\ \frac{F_s}{F_b} = \frac{1}{2}$$

$$\frac{F_s}{F_b} = \frac{A_s Y_s}{A_b Y_b} = \frac{(1\times10^{-5})\ (2\times10^{11})}{(2\times10^{-5})\ (1\times10^{11})} = \frac{1}{1} \quad \text{...(iii)}$$

Therefore, from Eqs. (i) and (iii), we get

$$x = \frac{L}{2} = \frac{2}{2} = 1\ \text{m}$$

For Problems 12–13

12. (1), 13. (2)

The restoring force (F) produced in the wire provides the necessary centripetal force i.e.

$$F = ML\,\omega^2 \quad \text{...(i)}$$

(since radius of circular path = length of wire = L)

We have, Young's modulus

$$Y = \frac{F/A}{\Delta L/L} \quad \text{or } F = YA\,.\,\frac{\Delta L}{L} \quad \text{...(ii)}$$

Comparing (i) and (ii), we get

$$YA\,.\,\frac{\Delta L}{L} = ML\omega^2 \quad \text{or} \quad Y = \frac{ML^2\omega^2}{A\Delta L}$$

Substituting given values

$$Y = \frac{(1\,\text{kg})\times(1\,\text{m})^2\times(20\ \text{rad/s})^2}{(10^{-6}\,\text{m}^2)(10^{-3}\,\text{m})} = 4\times10^{11}\ \text{N/m}^2$$

The breaking force, $F = ML\omega^2_{\text{max}}$

$= (1\text{kg})\ (1\text{m})\ (100\ \text{rad/s})^2 = 10^4$ newton

$$\therefore \text{Breaking stress} = \frac{\text{Breaking force}}{\text{Area}} = \frac{10^4}{10^{-6}} = 10^{10}\ \text{N/m}^2$$

For Problems 14–15

14. (3) $\Delta l = \frac{Fl_0}{YA} = \frac{(2000\text{ N})(20.0\text{ m})}{(2.0\times10^{11}\text{ Pa})(8.00\times10^{-4}\text{ m}^2)} = 2.5\times10^{-4}\text{ m}$

$= 0.25\text{ mm}$

15. (1) The force that each car exerts on the cable is

$$F = m\omega^2 l_0 = \frac{W}{g}\omega^2 l_0,\text{ so }\Delta l = \frac{Fl_0}{YA} = \frac{W\omega^2 l_0^2}{gYA}$$

$$= \frac{(2000\text{ N})\left(\frac{\sqrt{19}}{5}\text{ rad/s}\right)^2(20.0\text{m})^2}{(10\text{ m/s}^2)(2.0\times10^{11}\text{ Pa})(8.00\times10^{-4}\text{ m}^2)}$$

$$= 3.8\times10^{-4}\text{ m} = 0.38\text{ mm}$$

For Problems 16–17

16. (2), 17. (4)

Let tension in steel rod be T_1 and in brass rod be T_2, then

$$T_1 + 2T_2 = 5000 \qquad \text{...(i)}$$

Extension in both steel and brass rods will be same:

$$\Delta l_s = \Delta l_b$$

$$\frac{T_1 l_1}{AY_1} = \frac{T_2 l_2}{AY_2} \Rightarrow \frac{T_1\times30}{2\times10^{11}} = \frac{T_2\times20}{1\times10^{11}}$$

$$T_1 = \frac{4T_2}{3} \qquad \text{...(ii)}$$

From (i) and (ii), $T_1 = 2000$ N

$T_2 = 1500$ N

Stress in steel wire:

$$S_1 = \frac{T_1}{A} = \frac{2000}{10^{-4}} = 2\times10^7\text{ N/m}^2 = 20\text{ MPa}$$

Stress in brass wire:

$$S_2 = \frac{T_2}{A} = \frac{1500}{10^{-4}} = 1.5\times10^7\text{ N/m}^2 = 15\text{ MPa}$$

For Problems 18–19

18. (2) After loading the right wire, the zero of vernier scale coincide with 2nd division of the main scale. Hence the change in length of the right wire should be 0.20 cm. Hence,

$$\text{Strain produced in wire} = \frac{0.20\text{ cm}}{2.2\text{ m}} = 0.0010$$

19. (1) $Y = \frac{\frac{2g}{\pi\times(0.21\times10^{-3})^2}}{0.0010} = \frac{20\times10^9}{\pi\times0.21\times0.21}$

$$= \frac{2\times10^{11}}{\pi\times0.441} = 1.4\times10^{11}\text{ N/m}^2$$

Matrix Match Type

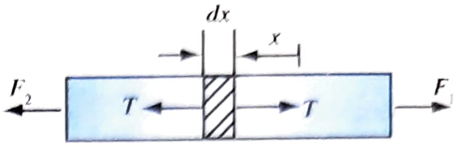
1. i.→a.,d.; ii.→a.,d.; iii.→a.,d.; iv.→a.,b.,c.
2. i.→a.,d.; ii.→b.,c.; iii.→a.,d.; iv.→a.,d.

   Directly from the theory. (Here, we assumed that in longitudinal and tensile stress that cubicle and parallelopiped are of the same shape, i.e., both are parallelopiped).
3. i.→a.,c.; ii.→a.,d.; iii.→b.,d.; iv.→b.,c.

   For A and B, the rod is in equilibrium and hence internal restoring force developed per unit area across any cross-section is same and stress developed in the rod is uniform, while in C and D, the case is reverse.
4. i.→a.,c.,d.; ii.→a.,c.,d.; iii.→a.; iv.→a.,b.,c.,d.

Numerical Value Type

1. (1) Consider an element of thickness dx. Change in the length of the element is $dl = \frac{T}{S}\frac{dx}{Y}$ and $T = F_1 - (F_1 - F_2)\frac{x}{l}$



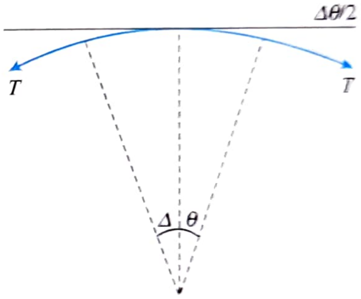
$$\int_0^{\Delta l} dl = \int_0^l \frac{F_1 - \frac{(F_1-F_2)x}{l}}{SY}dx$$

$$\Delta l = \frac{(F_1+F_2)}{2SY}l = \frac{200\times1}{2\times0.5\times2\times10^{11}}$$

$$= 1\times10^{-9}\text{ m}$$

$$n = 1$$

2. (2) $2T\sin\frac{\Delta\theta}{2} = dm\omega^2 r$



$$2T\left(\frac{\Delta\theta}{2}\right) = \rho\times A\times r^2\omega^2\Delta\theta$$

$$\sigma = \frac{T}{A} = \rho r^2\omega^2$$

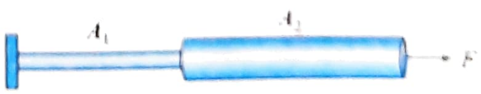
$$\therefore \quad \omega = \frac{1}{r}\sqrt{\frac{\sigma}{\rho}} = 2\text{ rad/s}$$

3. (3) $\Delta L_1 = \frac{F(L/2)}{AY} = \frac{FL}{2AY}$

$$\Delta L_2 = \frac{F(L/2)}{2AY} = \frac{FL}{4AY}$$

$$\text{Required ratio} = \frac{\Delta L_1 + \Delta L_2}{\Delta L_2} = \frac{\Delta L_1}{\Delta L_2} + 1 = 2 + 1 = 3$$

4. (8.00)



$$\frac{F}{A_1} = Y\frac{\Delta l_1}{l} \Rightarrow \Delta l_1 = \frac{Fl}{A_1Y}$$

$$\frac{F}{A_2} = Y\frac{\Delta l_2}{l} \Rightarrow \Delta l_2 = \frac{Fl}{A_2Y} = \frac{Fl}{4A_1Y}$$

$$\Delta l_1 + \Delta l_2 = 10\text{ mm}$$

$$\frac{Fl}{A_1Y} + \frac{Fl}{4A_1Y} = 10\text{ mm} \Rightarrow \frac{Fl}{A_1Y} = 8\text{ mm}$$

5. (8.00) $l = 5\times10^{-2}$ m, $\Delta l = 0.2$ cm $= 0.2\times10^{-2}$ m, $F = 8$ N

$$\text{Shearing strain } = \frac{\Delta l}{l} = \frac{0.2}{5} = 0.04$$

$$\text{Shearing stress } = \frac{F}{l\times l} = \frac{8}{(5\times10^{-2})^2} = 3200\ \text{Nm}^{-2}$$

$$\text{Modulus of rigidity, } \eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{3200}{0.04}$$

$$= 80000\ \text{Nm}^{-2} = 8\times10^4\,\text{N m}^{-2}$$

6. (2.00) We use the approximate from $S = F/(A\phi)$.

Putting $S = 8\times10^{10}$ N/m², $F = 4000$ N and $A = (0.25\text{ m})^2 = 6.25\times10^{-2}$ m² in the above equation and solving for ϕ, we get

$$\phi = \frac{(4000\text{ N})}{(6.25\times10^{-2}\text{ m}^2)(8\times10^{10}\text{ N/m}^2)} = 8.0\times10^{-7}\text{ rad}$$

The displacement of the upper surface is given by $d = L\phi$, where L is an edge of the cube.

$$\therefore\quad d = (8.0\times10^{-7})(25\text{ cm}) = 2.0\times10^{-5}\text{ cm}$$

7. (2.00) Shearing stress, $F/A = 10^6$ N/m²

Length of side, $L = 10$ cm $= 0.1$ m

Shearing displacement, $\Delta x = 0.05$ cm $= 0.005$ cm

The modulus of rigidity is

$$\eta = \frac{F}{A\theta} = \frac{FL}{A\Delta x} \qquad \left(\because \theta \cong \tan\theta = \frac{\Delta x}{L}\right)$$

$$= \frac{10^6\times0.1}{0.005} = 2\times10^8\text{ N/m}^2$$

8. (1025) The density of water at the ocean's surface where the pressure is 1 atm is about 1000 kg/m³. Thus, 1 m³ of water at the surface has a mass of 1000 kg. Let us calculate the change in volume ΔV of this water when it is at a pressure of 500 atm.

$$\Delta V = \frac{(\Delta p)V}{K}$$

The pressure change Δp is 500 atm – 1 atm = 500 atm, which is equivalent to 500 × 10⁵ N m⁻². Thus, the volume change ΔV is

$$\therefore\quad \Delta V = \frac{(500\times10^5\text{ N/m}^2)(1\text{ m}^3)}{2.0\times10^9\text{ N/m}^2} = 0.025\text{ m}^3$$

The density of the water is then

$$\rho = \frac{m}{V-\Delta V} = \frac{1000\text{ kg}}{1.00\text{ m}^3 - 0.025\text{ m}^3}$$

$$= \frac{1000\text{ kg}}{0.975\text{ m}^3} = 1025\text{ kgm}^{-3}$$

9. (32.00) $B = -\dfrac{\Delta p}{\Delta V/V_0} = -\dfrac{(0.25\text{ m}^3)(1.6\times10^7\text{ Pa})}{5.0\times10^9\text{ Pa}}$

$$= -8.0\times10^{-4}\text{ m}^3 = -0.80\text{ L}$$

Even through the pressure increase is very large, the fractional change in volume is very small.

$$\therefore\ \Delta V/V_0 = (-8.0\times10^{-4}\text{ m}^3)/(0.25\text{ m}^3) = -0.0032 \text{ or } -0.32\%$$

10. (0.75) The compressibility K is the reciprocal of the bulk modulus.

$$\therefore\quad \Delta V = -VK\Delta p$$

$$= -(100\text{ mL})(5\times10^{-10}\text{m}^2/\text{N})(15\times10^6\text{ N/m}^2)$$

$$= -0.75\text{ mL}$$

The decrease in volume is thus 0.75 mL.

11. (100.0) $K = \dfrac{(\Delta P)V}{\Delta V}$ or $\Delta P = \dfrac{K\Delta V}{V}$

Let h be the required depth and the density of water be ρ. Then

$$h\rho g = \frac{K\Delta V}{V}$$

$$\text{or}\quad h = \frac{K\Delta V}{\rho g V} = \frac{9.8\times10^8\times0.1}{10^3\times9.8\times100} = 100\text{ m}$$

12. (1.25) If T_b and T_s are tension developed in the steel and the brass rod then

$$T_b(2.5L - x) = T_s\cdot x \qquad \text{...(i)}$$

$$\text{and}\quad T_b + T_s = F \qquad \text{...(ii)}$$

Solving (i) and (ii) we get $T_b = \dfrac{xF}{2.5L}$

$$\text{and}\quad T_s = \left(1 - \frac{x}{2.5L}\right)F$$

For the bar to remain horizontal, extension in the two rods must be same.

$$\therefore\quad \frac{T_b(2L)}{4AY_B} = \frac{T_sL}{AY_s} \qquad [\text{where } A = \pi r^2]$$

$$\frac{T_s}{T_b} = \frac{Y_s}{2Y_B}$$

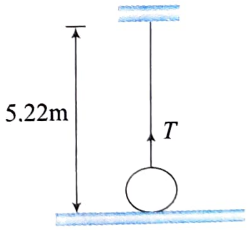
$$\Rightarrow\quad \frac{2.5L - x}{x} = 1 \Rightarrow x = 1.25L$$

$$\text{Hence}\quad \frac{x}{L} = 1.25$$

13. (8.76) Let ΔL be the extension of wire when the sphere is at mean position. Then,

$$L + \Delta L + 2r = 5.22$$

$$\Delta L = 5.22 - L - 2r = 5.22 - 5 - 2(0.1) = 0.02\text{ m}$$



Let T be the tension in the wire at mean position during oscillations. Then,

$$Y = \frac{T/A}{\Delta L/L}$$

$$\Rightarrow\quad T = \frac{YA\Delta L}{L} = \frac{Y(\pi r^2)\Delta L}{L}$$

$$\text{or}\quad T = \frac{(2\times10^{11})(\pi)(0.5\times10^{-3})^2\times0.02}{5} = 200\pi$$

Also, at mean position

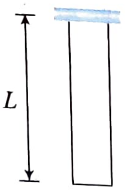
$$T - mg = \frac{mv^2}{R}$$

(here, $R = 5.22 - r = 5.22 - 0.1 = 5.12$ m)

$$\therefore\quad (200\pi) - (8\pi)(10) = \frac{(8\pi)v^2}{5.12}$$

$$\Rightarrow\quad v^2 = 5\times20.48 \Rightarrow v = 8.76$$

(100) If L is the maximum length of the wire that can be suspended. Then, $F = mg$, where $m = \rho LA$ (where, A = cross-sectional area)



$\therefore$ Maximum stress $= \dfrac{mg}{A} = g\cdot\dfrac{\rho LA}{A} = \rho Lg$

Given, breaking stress $\rho_b = 7.8\times10^9\ \text{Nm}^{-2}$

$$\therefore\quad \rho_b = 7.8\times10^9\ \text{Nm}^{-2}$$

$$\Rightarrow\quad L = 7.8\times10^9\times\frac{1}{7800\times10}$$

or $\quad L = 10^5\ \text{m} = 100\ \text{km}$

(1.36) The increase in pressure is $\Delta p = \rho gh$

or $\quad \Delta p = 10^3\times9.8\times3000 = 29.4\times10^6\ \text{Nm}^{-2}$

The volume strain $= \dfrac{\Delta V}{V} = \dfrac{\Delta p}{B} = \dfrac{29.4\times10^6}{2.2\times10^9} = 1.36$

(0.10) Increase in pressure is $\Delta p = \dfrac{Mg}{A}$

Bulk modulus is $B = \dfrac{\Delta p}{(\Delta V/V)}$

$$\therefore\quad \frac{\Delta V}{V} = \frac{\Delta p}{B} = \frac{Mg}{AB} \qquad \ldots\text{(i)}$$

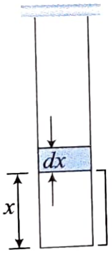
Also, the volume of the sphere is $V = \dfrac{4}{3}\pi R^3$

$$\Rightarrow\quad \frac{\Delta V}{V} = \frac{3\Delta R}{R}$$

or $\quad \dfrac{\Delta R}{R} = \dfrac{1}{3}\cdot\dfrac{\Delta V}{V}$, using Eq. (i) we get

$$\frac{\Delta R}{R} = \frac{Mg}{3AB} = \frac{0.3}{3} = 0.1$$

17. (2.00) Consider an element as shown in the figure.



Stress in the element $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{xA\rho g}{A} = x\rho g$

Now, electric potential energy stored in the wire is

$$dU = \frac{1}{2}\ \text{(stress) (strain) (volume)}$$

$$= \frac{1}{2}\cdot\frac{(\text{stress})^2}{Y}(\text{volume})$$

$$dU = \frac{1}{2}\cdot\frac{(x\rho g)^2}{Y}Adx = \frac{1}{2}\cdot\frac{\rho^2g^2A}{Y}x^2dx$$

Total electric potential energy $= \dfrac{1}{2}\cdot\dfrac{\rho^2g^2A}{Y}\displaystyle\int_0^L x^2dx = \dfrac{\rho^2g^2AL^3}{6Y}$

$$= \frac{12\times10^{-6}}{6} = 2\times10^{-6}\ \text{J}$$

18. (24.00) Energy per unit volume $= \dfrac{1}{2}$(stress)(strain)

$$u = \frac{U}{V} = \frac{1}{2}(\text{stress})\left(\frac{\text{stress}}{\text{Bulk modulus}}\right)$$

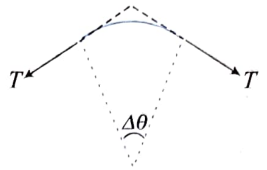
$$= \frac{1}{2}(\rho gh)^2\ \text{(Compressibility)}$$

$$= \frac{1}{2}(1000\times9.8\times1000)^2(5\times10^{-10})$$

$$= 2.4\times10^4\ \text{J m}^{-3} = 24\times10^3\ \text{J m}^{-3}$$

19. (3.00) $2T\sin\dfrac{\Delta\theta}{2} = dm\omega^2r \Rightarrow 2T\dfrac{\Delta\theta}{2} = \rho Ar\Delta\theta\omega^2r$

$$\Rightarrow\quad \sigma = \frac{T}{A} = \rho r^2\omega^2 \Rightarrow \omega = \frac{1}{r}\sqrt{\frac{\sigma}{\rho}} = 3$$



Archives

JEE Main

Single Correct Answer Type

1. (4) $A_1l_1 = A_2l_2$

$$\Rightarrow\quad l_2 = \frac{A_1l_1}{A_2} = \frac{A\times l_1}{3A}$$

$$\Rightarrow\quad \frac{l_1}{l_2} = 3$$

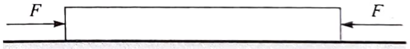
$$\Delta x_1 = \frac{F_1}{A\gamma}\times l_1$$

$$\Rightarrow\quad x_2 = \frac{F_2}{3A\gamma}\times l_1$$

Here $\Delta x_1 = \Delta x_2$

$$\frac{F_2}{3A\gamma}l_2 = \frac{F_1}{A\gamma}l_1 \quad\Rightarrow\quad F_2 = 3F_1\times\frac{l_1}{l_2} = 3F_1\times3 = 9F$$

2. (3) $0.10\times1.1\times10^{-5} = \dfrac{\frac{F}{A}}{2\times10^{11}}\times0.10$



$$\therefore\quad \frac{F}{A} = \text{Pressure} = 1.1\times10^{-5}\times100\times2\times10^{11}$$

$$= 2.2\times10^8\ \text{Pa}$$

3. (1) $T = 2\pi\sqrt{\dfrac{l}{g}} \qquad \ldots\text{(i)}$

$$T_M = 2\pi\sqrt{\frac{l+\Delta l}{g}} \qquad \ldots\text{(ii)}$$

$$Y = \frac{Fl}{A\Delta l} \Rightarrow \Delta l = \frac{Mgl}{AY} \qquad \ldots\text{(iii)}$$

$$\Rightarrow\quad \frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

4. (3) $\dfrac{V_f}{V_i} = 9^3$

$\because$ Density remains same

So, mass ∝ volume

$$\frac{m_f}{m_i} = 9^3 \text{ and } \frac{(\text{Area})_f}{(\text{Area})_i} = 9^2$$

$$\text{Stress} = \frac{(\text{Mass}) \times g}{\text{Area}}$$

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{m_f}{m_i}\right)\left(\frac{A_i}{A_f}\right) = \frac{9^3}{9^2} = 9$$

5. (4) Bulk Modulus$=\dfrac{\text{volumetric stress}}{\text{volumetric strain}}$

$$K = \frac{mg/a}{dV/V} \Rightarrow \frac{dV}{V} = \frac{mg}{Ka} \quad \text{...(i)}$$

volume of sphere $V = \frac{4}{3}\pi r^3$

Fractional change in volume $\dfrac{dV}{V} = \dfrac{3dr}{r}$ ...(ii)

Using Eqs. (i) and (ii), $\dfrac{3dr}{r} = \dfrac{mg}{Ka}$ hence $\dfrac{dr}{r} = \dfrac{mg}{3Ka}$

JEE Advanced

Multiple Correct Answers Type

1. (1), (2)

As (Breaking stress)$_P$ > (Breaking Stress)$_Q$

⇒ P is more tensile than Q.

We can increase the length of P by a larger amount than Q

⇒ P is more ductile than Q.

⇒ P is less brittle than Q.

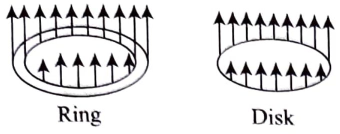
By checking the value of $\dfrac{\text{Stress}}{\text{Strain}}$ at any particular value of strain

⇒ $Y_Q > Y_P$

Concept Application Exercise

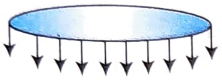
ercise 8.1

. (c)



Ring have double surface than that of disk.

. Force due to surface tension acts all along the circumference of the circular plate. Therefore, force required to take away the plate.



$T \times 2\pi R$

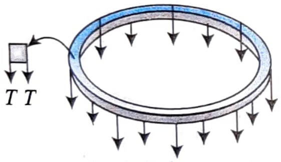
Force due to surface tension $F = T.\Delta L = T(2\pi R)$

$= (0.70 \text{N/m})(2 \times \pi \times 0.10 \text{m}) = 0.44 \text{ N}$

Hence force required to lift the plate = Force due to surface tension $= 0.44$ N

. Force due to surface tension $F = T.\Delta L = T \times 2(2\pi R)$

$= (0.70\text{N/m})2 \times (2 \times \pi \times 0.10\text{m}) = 0.88$ N

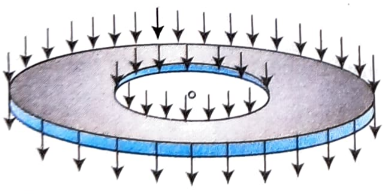


$T \times 2(2\pi R) = 4T\pi R$

Hence force required to lift the plate = Force due to surface tension $= 0.88$ N

4. Force due to surface tension, $F = T \times 2\pi(r + R)$

$= (0.70\text{N/m})(2 \times \pi \times (0.10 + 0.20)\text{m}) = 1.32$ N



$F_{\text{surface tension}} = T \times (2\pi R) + t(2\pi r)$

Hence force required to lift the plate = Force due to surface tension = 1.32 N

5. The weight supported in cases (b) and (c) is also 5.0×10^{-2} N. This is due to the reason that the weight is supported by the force (F) of surface tension (T) where $F \propto l$, l being the length of the film. Since T and l are the same, F is also the same in all the cases.

6. When the ring is about to leave the water surface, surface tension force on it is

$$F_{ST} = 2\pi RT + 2\pi rT = 2\pi(R + r)T$$

Spring force $F_s = kx$

$$\therefore \quad kx = 2\pi(R + r)T + mg$$

$$\therefore \quad T = \frac{kx - mg}{2\pi(R + r)}$$

7. The glass plate is held inside water as shown in figure. Here, $l = 50$ cm; $b = 40$ cm and $t = 5$ cm; $T = 50 \times 10^{-3}$ N/m.

The glass plate is under the action of the following forces

(i) weight mg acting vertically downwards,

(ii) force due to surface tension acting vertically downwards and

(iii) upthrust experienced due to the half immersed part of the plate.

Thus, apparent weight of the glass plate,

$$mg' = mg + 2(l + t) \times T - \left(l \times t \times \frac{b}{2}\right)\rho g$$

$$= 3 \times 10 + 2(50 + 5) \times 10^{-2} \times 0.5 - \left(50 \times 5 \times \frac{40}{2}\right) \times 10^{-6} \times 10^{3} \times 10$$

$$= 30 + 0.55 - 25 = 5.55 \text{N}$$

Exercise 8.2

1. (a) When two droplets merge with each other, their surface energy decreases.

$$W = T(\Delta A) = (\text{negative}) \text{ i.e. energy is released.}$$

2. (c) Work done to form a bubble of radius R

$$W_1 = 8\pi R^2 T_1$$

Work done to form a bubble of radius $2R$

$$W_2 = 8\pi(2R)^2 T_2 = 32\pi R^2 T_2 \quad \therefore \quad \frac{W_1}{W_2} = \frac{T_1}{4T_2}$$

If surface tension of soap solution is same then

$$W_2 = 4W_1$$

But in the problem temperature of solution is increased so its surface tension decreases.

$$\therefore \quad W_2 < 4W_1$$

3. Work done to form a soap bubble

$$W = 8\pi R^2 T \qquad (\text{As } V \propto R^3 \therefore R \propto V^{1/3})$$

$$\therefore \quad W \propto V^{2/3}$$

$$\frac{W_2}{W_1} = \left(\frac{V_2}{V_1}\right)^{2/3} = (2)^{2/3} \Rightarrow W_2 = (4)^{1/3} W$$

4. Here, $T = 30$ dyne cm^{-1}; initial surface area of the soap bubble = 0.50 cm^2; final surface area of the soap bubble = 1.10 cm^2. Since the soap bubble has two free surfaces, effective increase in the surface area of the soap bubble,

$$A = 2 \times (1.10 - 0.50) = 1.20 \text{ cm}^2$$

The work done in blowing up the soap bubble to the new size,

$$W = T \times A = 30 \times 1.20 = 36 \text{ erg}$$

5. Let r be the radius of each of the tiny drops. Clearly, volume of 27 tiny drops = volume of the liquid drop of diameter D

$$\text{or} \quad 27 \times \frac{4\pi}{3} r^3 = \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = \frac{4\pi}{3} \times \frac{D^3}{8}$$

$$\text{or} \quad r^3 = \frac{D^3}{8 \times 27} \Rightarrow r = \frac{D}{2 \times 3} = \frac{D}{6}$$

Original surface area of the drop $= 4\pi\left(\frac{D}{2}\right)^2 = \pi D^2$

Final surface area of 27 droplets $= 27 \times 4\pi r^2$

$$= 27 \times 4\pi\left(\frac{D}{6}\right)^2 = 3\pi D^2$$

Increase in surface area, $\Delta A = 3\pi D^2 - \pi D^2 = 2\pi D^2$

Increase in potential energy, $W = T\Delta A = 2\pi D^2 T$

Exercise 8.3

1. Pressure difference on two sides of a curved surface is inversely proportional to the radius of curvature.

2. $p + \rho gh = \dfrac{2T}{R} + \rho gh = \dfrac{2\times 0.075}{1.0\times 10^{-3}} + (1000\times 10\times 0.1)$

   Or $P = 150 + 1000 = 1150$ Pa

3. Here, $\dfrac{r}{r'} = \dfrac{1}{2}$

   Now, excess of pressure inside a bubble,

$$p_i - p_0 = \frac{4T}{r}$$

$$\frac{p_i - p_0}{p_i' - p_0} = \frac{4T}{r}\times\frac{r'}{4T} = \frac{r'}{r} = \frac{1}{1/2} = \frac{2}{1} \text{ i.e., } 2:1$$

   Also, work done in blowing a bubble of radius r,

$$W = T\times 4\pi r^2$$

$$\therefore \quad \frac{W}{W'} = \frac{T\times 4\pi r^2}{T\times 4\pi r'^2} = \left(\frac{r}{r'}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \text{ i.e., } 1:4$$

4. Since the drag force balances the gravity, the necessary condition must be that the variation in hydrostatic pressure inside the drop should be negligible compared to the excess pressure due to surface tension

$$\rho g 2R << \frac{2T}{R} \Rightarrow R << \sqrt{\frac{T}{\rho g}}$$

5. Let P_1 and P_2 be pressures in the tubes 1 and 2 just above the meniscus. Then, at the points just below the meniscus of the liquid in the two tubes, the pressures will be

$$P_1' = P_1 - \frac{2T}{R_1} \text{ and } P_2' = P_2 - \frac{2T}{R_2}$$

   Here, R_1 and R_2 are radii of the meniscus of the liquid in the two tubes.

   As shown in Fig. (b), the radius (R) of the meniscus of liquid, the radius (r) of the tube and the angle of contact (θ) are related to each other as

$$\frac{r}{R} = \cos\theta \text{ or } R = \frac{r}{\cos\theta}$$

$$R_1 = \frac{r_1}{\cos\theta} \text{ and } R_2 = \frac{r_2}{\cos\theta}$$

$$P_1' = P_1 - \frac{2T\cos\theta}{r_1} \text{ and } P_2' = P_2 - \frac{2T\cos\theta}{r_2}$$

   Hence, $P_2' - P_1'$ = pressure exerted by height h of the water column

$$\text{Or} \quad \left(P_2 - \frac{2T\cos\theta}{r_2}\right) - \left(P_1 - \frac{2T\cos\theta}{r_1}\right) = h\rho g$$

   or true pressure difference,

$$P_2 - P_1 = h\rho g - 2T\cos\theta\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

   On substituting the corresponding values we get

$$P_2 - P_1 = 1850 \text{ N/m}^2$$

Exercise 8.4

1. (d) If radius of capillary is reduced to half, the rise of liquid column will be two times as $h \propto 1/r$.

2. (a) $h = \dfrac{2T}{Rdg}$

$$hR = \frac{2T}{dg} = \text{constant}$$

   When h decreases, R increases.

3. Here, $T = 72$ dyne cm^{-1};

   length of the tube above the surface of water,

$$H = 80 \text{ mm} = 8 \text{ cm}$$

   Now, $h = \dfrac{2T\cos\theta}{r\rho g}$

   For water : $\theta = 0°$ and $\rho = 1$ g cm^{-1}

$$h = \frac{2\times 75\times\cos 0°}{0.03\times 1\times 1000} = 5.0 \text{ cm}$$

   When the tube is lowered:

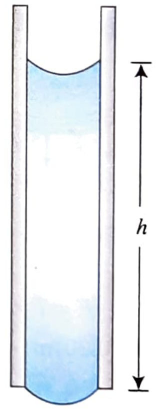
   Length of the tube above the surface of water,

$$H' = 30 \text{ mm} = 3 \text{ cm}$$

   Since $h > H'$, it follows that tube is of insufficient length. The water will rise upto the top end of the tube. However, it will not overflow. It will change the radius of curvature of the meniscus from r to R, where $RH' = rh$

$$\text{or} \quad R = \frac{rh}{H'} = \frac{0.03\times 5.0}{3} = 0.05 \text{ cm}$$

4. When a capillary tube filled with water is placed vertically, a column of water of length h will be held by the upper and the lower meniscus of water as shown in figure.



   If r is radius of the tube, then weight of the water in the tube,

$$mg = \pi r^2 h\rho g$$

   The weight of the liquid is supported by the force due to surface tension in the two menisci i.e.

$$F = 2\pi r T + 2\pi r T = 4\pi r T,$$

   where T is surface tension of the water.

   In equilibrium, $mg = F$

   i.e. $\pi r^2 h\rho g = 4\pi r T$

$$\text{or} \quad h = \frac{4T}{r\rho g} = \frac{4\times 72}{0.1\times 1\times 980} = 2.94 \text{ cm}$$

5. Let $r_1 = r$. Then, $r_2 = r/3$

   Here, $h_1 = 2$ cm

   Let h_2 be the height, to which liquid rises in the second tube

   Now, $h = \dfrac{2T\cos\theta}{r\rho g}$ or $h \propto \dfrac{1}{r}$

$$\frac{h_1}{h_2} = \frac{r_2}{r_1} \text{ or } h_2 = \frac{r_1 h_1}{r_2}$$

$$\Rightarrow h_2 = \frac{r \times 2}{(r/3)} = 6 \text{ cm}$$

When the capillary tube is inclined: In the ascent formula, h refers to the vertical height of the liquid in the capillary tube. If the tube is inclined at an angle θ with the vertical, then length of liquid column in the tube,

$$L = \frac{h}{\cos\theta} = \frac{6}{\cos 60°} = \frac{6}{0.5} = 12 \text{ cm}$$

Exercise 8.5

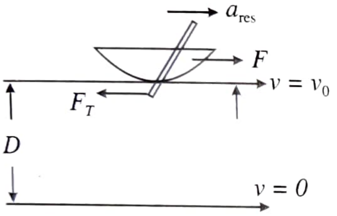
1. (d) A longer pipe offers a greater resistance to the flow of a viscous fluid than a shorter pipe does. The volume flow rate depends inversely on the length of the pipe. For a given pipe radius and pressure difference between the ends of the pipe, the volume flow rate is less in longer pipes. In this case the longer pipe is twice as long, so its volume flow rate Q_B is one-half that of Q_A.

2. $F - F_T = ma_{res}$

As boat moves with constant velocity $a_{res} = 0$

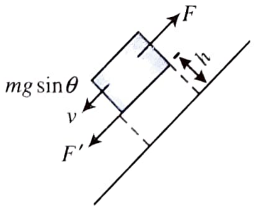
$$F = F_T$$

We know $F_T = \eta A \frac{dv}{dy}$, but $\frac{dv}{dy} = \frac{v_0 - 0}{D} = \frac{v_0}{D}$



then $F = F_T = \frac{\eta A v_0}{D}$

3. $F = F' = \eta A \frac{dv}{dy} = mg \sin\theta$ $\left[\frac{dv}{dy} = \frac{v}{h}\right]$



$$20 \times 10 \times \sin 30° = \eta \times 4 \times \frac{10}{h}$$

$$h = \frac{40 \times 10^{-2}}{100}\eta \quad [\eta = 10^{-1} \text{ poise} = 10^{-2} \text{ Ns/m}^2]$$

$$= 4 \times 10^{-3} \text{ m} = 4 \text{ mm}$$

4. By Stoke's law, the terminal velocity of a water drop of radius r is given by

$$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

where ρ is the density of water, σ is the density of air and η the coefficient of viscosity of air. Here σ is negligible and $r = 0.0015$ mm $= 1.5 \times 10^{-3}$ mm $= 1.5 \times 10^{-6}$ m. Substituting the values:

$$v = \frac{2}{9} \times \frac{(1.5 \times 10^{-6})^2 \times (1.0 \times 10^3) \times 9.8}{1.8 \times 10^{-5}} = 2.72 \times 10^{-4} \text{ m/s}$$

5. The velocity attained by the sphere in falling freely from a height h is

$$v = \sqrt{2gh} \qquad \text{...(i)}$$

This is the terminal velocity of the sphere in water. Hence by Stoke's law, we have

$$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

where r is the radius of the sphere, ρ is the density of the material of the sphere, σ (= 1.0 × 10³ kg/m³) is the density of water and η is coefficient of viscosity of water.

$$\therefore \quad v = \frac{2 \times (1.0 \times 10^{-3})^2 (1.0 \times 10^4 - 1.0 \times 10^3) \times 10}{9 \times 1.0 \times 10^{-3}} = 20 \text{ m/s}$$

From Eq. (i), we have $h = \frac{v^2}{2g} = \frac{20 \times 20}{2 \times 10} = 20$ m

6. $F = \eta A \frac{dy}{dt} = \eta A \frac{v}{h} = 15 \times 2 \times 10^4 \times \frac{10}{0.25} = 1.2 \times 10^7$ dyne

7. $F = 6\pi\eta r v$

$$\eta = \frac{F}{6\pi r v} = \frac{3.14}{(6)(3.14)(4)(5 \times 10^2)} = 8.3 \times 10^{-5} \text{ poise}$$

8. Here, coefficient of viscosity, $\eta = 0.01$ poise; viscous force, $F = 200$ dyne; velocity of one plate w.r.t. the other plate, $dv = 10$ cm⁻¹; and side of the square plate =10 cm.

Therefore, area of the plate =10 × 10 = 100 cm²

Now, $F = \eta A \frac{dv}{dx}$

$$dx = \frac{\eta A dv}{F} = \frac{0.01 \times 100 + 10}{200} = 0.05 \text{ cm}$$

9. Here, length of the needle, $l = 5$ cm; radius of the needle, $r = 0.03$ cm; viscosity of blood, $\eta = 0.017$ poise; density of blood, $\rho = 1.02$ g cm⁻³; and height of the blood bag, $h = 85$ cm.

Pressure head, $p = h \rho g = 85 \times 1.02 \times 980$ dyne cm⁻²

Therefore, volume of the blood given to the patient per second,

$$V = \frac{\pi p r^4}{8\eta l} = \frac{\pi \times 85 \times 1.02 \times 980 \times (0.03)^4}{8 \times 0.017 \times 5} = 0.327 \text{ cm}^3\text{s}^{-1}$$

Therefore, time for which the transfusion takes place,

$$t = \frac{\text{total volume of the blood}}{\text{rate of flow of the blood}} = \frac{500}{0.327} = 1529 \text{ s}$$

Exercises

Single Correct Answer Type

1. (2) Suppose, R = radius of water drop and r = radius of droplets

$$\therefore \quad \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

(Since volume remains constant)

$$\therefore \quad r = \frac{R}{2}$$

Since excess pressure inside drop $= \frac{2T}{R}$

(T—surface tension, R—radius)

Therefore, pressure difference between inner and outer surface of big drop will be half of that for smaller droplet.

2. (3) The air pressure is greater inside the smaller bubble ($4T/r$). Hence, air flows from the smaller to the larger bubble.

3. (2) When two bubbles come into contact, they try to minimize the surface area for which air flows from bubble having higher pressure (smaller radius) to bubble having lower pressure.

4. (4) Surface tension force, $F = S \times$ length or $F = S \times [2\pi R + 2\pi r]$ as liquid surface is on the both sides.

5. **(3)** Work done = (increase in surface area) × surface tension

$$= 2 \times \left[4\pi\left(\frac{2D}{2}\right)^2 - 4\pi\left(\frac{D}{2}\right)^2\right] \times T$$

(Since for sap bubble, there are two free surfaces)

$$= 2 \times (4\pi D^2 - \pi D^2)\, T = 6\,\pi D^2 T$$

6. **(4)** $V \times \frac{4}{3}\pi R^3;\ 2V = \frac{4}{3}\pi R'^3;\ 2 \times \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R'^3;$

$R' = 2^{1/3} R$

$W' = 2 \times 4\pi\, [2^{1/3} R]^2\, \sigma = 2^{2/3} \times 2 \times 4\pi R^2 \sigma = 4^{1/3}\, W$

7. **(3)** Final surface energy = $1000 \times 4\pi r^2 \sigma$

Initial surface energy, $E = 4\pi R^2 \sigma$

Again, $\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$ or $R = 10r$

Now, final energy

$$1000 \times 4\pi r^2 \sigma = 10 \times 4\pi R^2 \sigma = 10E$$

8. **(3)** Work done = $10E - E = 9E$.

9. **(2)** $v_0 = \frac{2}{9}\frac{r^2(\rho - \rho')g}{\eta}$ or $\eta = \frac{2r^2(\rho - \rho')g}{9v_0}$

$$= \frac{2 \times 0.2 \times 0.2 \times 9 \times 980}{9 \times 8} \text{ poise} = 9.8 \text{ poise}$$

10. **(2)** For the same radius, terminal velocity is proportional to the density difference.

11. **(1)** $F = \frac{0.01 \times 100 \times 10}{0.1} \text{dyn} = 100 \text{dyn}$

12. **(3)** Shearing stress $= \frac{|\text{viscous force}|}{\text{area}}$

$$= \frac{\eta A \frac{dv}{dx}}{A} = \eta \frac{dv}{dx}$$

$$= 10^{-3} \times \frac{5}{10} \text{N/m}^2 = 0.5 \times 10^{-3} \text{ N/m}^2$$

13. **(1)** In the condition of weightlessness, water rises to the whole of the available length.

14. **(1)** Here, surface tension force support the weight of liquid.

$$S \times 2\pi\left[r + \frac{d}{2}\right] = \left(r^2 - \frac{d^2}{4}\right) h\rho g$$

$\Rightarrow h = 0.0144 \text{ m} = 1.44 \text{ cm}$

15. **(2)** $\left[P_0 + \frac{4\sigma}{R_2}\right] - \left[P_0 + \frac{4\sigma}{R_1}\right] = \frac{4\sigma}{R}$

or $\frac{1}{R} = \frac{1}{R_2} - \frac{1}{R_1}$

or $R = \frac{R_1 R_2}{R_1 - R_2} = \frac{50 \times 80}{30} \text{mm} = \frac{400}{3} \text{mm}$

$$= \frac{400}{3 \times 1000} \text{m} = \frac{4}{30} \text{m} = 0.133 \text{m}$$

16. **(1)** $F = (\sigma_1 - \sigma_2)\, l$

17. **(3)** $F = \frac{2A\sigma}{t} = \frac{2 \times 8 \times 75 \times 10}{0.12} \text{dyn} = 10^5 \text{ dyn}$

18. **(1)** $\left[2\pi \times \frac{8.7}{2} + 2\pi \times \frac{8.5}{2}\right] \sigma = 3.97 \times 980$

or $\sigma = \frac{3.97 \times 980 \times 7}{22 \times 17.2} \text{dyn/cm} = 72 \text{ dyn/cm}$

19. **(3)** $m \times 10 = 2 \times 3 \times 10^{-2} \times \frac{10}{100}$

or $m = 6 \times 10^{-4} \text{ kg} = 6 \times 10^{-4} \times 10^3 \text{ g} = 0.6 \text{ g}$

20. **(2)** Pressure due to 15 cm long liquid column needs to be balanced.

21. **(4)** $25 \times 10^{-3} \times 2 \times 0.1 = m \times 10$

or $m = \frac{5 \times 10^{-3}}{10} \text{kg} = 0.5 \text{g}$

22. **(3)** $\frac{h_2}{h_1} = \frac{\sigma_2 \cos\theta_2}{\rho_2} \times \frac{\rho_1}{\sigma_1 \cos\theta_1}$

or $\frac{h_2}{h_1} = \frac{140 \times \frac{1}{2}}{2} \times \frac{1}{70 \times 1}$ or $h_2 = \frac{h_1}{2} = \frac{6}{2} \text{cm} = 3 \text{cm}$

23. **(3)** $\frac{40}{100} \times 1000 \times 9.8 = \frac{2 \times 7 \times 10^{-2}}{R}$

or $R = \frac{14 \times 10^{-2} \times 100}{40 \times 1000 \times 9.8} \text{m} = \frac{14 \times 1000}{40 \times 1000 \times 9.8} \text{mm} = \frac{1}{28} \text{mm}$

Diameter $= 2R = \frac{1}{14} \text{mm}$

24. **(2)** $V \propto r^4, V' \propto \left(r + \frac{1}{10}r\right)^4$

$$\frac{V'}{V} = \frac{11 \times 11 \times 11 \times 11}{10 \times 10 \times 10 \times 10} = 1.4641$$

or $\left(\frac{V'}{V} - 1\right) \times 100 = 46.4\%$

25. **(2)** $V\rho g + F = V(4\rho)g$

or $F = 3V\rho g = 3mg$ or $\frac{F}{mg} = 3$

26. **(2)** $m = \frac{4}{3}\pi r^3 \rho m$

Keeping m constant, if r is halved, ρ will increased by a factor of 8.

Now, $v_0 \propto r^2 \rho$

$$v'_0 \propto \frac{r^2}{4}(8\rho) \text{ or } v'_0 \propto 2r^2\rho$$

Dividing, $\frac{v'_0}{v_0} = 2$ or $v'_0 = 2v_0$

or $v'_0 = 2v$

27. **(1)** As we increase the separation between the wires by 1 mm, the increase in surface area of the film is

$$\Delta A = 2 \times 0.1 \times 10^{-3} = 2 \times 10^{-4} \text{m}^2$$

(as there are two surfaces of film)

So, increase in surface energy, $\Delta U = S\Delta A = 1.44 \times 10^{-5}$ J

Work done = $\Delta U = 1.44 \times 10^{-5}$ J

28. **(1)** Surface tension force,

$$F = S \times 2l = 0.072 \times 2 \times \frac{2.5}{100} = 3.6 \times 10^{-3} \text{N}$$

29. **(4)** As in vacuum, there is no viscosity, no resistive force and hence no terminal velocity can be acquired.

(4) $v_0 = \frac{2}{9}\frac{r^2(\rho-\rho')g}{\eta}$

Now, $x = \frac{4}{3}\pi r^3 \rho$ or $\rho \propto \frac{x}{r^3}$

Similarly, $\rho' \propto \frac{y}{r^3}$

$\therefore \quad v_0 \propto \frac{x-y}{r}$

(2) $h = \frac{2S\cos\theta}{r\rho g}$

Cross-sectional area increases four times, which means radius gets doubled.

So, $h' = \frac{2S}{2(r\rho g)} = \frac{h}{2}$

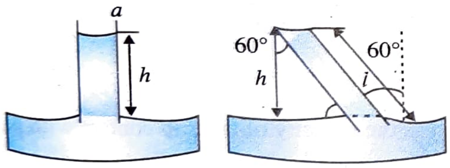
(1) The terminal velocity of the spherical raindrop of radius r is given by

$$v_t = \frac{2r^2\rho g}{9\eta}$$

where ρ is the density of water and η the viscosity of air. Substituting $r = 0.3$ mm $= 0.3 \times 10^{-3}$ m, $\rho = 10^3$ kg/m³; $g = 9.8$ ms⁻² and $\eta = 1.8 \times 10^{-5}$ Ns/m².

We get $v_t = \frac{2\times(0.3)^2\times10^{-3}\times9.8}{9\times1.8\times10^{-5}} = 10.9$ m/s

(1) Since water rises to height of 2 cm in a capillary



If tube is at 60°, in this case height must be equal to

$h = 2$ cm $\Rightarrow \cos 60° = \frac{h}{l}$

$\therefore \quad l = \frac{h}{\cos 60°} = \frac{2}{1/2} = 4$ cm

(4) For equilibrium,

Total upward force by surface tension

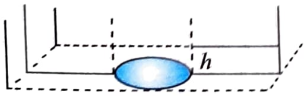
= Weight of the water in tube

⇒ $\pi \times D \times$ surface tension (circumference)

= $\pi\,(D/2)^2 \times h \times$ density $\times\, g$ (cross section)

where D (diameter) = 0.1 mm = 0.01 cm

Density of water = 1×10^{-3} g/cm³



$$\Rightarrow \pi \times (0.01) \times 75 \times 10^{-3} = \pi \times \left(\frac{0.01}{2}\right)^2 \times h \times 1 \times 10^{-3} \times 1000$$

$$\therefore \quad h = \frac{0.75\times0.01\times4}{0.01\times0.01} = 0.3\,\text{m} = 30\,\text{cm}$$

35. (3) Height, $h = \frac{2T\cos\theta}{r\rho g}$

$\therefore$ For water, $h_w = \frac{2\times T_w\times\cos 0°}{r\times1\times g}$

And, for mercury,

$$h_m = -\frac{2\times T_m\times\cos 135°}{r\times13.6\times g}$$

$$\therefore \quad \frac{h_w}{h_m} = \frac{2\times T_w\times1}{r\times1\times g}\times\frac{r\times13.6\times g\times\sqrt{2}}{2\times T_m\times1} \quad \left[\because \cos 135° = -\frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow \frac{10}{3.42} = \frac{T_w}{T_m}\times13.6\times\sqrt{2}$$

$$\therefore \quad \frac{T_w}{T_m} = \frac{10}{3.42\times13.6\times1.414} \approx \frac{1}{6.5} = \frac{2}{13}$$

$\therefore \quad T_w : T_m = 2/13$

36. (2) Since, ball gains the constant velocity. Therefore, net force on the ball = zero.

⇒ Weight of the ball = Buoyancy force + Viscous force (F_v)

$$\Rightarrow Mg = \frac{M}{d_1}\times d_2\times g + F_v$$

$$\therefore \quad F_v = Mg\left(1-\frac{d_2}{d_1}\right)$$

37. (1) If y is the required distance, then

$a^2 y\rho g = mg + 4\,a\sigma$

or $\quad y = \frac{mg\times4a\sigma}{a^2\rho g} = \frac{20\times980+4\times3\times70}{3\times3\times1\times980}$ cm = 2.3 cm

38. (1) Effective length of flat part = $2r$. Effective length of curved part is πr.

$F = \sigma l$

So, required ratio is $2:\pi$.

39. (1) Total upward force due to surface tension = $2\sigma l$

Weight of lifted liquid = $(hld)\,\rho g$

Equating, we get $h = \frac{2\sigma}{\rho g d}$

40. (1) Increase in PE $= hld\,\rho g\left(\frac{h}{2}\right) = \frac{ld\rho g}{2}h^2$

If F is the force of attraction between the plates, then

$$Fd = \frac{ld\rho g}{2}h^2 \quad \text{or} \quad F = \frac{l\rho g}{2}h^2$$

41. (2) $F \propto h^2$

But $h^2 \propto \sigma^2$

$\therefore \quad F \propto \sigma^2$

If σ is doubled, then F will become $4F$.

42. (4) $\frac{4}{3}\pi R^3 = n\times\frac{4}{3}\pi r^3$ or $R^3 = nr^3$

Energy evolved = $(n \times 4\pi r^2 - 4\pi R^2)\,T$

Now, $\theta = \frac{Q}{V\rho S} = \frac{Q}{V}$ (In CGS system, $\rho = 1$ and $S = 1$)

$$\therefore \quad \theta = \frac{[n\times4\pi r^2 - 4\pi R^2]T}{\frac{4}{3}\pi R^3}$$

$$= \frac{3T[nr^2 - R^2]}{R^3} = 3T\left[\frac{nr^2}{nr^3} - \frac{R^2}{R^3}\right] = 3T\left[\frac{1}{r} - \frac{1}{R}\right]$$

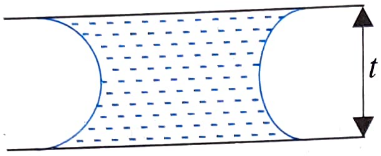
43. (4) $2\pi r\sigma + \frac{1}{2}\times\frac{4}{3}\pi r^3 dg = \frac{4}{3}\pi r^3 \rho g$

or $2\pi r\sigma = \frac{\pi r^3 g}{3}[4\rho - 2d]$ or $r^2 = \frac{3\times 2\pi\sigma}{\pi g(4\rho - 2d)}$

or $r^2 = \frac{3\sigma}{g(2\rho - d)}$ or $r = \sqrt{\frac{3\sigma}{g(2\rho - d)}}$

Diameter $= 2r = \sqrt{\frac{12\sigma}{g(2\rho - d)}}$

44. (2)



Pressure difference $= \frac{T}{R} = \frac{T}{t/2} = \frac{2T}{t}$

Force $= \frac{2T}{t}A$

But $V = At$ or $t = \frac{V}{A}$

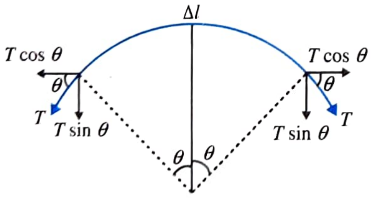
∴ Force $= \frac{2TA^2}{V}$

45. (2) Difference in apparent weights is due to differences in forces of surface tension. Due to 180°, the force surface tension in one case is opposite to the force of surface tension in the other case.

∴ $2\times\sigma_w\times\frac{10}{100} = 0.044 - 0.03$

or $\sigma_w\times\frac{0.014}{1}\times 5\ \text{N/m} = 0.07\,\text{N/m}$

46. (3) When the soap film enclosed by the thread is pricked, the thread would take up a circular shape. Consider a small element of the thread. While horizontal components of tension get balanced, the vertical components get added up.



or $2T\sin\theta = \sigma\,\Delta l$

or $2T\theta = \sigma\,(2\theta\times r)$

or $T = \sigma r = \sigma\times\frac{2\pi r}{2\pi} = 0.030\times\frac{6.28\times 10^{-2}}{2\pi}\,\text{N}$

$= 3\times 10^{-4}\ \text{N}$

47. (1) Total upward force due to surface tension $= \sigma(2r_1 + 2r_2)$. This supports the weight of the liquid column of height h. Weight of liquid column $= h\,[\pi r_2^2 - \pi r_1^2]\,\rho g$

Equating, we get $h\pi\,(r_2 + r_1)\,(r_2 - r_1)\,\rho g = 2\pi\sigma\,(r_1 + r_2)$

or $h\,(r_2 - r_1)\,\rho g = 2\sigma$ or $h = \frac{2\sigma}{(r_2 - r_1)\rho g}$

48. (4) Energy released $= [n\times 4\pi a^2 - 4\pi b^2]\,\sigma$

Now, $n\times\frac{4}{3}\pi a^3 = \frac{4}{3}\pi b^3$ or $n = \frac{b^3}{a^3}$

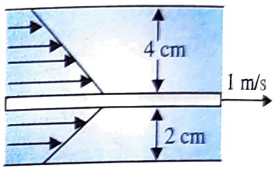
Therefore, energy released

$$= \left[\frac{b^3}{a^3}\times 4\pi a^2 - 4\pi b^2\right]\sigma = 4\pi b^2\left[\frac{b}{a} - 1\right]\sigma$$

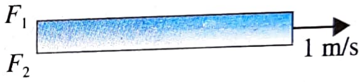
Now, $\frac{1}{2}\left(\frac{4}{3}\pi b^3\right)\rho v^2 = 4\pi b^2\left[\frac{b}{a} - 1\right]\sigma$

or $v = \left[\frac{6\sigma}{\rho}\left(\frac{1}{a} - \frac{1}{b}\right)\right]^{1/2}$

49. (2) The profile of the velocity of fluid is as shown in the figure. The velocity of the layer of fluid, which is in contact with metal plates (fixed), is zero. As we move towards the centre from either plate, the velocity of the layer of fluid increases and it becomes maximum at the location of moving plate. This maximum value is same as that of the velocity of plate.



50. (1) The force experienced by the plate due to fluid is shown in the figure.



F_1 is the force exerted by the layer in contact with plate on upper side and F_2 is the force exerted by the layer in contact with plate on lower side.

$$F_1 = \eta A\times\frac{v}{4\,\text{cm}} = 0.8\times 0.01\times\frac{1}{0.04} = 0.2\,\text{N}$$

where v is the velocity of layer in contact with plate.

$$F_2 = \eta A\times\frac{v}{2\,\text{cm}} = 0.8\times 0.01\times\frac{1}{0.02} = 0.4\,\text{N}$$

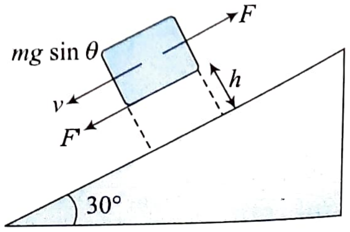
Total force, $F = F_1 + F_2 = 0.6$ N in backward direction.

51. (4) A longer pipe offers a greater resistance to the flow of a viscous fluid than a shorter pipe does. The volume flow rate depends inversely on the length of the pipe. For a given pipe radius and pressure difference between the ends of the pipe, the volume flow rate is less in longer pipes. In this case the longer pipe is twice as long, so its volume flow rate Q_B is one-half that of Q_A.

52. (1) Taking g constant, work done in both the cases is $-mg\,(1000)$ since the max velocity of the journey is attained at a height of 1 km, and the second half journey is travelled by max velocity, the time taken for second half journey will be less.

53. (2) $F = F' = \eta A\frac{dv}{dz} = mg\sin\theta$

Velocity gradient $\frac{dv}{dz} = \frac{v}{h}$

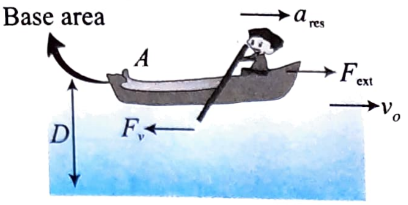


Hence $20\times10\times\sin30^\circ=\eta\times4\times\frac{10}{h}$

$$h=\frac{10^{-2}\times4\times10}{20\times10\times\frac{1}{2}}=\frac{40\times10^{-2}}{100}\quad[\eta=10^{-1}\text{ poise}=10^{-2}\text{ N s m}^{-2}]$$

$$=4\times10^{-3}\text{ m}=4\text{ mm}$$

4. (3) We can draw the free-body diagram of boat



$$F_{ext}-F_v=ma_{res}$$

As boat moves with constant velocity $a_{res}=0$

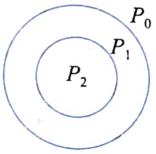
$$F_{ext}=F_v$$

But $F_V=\eta A\frac{dv}{dz}$ but $\frac{dv}{dz}=\frac{v_0-0}{D}=\frac{v_0}{D}$

then $F_{ext}=F_V=\frac{\eta Av_0}{D}$

55. (3) $P_1-P_0=\frac{4S}{2R}$ ...(i)

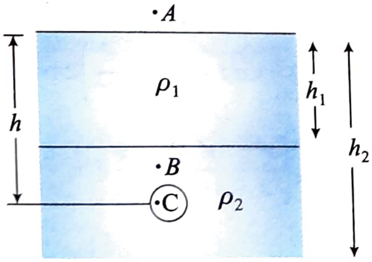
$P_2-P_1=\frac{4S}{R}$ ...(ii)



From (i) and (ii) $P_2-P_0=\frac{6S}{R}$

56. (2) $P_C=P_B+\frac{2T}{r}$

$$P_B=P_A+\rho_1gh_1+\rho_2g(h-h_1)$$



where $P_A=P_0$

$$P_C=P_0+\rho_1gh_1+\rho_2g(h-h_1)+\frac{2T_2}{r}$$

Then evaluate by putting the numerical values.

57. (3) $P_0+\rho_1gh-\rho_2gh+\frac{2T}{r}=P_0$

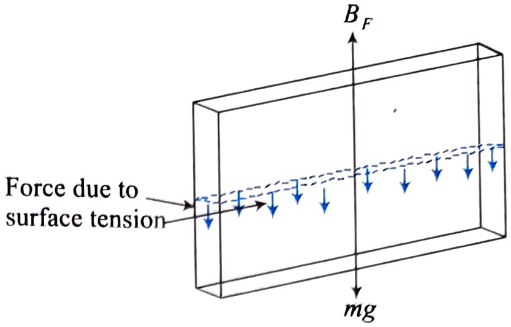
$$\Rightarrow\ T=\frac{r}{2}(\rho_2-\rho_1)gh$$

58. (2) The forces acting on the plate are

(i) mg (↓)

(ii) force due to surface tension (F_T)(↓)

(iii) Buoyant force (B_F)(↑)



$$w=mg=(8.2\times10^{-3})\times9.8=80.36\times10^{-3}\text{ N}$$

The water is in contact along the length,

$L=2(10+0.2)=20.4$ cm

$\therefore\ F_T=(20.4\times10^{-2})(7.3\times10^{-2})=148.92\times10^{-4}$ N

$$B_F=\left[\frac{10\times0.2\times1.54}{2}\times10^{-6}\right]\times1000\times9.8=15.092\times10^{-3}\text{ N}$$

∴ Apparent weight

$$w+F_T-B_F=(80.36+14.892-15.092)\times10^{-3}$$
$$=80.16\times10^{-3}\text{ N}$$

59. (3) Viscous force $F\propto$ (area)

So let $F=kA$

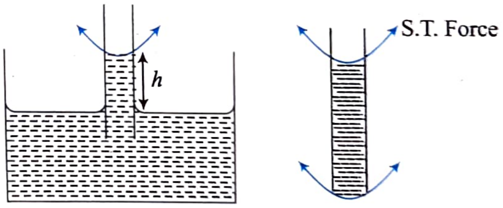
$F_0=k(A_1+A_2)$ ...(i)

and $T=kA_1$ ...(ii)

Dividing Eq. (ii) by Eq. (i) we get,

$$\frac{T}{F_0}=\frac{A_1}{A_1+A_2}\quad\therefore\ T=\frac{F_0A_1}{A_1+A_2}$$

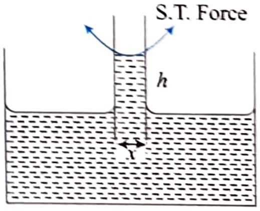
60. (1) When the capillary is inside the liquid, the surface tension force supports the weight of liquid of height '*h*'.



When the capillary is taken out from the liquid similar type of surface tension force acts at the bottom also, as shown in second figure. Hence, now it can support weight of a liquid of height 2*h*.

61. (2) Let L be the width of plates (perpendicular to paper inwards).

Surface tension force in upward direction = weight of liquid of height h



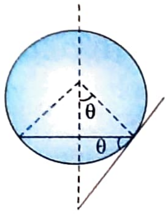
$\therefore\ Tl\cos\theta=Vdg$ (V = volume)

or $T(2L)\cos\theta=(Lxh)dg$

$$h=\frac{2T\cos\theta}{xdg}$$

62. (2) $\cos\theta=\frac{R-h}{R}$

$$\Rightarrow\ \theta=\cos^{-1}\left(\frac{R-h}{R}\right)$$



Multiple Correct Answers Type

1. (1), (2), (3), (4)

i. Excess pressure inside a spherical liquid drop = $2T/R$, because there is only one free surface here

ii. In case of spherical meniscus of radius of curvature R, also excess pressure = $2T/R$ because again there is only one free surface.

iii. Excess pressure inside a cylindrical drop of radius R = T/R. Hence, for a cylindrical bubble in air, excess pressure is $2T/R$, because there exist two free surfaces in this case.

iv. For a spherical bubble in water, excess pressure = T/R, as there is only one free surface.

Hence, all the four options are correct.

2. (1), (2)

$$h = \frac{T}{Rdg}$$

Given that h, T, d and g are fixed, hence R must be same.

As weight of the liquid is balanced by force due to surface tension ($T \cos \theta$ and R are fixed for a given liquid, given material of capillary at a constant temperature) hence, weights of liquids in both the capillaries must be equal

So, options (1) and (2) are correct.

3. (1), (3)

Viscous force is somewhat like a frictional force but not exactly the same because it differs from it due to the two main following statements:

i. Viscous force is velocity dependent while frictional force is not.

ii. Viscous force is temperature dependent while frictional force is not.

So, options (1) and (3) are correct.

4. (3), (4)

The angle of contact at the free liquid surface inside the capillary tube will change such that the vertical component of the surface tension forces just balances the weight of the liquid column.

5. (1), (2), (3)

The two free liquid surfaces must provide a net upward force due to surface tension to balances the weight of liquid column.

6. (1), (3)

Let S = surface tension = surface energy per unit area; r = radius of each small drop; and R = radius of a single drop

$$n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \quad \text{or} \quad R = rn^{1/3}$$

Initial surface energy, $E_i = n \times 4\pi r^2 \times S = nE$

Final surface energy, $E_f = 4\pi R^2 S = 4\pi r^2 n^{2/3} S = n^{2/3} E$

Therefore, energy released = $E_i - E_f = E(n - n^{2/3})$

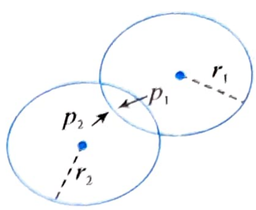
7. (1), (4)

Volume becomes eight times, so radius r becomes double. We know that terminal velocity: $v_t \propto r^2$, hence terminal velocity becomes four times. And drag force: $F \propto rv_t$, hence drag becomes eight times

8. (1), (2), (3)

Acceleration will be less than $g\left(\frac{\rho - \sigma}{\rho}\right)$. Time will depend on density ρ. Viscous force may be maximum or minimum both are possible.

9. (1), (3)



Excess pressure is always present on concave side.

So, interface is concave towards small bubble.

$$p_1 = p_0 + \frac{4T}{r_1}$$

$$p_2 = p_0 = \frac{4T}{r_2}$$

Excess pressure $= p_1 - p_2 = 4T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

This excess pressure for the interface is also equal to $\frac{4T}{R}$.

$$\therefore \quad 4T\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{4T}{R}$$

$$R = \frac{r_1 r_2}{r_2 - r_1} = \frac{(0.002)(0.004)}{0.004 - 0.002} = 0.004\,\text{m}$$

10. (1), (2), (3)

Magnitude of viscous force, $F = \eta A \frac{dv}{dr}$

⇒ Viscous force per unit area $\sigma = \frac{F}{A} = \eta \frac{dv}{dr}$

$$v = v_0\left(1 - \frac{r^2}{R^2}\right)$$

$$\frac{dv}{dr} = -\frac{2v_0 r}{R^2} \Rightarrow \sigma = \eta \cdot \frac{2v_0 r}{R^2} \qquad \text{...(i)}$$

Consider an annular element at r from axis, width dr.

$$dA = 2\pi r dr; \; dQ = v.dA = v_0\left(1 - \frac{r^2}{R^2}\right) 2\pi r dr$$

$$Q = \int dQ = 2\pi v_0 \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R = \frac{\pi}{2} R^2 v_0$$

$$\Rightarrow \quad v_0 = \frac{2Q}{\pi R^2}$$

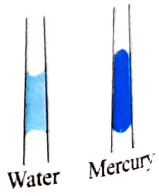
$$\therefore \quad \sigma = \eta \frac{4Q}{\pi R^4} r, R = 0.1\,\text{m} \qquad \text{(from Eq. (i))}$$

At $r = 0.04$ m, $\sigma = (0.75) \times 4 \times \frac{\pi}{2} \times 10^{-2} \times \frac{0.04}{\pi \times 10^{-4}} = 6\,\text{Nm}^{-2}$

11. (1), (3)

$\Delta P = \frac{2\sigma}{r}$, where σ is surface tension.

Pressure difference is reciprocal to radius of free surface. Check the movement of liquid by pressure difference.



ked Comprehension Type

Problems 1–2

(4) If ρ_w is the density of water, then the density of the lead sphere and the density of the liquid are given by

$$\rho_0 = 11.20\,\rho_w \quad \text{and} \quad \rho = 1.26\,\rho_w$$

Thus, the equation $T = 2\pi\sqrt{\dfrac{l}{2g}}$ may be written as

$$\eta = \frac{2}{9}\frac{r^2}{v}(11.20 - 1.26)\rho_w g$$

Here, $r = 0.5 \times 10^{-3}$ m; $v = 0.7 \times 10^{-2}$ m s^{-1}; $g = 9.8$ m s^{-2}; and $\rho_w = 10^3$ kg/m^3.

$$\eta = \frac{2}{9}\frac{(0.5\times10^{-3})^2}{0.7\times10^{-2}}(11.20-1.26)(10^3)(9.8) = 0.77 \text{ N s/m}^2$$

(1) Reynolds number, $R_e = \dfrac{\rho v d}{\eta}$

$$R_e = \frac{(1.26\times10^3)(0.7\times10^{-2})(1\times10^{-3})}{0.77}$$

$R_e = 0.01 < 0.20$ (maximum limit of Stokes' law).

For Problems 3–4

3. (3) Here $s = 7.0 \times 10^{-2}$ N/m; $r = 0.2 \times 10^{-3}$ m; $\rho = 10^3$ kg/m^3

We know that the height of liquid rise in the tube is

$$h = 2\sigma\cos\theta/\rho R g$$

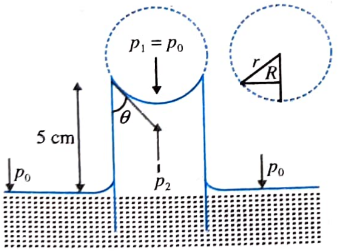
where R is the radius of the meniscus, and θ is the angle of contact.

Since $\theta = 0$ (given), radius of the meniscus is equal to the radius of the capillary tube, i.e., $R = r$.

$$\therefore \quad h = \frac{2(7.0\times10^{-2})}{(10^3)(0.2\times10^{-3})(10)} = 0.07\,\text{m}$$

or $h = 7.0$ cm

4. (2) When the length of the capillary tube above the free surface of the liquid is less than the height of liquid that rise in the tube, radius R of the free surface is not equal to the radius of the tube. It is greater than r as the surface tends to be flatter.



According to the equation, $p_1 - p_2 = (4\sigma/R)$, the pressure difference across the surface is given by

$$\Delta p = \frac{2\sigma\cos\theta}{R}$$

If p_1 and p_2 are the pressure just above and below the meniscus, respectively, then $p_1 - p_2 = \rho g h_0$

$$\therefore \quad \rho g h_0 = \frac{2\sigma}{R} \qquad \text{...(i)}$$

In part (a) we have seen that when $h_0 = h$; $\theta = 0$, $R = r$.

$$\text{and} \quad \rho g h = \frac{2\sigma}{r} \qquad \text{...(ii)}$$

Therefore, dividing Eq. (i) by Eq. (ii), we have $\dfrac{r}{R} = \dfrac{h_0}{h}$

From the figure, it is clear that $\cos\theta = \dfrac{r}{R}$

Therefore, the angle of contact is

$$\theta = \cos^{-1}\left(\frac{h_0}{h}\right) = \cos^{-1}\left(\frac{5}{7}\right) \approx 44°$$

For Problems 5–7

5. (3) Using Poiseuille's equation, we get

$$Q = \frac{(p_1 - p_2)\pi D^4}{128\eta L}$$

Here, $p_1 - p_2 = 38.4 \times 10^3$ N/m^2
$D = 2.5$ cm $= 2.5 \times 10^{-2}$ m
$L = 30$ m; $h = 0.12$ N/m^2

$$\therefore \quad Q = \frac{(38.4\times10^3)(3.14)(2.5\times10^{-2})^4}{128(0.12)(30)} = 1.0\times10^{-4}\,\text{m}^3/\text{s}$$

6. (1) We know that $\tau = -\dfrac{dp}{dx}\dfrac{r}{2}$

At the wall of the pipe, i.e., $r = R$

$$\tau_{\max} = -\frac{dp}{dx}\frac{R}{2} = \left(\frac{p_1 - p_2}{L}\right)\frac{D}{4}$$

$$= \left(\frac{38.4\times10^{-3}}{30}\right)\frac{2.5\times10^{-2}}{4} = 8\times10^{-6}\,\text{N/m}^2$$

7. (2) We know that $P = \dfrac{128\eta Q^2 L}{\pi D^4} = (p_1 - p_2)Q$

$$\therefore \quad P = (38.4\times10^3)(1\times10^{-4}) \quad \text{or} \quad P = 3.84 \text{ W}$$

For Problems 8–10

8. (1) Excess pressure is always on the concave side of the surface. For angle of contact of $2\pi/3$, the liquid should have a convex surface. So, the excess pressure should be in the upward direction.

9. (1) In soap bubble, there will be two free surfaces. So, excess pressure will be double than that in an air bubble. So, air bubble is $p/2$.

10. (3) Difference in pressure between the inner and outer surface is $(p_i - p_0)$.

$\therefore$ Force $= (p_i - p_0)\pi r^2$

Since excess pressure is given, $p = (p_i - p_0)$

$\therefore$ $F = p\pi r^2$

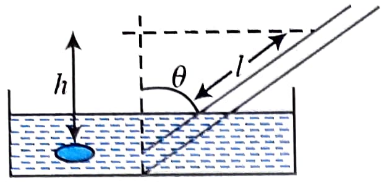
So option (3) is correct and the rest are incorrect.

For Problems 11–13

11. (3) $P_A = P_0 - \dfrac{2s}{r}$

The pressure inside a concave meniscus is less than the pressure outside (atmospheric). Assuming the meniscus to be spherical (as for thin capillaries), excess pressure is $2s/r$ where r is the radius of the hemispherical surface.

12. (2) When the capillary tube is tilted by an angle θ with the vertical, the capillary rise to distance l in the tube will be such that the meniscus will remain at the same vertical height above the level of water in the container.



Hence, $h = l\cos\theta$

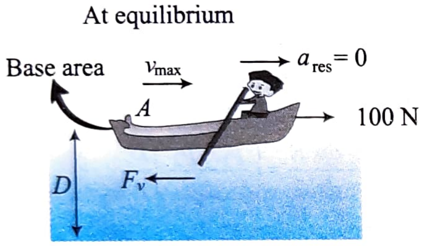
$$l = \frac{h}{\cos\theta} = \frac{10}{\cos 45^\circ} = 10\sqrt{2}\ \text{cm}$$

13. (3) The capillary rise h is given by $h = \frac{2S\cos\theta}{\rho g r}$

where hr = constant for a given glass and liquid. This is shown correctly in (3).

For Problems 14-15

14. (4) At equilibrium

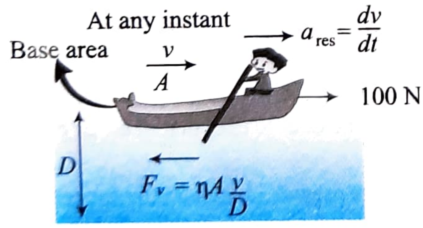


$$F_{ext} = F_v$$

$$F_{ext} = 100\,\text{N} \text{ and } F_v = \eta A\frac{v_{max}}{D} = 1.5\times 10\times\frac{v_{max}}{D}$$

$$100 = 1.5\,v_{max} \Rightarrow v_{max} = \frac{200}{3}\,\text{m/s}$$

15. (3) At any instant



$$F_{ext} - F_v = m\frac{dv}{dt} \Rightarrow 100 - \eta A\frac{v}{D} = m\frac{dv}{dt}$$

$$100 - 1.5\,v = 150\frac{dv}{dt}$$

$$\int_0^t dt = \int_0^{\frac{v_{max}}{2}} 150\frac{dv}{100-1.5v}$$

$$t = 150\left[\frac{\ln(100-1.5v)}{-1.5}\right]_0^{100/2}$$

$$t = 100\ln 2 \text{ seconds}$$

Matrix Match Type

1. i. → b., c.; ii. → a., d.; iii. → b., c.

For A and C: In this case, the final surface area is greater than the initial surface area, and hence surface energy increases the expanse of internal energy; hence it causes cooling.

For B, reverse of the above reasoning.

2. i. → b.; ii. → c.; iii. → d.

The forces acting on the plate are

weight of the plate vertically downwards

$$W = mg = 20\text{ gf}$$

i. Buoyant force due to liquid,

$$F_b = V\rho_l g = \left(\frac{lbt}{2}\right)\rho_l g = \left(\frac{10\times4\times0.4}{2}\right)\times 1\text{gf} = 8\text{ gf}$$

ii. Force due to surface tension, vertically downward

$F = T\times$ perimeter of plate in contact with water

$$= \frac{70}{980}\times 2(10+0.4)\,\text{gf} = 1.5\,\text{gf}$$

iii. Apparent weight of the plate $= (W+F) - F$

$$= (20 + 1.5) - 8 = 13.5\text{ gf}$$

3. (1) (i) Stroke's law, $F = 6\pi\eta r v$

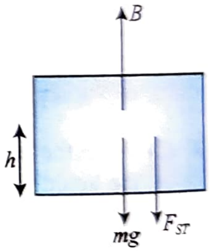
(ii) Terminal velocity, $v_T = \frac{2}{9}\cdot\frac{r^2 g(\rho_S - \rho_L)}{\eta}$

(iii) Excess pressure inside a mercury drop is $\Delta p = \frac{2T}{r}$

(iv) Viscous force is $F_v = -\eta A\frac{dv}{dy}$

Numerical Value Type

1. (2) $Mg + F_{ST} = B$

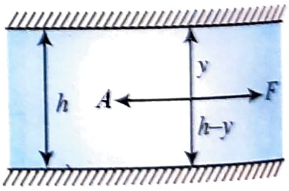


$$mg + 4aT = a^2 h\rho_w g$$

$$10 + 4\times\frac{10}{4} = 10h$$

$$h = 2\text{ m}$$

2. (5) Case I: $y_1 = y_2 = h/2$



$$\frac{\partial u}{\partial y} = \frac{v}{h/2} = \frac{2v}{h}$$

Viscous force $= \frac{\eta_0 2v}{h}A$

The plate has two sides.

Total force $F = \frac{4\eta_0 vA}{h}$

Case II: $F = \frac{\eta vA}{y} + \frac{\eta vA}{h-y} = \frac{\eta vAh}{y(h-y)}$

$$\therefore \frac{\eta vAh}{y(h-y)} = \frac{4\eta_0 Av}{h}$$

$$\frac{3}{16}h^2 = y(h-y) \qquad \left(\because \eta = \frac{3}{4}\eta_0\right)$$

Solving, we get $y = \frac{h}{4}, \frac{3h}{4}$

Hence, for minimum distance from top $y = 20/4 = 5$ cm

We have $pV = mRT$

$$\left(p_0 + \rho_w gh - \frac{4T}{d}\right)\frac{\pi}{6}d^3 = \left(p_0 + \frac{4T}{d}d\right)\frac{\pi}{6}n^3d^3$$

(the bubble has only one surface)

$$\therefore \quad h = \frac{d(n^3-1)p_0 + 4T(n^2-1)}{\rho_w gd}$$

$$\text{or} \quad h = \frac{4\times10^{-6}(1\times1^3-1)\times10^5 + 4\times0.07(1\times1^2-1)}{1000\times9.8\times4\times10^{-6}} = 5\text{ m}$$

8) $\frac{4}{3}\pi R^3 = n\times\frac{4}{3}\pi r^3$

or $R^3 = nr^3$ or $R = n^{1/3}r$

or $R = 2n^{1/3}$ mm

$$v_0 \propto r^2, v_0' \propto R^2$$

$$\text{Now,} \quad \frac{v_0'}{v_0} = \frac{R^2}{r^2} = \frac{4n^{2/3}}{4}$$

or $\frac{32}{8} = n^{2/3}$ or $n^{2/3} = 4$

or $n = 4^{3/2} = \sqrt{64}$

or $n = 8$

(5) $Mg = PA$; A is sectional area

$$Al\rho g = 10^5 A$$

$$l = \frac{10^5}{2\times10^3\times10} = 5\text{m}$$

(8.00) $\frac{\pi r^4 P_1}{8\eta l} = \frac{\pi(2r)^4 P_2}{8\eta(2l)}$

$$\Rightarrow \quad \frac{P_1}{P_2} = 8$$

7. (4.00) $D = 2r = 2\times2\times10^{-3}\text{ m} = 4\times10^{-3}\text{ m}$

For laminar flow with maximum velocity. Reynold's number is

$$N_R = 2000$$

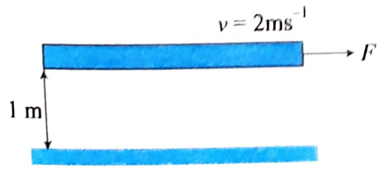
$$N_R = \frac{\rho vD}{h}$$

$$\Rightarrow \quad v = \frac{N_R\eta}{\rho D} = \frac{(2000)(2.12\times10^{-3})}{(1.06\times10^3)(4\times10^{-3})} = 1\text{ ms}^{-1}$$

Corresponding flow rate is $Q = Av = (\pi r^2)v$

$$Q = \pi(2\times10^{-3})^2\times1 = 4\pi\times10^{-6}\text{ m}^3\text{ s}^{-1}$$

8. (0.004) Velocity gradient $= \frac{\Delta v}{\Delta y} = \frac{2-0}{1-0} = 2\text{ ms}^{-1}\text{m}^{-1}$



Viscous force,

$$|F| = \eta A\frac{\Delta v}{\Delta y} = (0.01\times10^{-1})(2)(2) = 0.004\text{ N}$$

So, to keep the plate moving with constant velocity, a force of 4×10^{-3} N has to be applied.

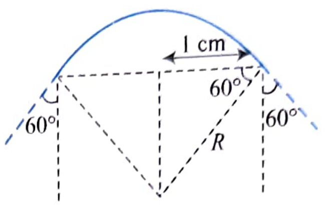
9. **(25)** Velocity gradient $= \frac{\Delta v}{\Delta y} = \frac{0.5-0}{1.25\times10^{-2}} = 40\text{ ms}^{-1}\text{m}^{-1}$

$$F = 2\left[\eta A\frac{\Delta v}{\Delta y}\right] \Rightarrow 1 = 2\times\eta\times0.5\times40$$

$$\Rightarrow \quad \eta = \frac{1}{40}\text{kg–sm}^{-2}$$

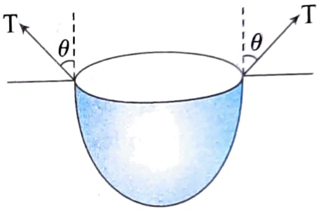
or $\quad \eta = 0.25\times10^{-1} = 25\times10^{-3}$ kg- sm^{-2}

10. **(10)** $\cos 60° = \frac{1}{R} \Rightarrow R = 2\text{ cm}$



$$\Delta p = \frac{4T}{R} = \frac{4(0.05)}{0.02} = 10\text{ Pa}$$

11. **(3.00)** The vertical force due to surface at the hole



$$= T\cos\theta\times L = T\cos\theta\times2\pi r$$

This will balance weight $mg\,(=\pi r^2 h\rho g)$

$\therefore \quad T\cos\theta\times2\pi r = \pi r^2 h\pi g$

$$\Rightarrow \quad h = \frac{2T\cos\theta}{r\rho g}$$

h is maximum, when cos θ = 1

$$\therefore \quad h_{\max} = \frac{2T}{r\rho g} = \frac{2\times0.075}{0.0005\times1000\times10} = 0.03\text{m} = 3\text{ cm}$$

12. **(4.00)** Mass of air entering in 1s is $m = \pi b^2 v\rho$

Change in momentum in 1s $= mv - 0 = \pi b^2\rho v^2$

Thus, $F = \pi b^2\rho v^2$

Pressure due to this force is $\Delta p = \frac{F}{A} = \frac{\pi b^2\rho v^2}{\pi b^2} = \rho v^2$

When this excess pressure equals to $\frac{4T}{R}$, the bubble gets separated.

$$\therefore \quad \rho v^2 = \frac{4T}{R} \Rightarrow R = \frac{4T}{\rho v^2}$$

So, $\beta = 4$

13. **(20)** The velocity attained by the sphere in falling freely from a height h is

$$v = \sqrt{2gh} \qquad \text{...(i)}$$

This is the terminal velocity of the sphere in water. Hence by Stokes's law, we have

$$v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

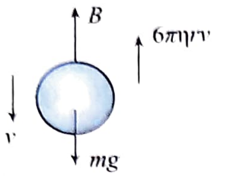
where r is the radius of the sphere, ρ is the density of the material of the sphere

$\sigma\,(=1.0\times10^3\ \text{kg/m}^3)$ is the density of water and η is coefficient of viscosity of water.

$$v=\frac{2\times(1.0\times10^{-3})^2(1.0\times10^4-1.0\times10^3)\times10}{9\times1.0\times10^{-3}}=20\ \text{m/s}$$

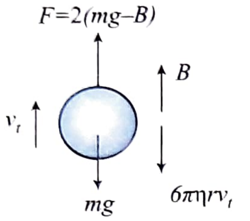
From Eq. (i), we have $h=\frac{v^2}{2g}=\frac{20\times20}{2\times10}=20\ \text{m}$

14. **(10)** First case:



$$B+6\pi\eta rv=mg \qquad \text{...(i)}$$

For second case:



$$F+B=mg+6\pi\eta rv_t \Rightarrow 2mg-2B+B=mg+6\pi\eta rv_t$$

$$\Rightarrow \quad mg-B=6\pi\eta rv_t \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), $v_t=v=10$ cm/s

15. **(0.08)** When the ring is about to leave the water surface, surface tension force on it is

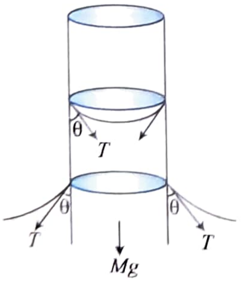
$$F_{ST}=2\pi RT+2\pi rT=2\pi(R+r)T$$

Spring force $F_s=kx$

$\therefore \quad kx=2\pi(R+r)T+mg$

$$\therefore\ T=\frac{kx-mg}{2\pi(R+r)}=\frac{0.9\times4\times10^{-2}-8\times10^{-4}\times10}{2\times3.14\times(50+20)\times10^{-3}}=0.080\,\text{Nm}^{-1}$$

16. **(0.133)** Force = Mg + force due to surface tension on the inner wall + force due to surface tension on the outer wall

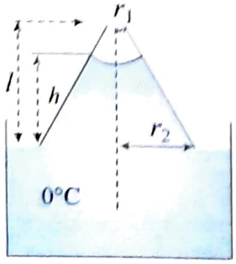


$$= Mg+2\pi rT\cos\theta+2\pi rT\cos\theta=Mg+4\pi rT\cos\theta$$

$$=10\times10^{-3}\times10+4\times\frac{22}{7}\times7\times10^{-3}\times0.75\times\frac{1}{2}$$

$$=0.133\ \text{N}$$

17. **(1.4)** The situation is shown in figure.



Let r_1 and r_2 be radii of upper and lower ends of the conical capillary tube. The radius r at the meniscus is given by

$$r=r_1+(r_2-r_1)\left(\frac{l-h}{l}\right)$$

$$=(2.5\times10^{-4})+(2.5\times10^{-4})\left(\frac{0.1-0.08}{0.1}\right)=3.0\times10^{-4}\ \text{m}$$

The surface tension at 0°C is given by

$$T_0=\frac{rh\rho g}{2}=\frac{(3.0\times10^{-4})(8\times10^{-2})(1/4\times10^4)(9.8)}{2}=0.084$$

For tube B: $\frac{T_0}{T_{50}}=\frac{h_0}{h_{50}}=\frac{6\times10^{-2}}{5.5\times10^{-2}}=\frac{12}{11}$

or $\quad T_{50}=\frac{11}{12}\times T_0=\frac{11}{12}\times0.084=0.077\ \text{N/m}$

Considering that change in surface tension as linear, the change in surface tension with temperature is given by

$$\alpha=\frac{T_{50}-T_0}{50}=\frac{0.077-0.084}{50}=-1.4\times10^{-4}\frac{\text{N}}{\text{m°C}}$$

18. **(1130)** Pressure inside the bubble $=p_0+\rho gh+\frac{2T}{r}$

$\therefore$ Amount of pressure inside the bubble greater than the atmospheric pressure $=\rho gh+\frac{2T}{r}$

Substituting the values, we get

$$\Delta p=10^3\times9.8\times0.1+\frac{2\times0.075}{10^{-3}}$$

$$=980+150=1130\ \text{N/m}^2=1130\ \text{Pa}$$

19. **(20.00)** When the tube is taken out, a convex meniscus is formed at the bottom end. The total upward force due to surface tension is

$$F=2\pi rT+2\pi rT=4\pi rT$$

This balances the weight of water column of length H

$$\Rightarrow \quad 4\pi rT=(\pi r^2H)\rho g\Rightarrow H=\frac{4T}{r\rho g}$$

But $h=\frac{2T}{r\rho g}$ therefore $H=2h=20$ cm

The length of the liquid column remaining = 20 cm

Archives

JEE Main

Single Correct Answer Type

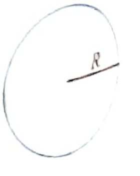
1. **(4)** $W=T\Delta A=0.03[2\times4\pi(5^2-3^2)]\times10^{-4}$

$$=0.384\,\pi\times10^{-3}\ \text{joule}\cong0.4\pi\ \text{mJ}$$

2. **(4)** $2TL=mg$

$$T=\frac{mg}{2L}=\frac{1.5\times10^{-2}}{2\times30\times10^{-2}}=\frac{1.5}{600}=0.025\ \text{N/m}$$

3. **(3)** $\rho4\pi R^2\Delta RL=T4\pi[R^2-(R-\Delta R)^2]$



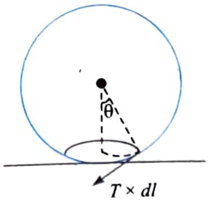
$$\rho R^2\Delta RL=T[R^2-R^2+2R\Delta R-\Delta R^2]$$

$$\rho R^2\Delta RL=T2R\Delta R\quad(\Delta R\text{ is very small})$$

$$R=\frac{2T}{\rho L}$$

None)

$$(2\pi rT)\sin\theta = \frac{4}{3}\pi R^3\rho_w g$$



$$T\times\frac{r}{R}\times 2\pi r = \frac{4}{3}\pi R^3\rho_w g$$

$$r^2 = \frac{2}{3}\frac{R^4\rho_w g}{T}$$

$$r = R^2\sqrt{\frac{2\rho_w g}{3T}}$$

(4) $T = \frac{rhg}{2}\times 10^3$

$$\frac{\Delta T}{T} = \frac{\Delta r}{r}+\frac{\Delta h}{h}+0$$

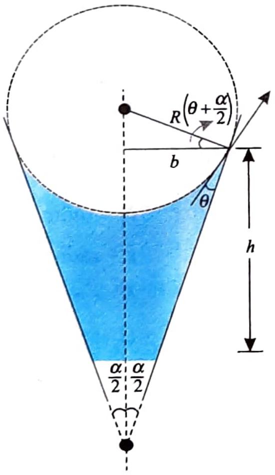
$$100\times\frac{\Delta T}{T} = \left(\frac{10^{-2}\times .01}{1.25\times 10^{-2}}+\frac{10^{-2}\times .01}{1.45\times 10^{-2}}\right)100$$

$$= (0.8 + 0.689) = (1.489)$$

$$100\times\Delta T = 1.489\% \simeq 1.5\%$$

Advanced

gle Correct Answer Type



(4) If R be the meniscus radius

$$R\cos(\theta + \alpha/2) = b$$

Excess pressure on concave side of meniscus $= \frac{2S}{R}$

$$h\rho g = \frac{2S}{R} = \frac{2S}{b}\cos\left(\theta+\frac{\alpha}{2}\right)$$

$$\Rightarrow\ h = \frac{2S}{b\rho g}\cos\left(\theta+\frac{\alpha}{2}\right)$$

(3) Density of sphere is $\rho = \frac{m}{V} = \frac{3m}{4\pi R^3}$

$$\Rightarrow\ \frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{R}\frac{dR}{dt}$$

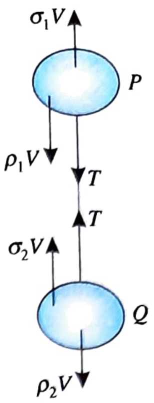
Since $\frac{1}{\rho}\frac{d\rho}{dt}$ is constant

$$\therefore\ \frac{dR}{dt}\propto R$$

Velocity of any point on the circumference v is equal to $\frac{dR}{dt}$ (rate of change of radius of outer layer).

Multiple Correct Answers Type

1. (1), (3)



Free body diagram of P and Q when they are connected by a string.

For P: $\sigma_1 V = \rho_1 V + T$ ...(i)

For Q: $T + \sigma_2 V = \rho_2 V$ ...(ii)

Adding Eqs. (i) and (ii)

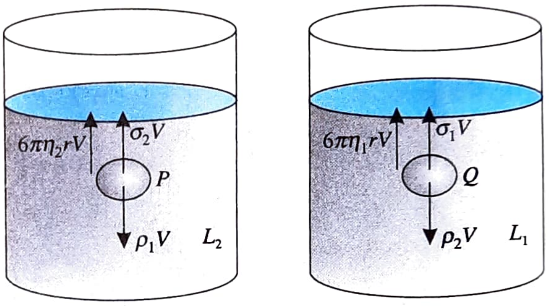
$$\sigma_1 - \rho_2 = \rho_1 - \sigma_2 \quad \text{...(iii)}$$

If the sphere P along in L_2 (in Fig. (a))

Terminal velocity of P; $V_P = \frac{2r^2}{9\eta_2}(\rho_1 - \sigma_2)$

If the sphere Q along in L_1 (in Fig. (b))

Terminal velocity of Q; $V_Q = \frac{2r^2}{9\eta_1}(\rho_2 - \sigma_1)$



(a) (b)

$$\left|\frac{V_P}{V_Q}\right| = \frac{\eta_1}{\eta_2} \text{ and } \vec{V}_P\cdot\vec{V}_Q < 0$$

as if $\rho_1 - \sigma_2 > 0$

$\Rightarrow \rho_2 - \sigma_1 < 0$

2. (1), (3)

The height of liquid in capillary is given by

$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

Here $r \rightarrow$ radius of capillary; $\theta \rightarrow$ contact angle; $\sigma \rightarrow$ surface tension; and $\rho \rightarrow$ density of liquid.

For given material of capillary tube, $\theta \rightarrow$ constant

Hence, $h \propto \frac{1}{r}$

Hence option (1) is correct. h depends on σ, hence option (2) is wrong.

If list is going up with constant acceleration.

$$g_{\text{eff}} = (g + a)$$

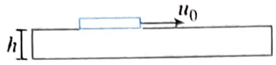
In this case the height of liquid in capillary

$h = \frac{2\sigma\cos\theta}{r\rho(g+a)}$, it means h decreases

It means option (3) is correct. h is proportional to cos θ not θ, hence option (4) is wrong.

3. (1), (3), (4)

Viscous force is given by $F = -\eta A\frac{dv}{dy}$



Since h is very small therefore, magnitude of viscous force is given by

$$F = \eta A \frac{\Delta v}{\Delta y}$$

$$\therefore \quad F = \frac{\eta A u_0}{h} \Rightarrow F \propto \eta \text{ and } F \propto u_0; \quad F \propto \frac{1}{h}, F \propto A$$

Since plate is moving with constant velocity, same force must be acting on the floor.

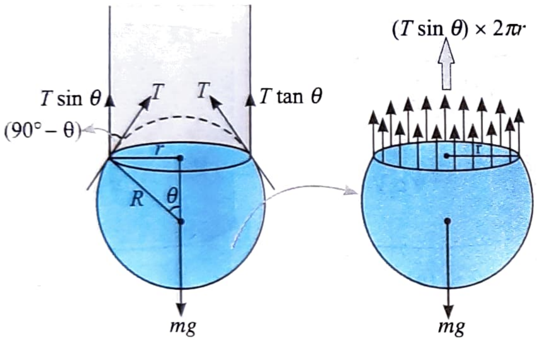
Linked Comprehension Type

1. **(3)** From force body diagram of liquid drop, the vertical component of surface tension on the drop is

$$(F_{\text{surface}})_{\text{vertical}} = (T \sin\theta) - 2\pi r$$

$$= T \cdot 2\pi r \sin\theta = T \cdot 2\pi r \times \frac{r}{R}$$

$$\Rightarrow \quad (F_{\text{surface}})_{\text{vertical}} = \frac{2\pi r^2 T}{R}$$



2. **(1)** The liquid drop detaches form the dropper when the vertical component of the force due to surface tension equals the weight of the drop. At this time

$$\frac{2\pi r^2 T}{R} = mg = \frac{4}{3}\pi R^3 \rho g$$

$$\text{or} \quad R^4 = \frac{3}{2}\frac{\pi^2 T}{\rho g}$$

$$\Rightarrow \quad R = \left(\frac{3}{2}\frac{r^2 T}{\rho g}\right)^{1/4}$$

Substituting the values, we get

$$R = 1.4 \times 10^{-3} \text{ m}$$

3. **(2)** The surface energy of the drop

$$E_{\text{surface}} = 4\pi R^2 \times T$$

$$\text{or} \quad E_{\text{surface}} = 4 \times \frac{22}{7} \times \pi \times (1.4 \times 10^{-3})^2 \times 0.11$$

$$\Rightarrow \quad E_{\text{surface}} = 2.7 \times 10^{-6} \text{ J}$$

Numerical Value Type

1. **(6)** The excess of pressure above atmospheric pressure, due to surface tension in a bubble $\Delta p = \frac{4T}{r}$

The surrounding pressure for 1st bubble

$$P_A = P_0 + \frac{4T}{r_A} = 8 + \frac{4 \times 0.04}{0.02} = 16 \text{ N/m}^2$$

Similarly for 2nd bubble

$$P_B = P_0 + \frac{4T}{r_B} = 8 + \frac{4 \times 0.04}{0.04} = 12 \text{ N/m}^2$$

Using, $PV = nRT$

$$(16)\frac{4}{3}\pi(0.02)^3 = n_A RT \qquad \text{...(i)}$$

$$(12)\left(\frac{4}{3}\pi(0.04)^3\right) = n_B RT \qquad \text{...(ii)}$$

Dividing Eq. (ii) with Eq. (i), we get $\frac{n_B}{n_A} = 6$

2. **(3)** $V_T \propto \frac{r^2[d_m - d_L]}{n}$

$$\frac{V_{TP}}{V_{TQ}} = \left(\frac{r_P}{r_Q}\right)^2 \times \frac{n_{L_2}}{n_{L_1}} \times \left[\frac{d_m - d_{L_1}}{d_m - d_{L_2}}\right]$$

$$\frac{V_{TP}}{V_{TQ}} = \left(\frac{2}{1}\right)^2 \times \frac{2}{3} \times \left[\frac{8 - 0.8}{8 - 1.6}\right]$$

$$\frac{V_{TP}}{V_{TQ}} = 3$$

3. **(6)** $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 \times K$ (Conservation of volume)

R = radius of bigger drop

r = radius of smaller drop

$$R^3 = r^3 \times K \qquad \text{...(i)}$$

$$U_i = S\,(4\pi R^2)$$

$$U_f = KS\,(4\pi r^2)$$

$$KS\,(4\pi r^2) - S \times 4\pi R^2 = 10^{-3}$$

$$4\pi S[Kr^2 - R^2] = 10^{-3}$$

$$K^{1/3} R^2 - R^2 = 10^{-2}$$

$$10^{\alpha/3} - 1 = 100$$

$$10^{\alpha/3} = 101$$

$$\alpha \approx 6$$

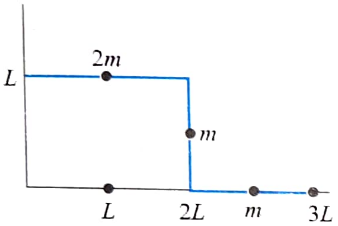
1

APPENDIX

Chapterwise Solved JEE Main 2019 Questions (16 Sets)

Chapter 1
System of Particles and Centre of Mass

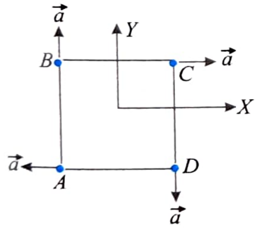
1. The position vector of the centre of mass $\vec{r}_{cm}$ of a symmetric uniform bar of negligible area of cross-section as shown in figure is



(1) $\vec{r}_{cm} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$ (2) $\vec{r}_{cm} = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$

(3) $\vec{r}_{cm} = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$ (4) $\vec{r}_{cm} = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$

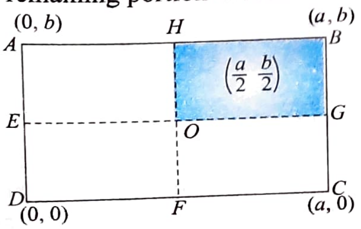
2. Four particles A, B, C and D with masses $m_A = m, m_B = 2m, m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is



(1) $\frac{a}{5}(\hat{i} - \hat{j})$ (2) $\frac{a}{5}(\hat{i} + \hat{j})$

(3) zero (4) $a(\hat{i} + \hat{j})$

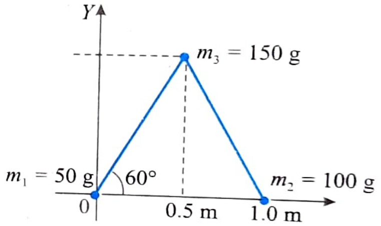
3. A uniform rectangular thin sheet $ABCD$ of mass M has length a and breadth b, as shown in the figure. If the shaded portion $HBGO$ is cut-off, the coordinates of the centre of mass of the remaining portion will be



(1) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (2) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

(3) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (4) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

4. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be



(1) $\left(\frac{7}{12}\text{m}, \frac{\sqrt{3}}{8}\text{m}\right)$ (2) $\left(\frac{\sqrt{3}}{4}\text{m}, \frac{5}{12}\text{m}\right)$

(3) $\left(\frac{7}{12}\text{m}, \frac{\sqrt{3}}{4}\text{m}\right)$ (4) $\left(\frac{\sqrt{3}}{8}\text{m}, \frac{7}{12}\text{m}\right)$

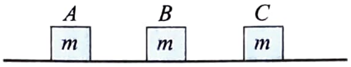
Chapter 2
Impulse and Collision

5. A simple pendulum, made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then M is given by

(1) $\frac{m}{2}\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$ (2) $\frac{m}{2}\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$

(3) $m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$ (4) $m\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$

6. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an brutal speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is the value of M/m?



(1) 4 (2) 5
(3) 3 (4) 2

7. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is ($g = 10\ \text{ms}^{-2}$)

(1) 30 m (2) 10 m
(3) 40 m (4) 20 m

8. A body of mass 1 kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10\ \text{ms}^{-2}$, the value of x will be close to

(1) 4 cm (2) 8 cm
(3) 80 cm (4) 40 cm

9. An alpha-particle of mass m suffers 1-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is

(1) $4m$ (2) $3.5m$
(3) $2m$ (4) $1.5m$

10. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively. If $m_2 = 0.5m_1$ and $v_3 = 0.5v_1$, then v_1 is

(1) $v_4 - \frac{v_2}{4}$ (2) $v_4 - \frac{v_2}{2}$
(3) $v_4 - v_2$ (4) $v_4 + v_2$

11. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body?

(1) 1.8 kg (2) 1.2 kg
(3) 1.5 kg (4) 1.0 kg

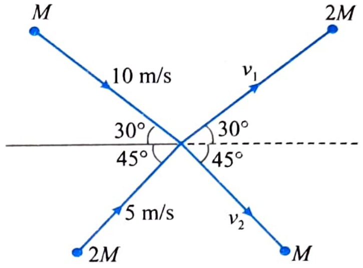
12. A particle of mass 'm' is moving with speed '$2v$' and collides with a mass '$2m$' moving with speed V in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the original direction. The speed of each of the moving particle will be

(1) $\frac{v}{(2\sqrt{2})}$ (2) $2\sqrt{2}\,v$
(3) $\sqrt{2}\,v$ (4) $\frac{v}{\sqrt{2}}$

13. A wedge of mass $M = 4m$ lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by

(1) $\frac{2v^2}{7g}$ (2) $\frac{v^2}{g}$
(3) $\frac{2v^2}{5g}$ (4) $\frac{v^2}{2g}$

14. Two particles, of masses M and $2M$, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2, respectively. The values of v_1 and v_2 are nearly



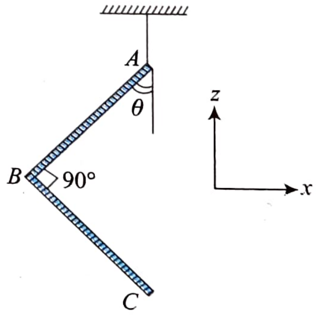
(1) 3.2 m/s and 6.3 m/s (2) 3.2 m/s and 12.6 m/s
(3) 6.5 m/s and 6.3 m/s (4) 6.5 m/s and 3.2 m/s

15. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is

(1) 0.20 ms^{-1} (2) 0.14 ms^{-1}
(3) 0.47 ms^{-1} (4) 0.28 ms^{-1}

Chapter 3
Rigid Body Dynamics: Part 1

16. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If $AB = BC$, and the angle made by AB with downward vertical is θ, then



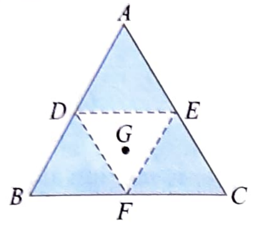
(1) $\tan\theta = \frac{2}{\sqrt{3}}$ (2) $\tan\theta = \frac{1}{3}$
(3) $\tan\theta = \frac{1}{2}$ (4) $\tan\theta = \frac{1}{2\sqrt{3}}$

17. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is

(1) 12 cm (2) 18 cm

(3) 16 cm (4) 14 cm

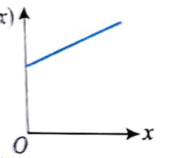
8. An equilateral triangle ABC is cut from a thin solid sheet of wood (see figure). D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then

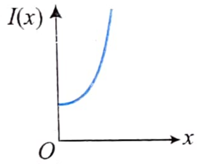


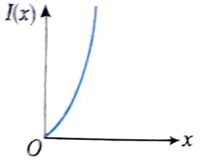
(1) $I = \frac{9}{16} I_0$ (2) $I = \frac{3}{4} I_0$

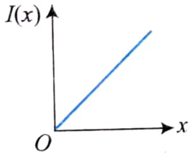
(3) $I = \frac{I_0}{4}$ (4) $I = \frac{15}{16} I_0$

19. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is $I(x)$. Which one of the graphs represents the variation of $I(x)$ with x correctly?

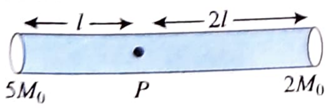
(1)


(2)


(3)


(4)


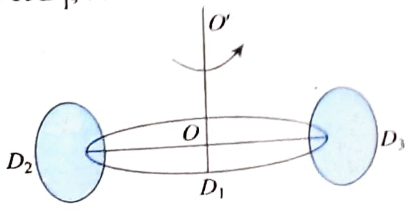
20. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be



(1) $\frac{g}{2l}$ (2) $\frac{7g}{3l}$

(3) $\frac{g}{13l}$ (4) $\frac{g}{3l}$

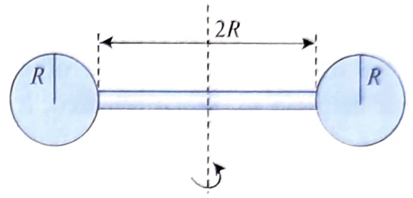
21. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1, as shown in the figure, will be



(1) $3MR^2$ (2) $\frac{2}{3}MR^2$

(3) MR^2 (4) $\frac{4}{5}MR^2$

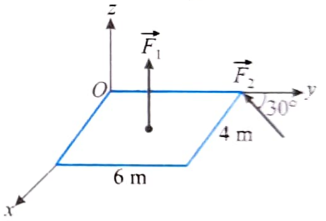
22. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is



(1) $\frac{152}{15}MR^2$ (2) $\frac{17}{15}MR^2$

(3) $\frac{137}{15}MR^2$ (4) $\frac{209}{15}MR^2$

23. A slab is subjected to two forces $\vec{F_1}$ and $\vec{F_2}$ of same magnitude F as shown in the figure. Force F_2 is in XY-plane while force F_1 acts along z-axis at the point $(2\hat{i} + 3\hat{j})$. The moment of these forces about point O will be



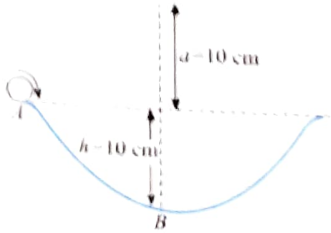
(1) $(3\hat{i} - 2\hat{j} - 3\hat{k})F$ (2) $(3\hat{i} + 2\hat{j} + 3\hat{k})F$

(3) $(3\hat{i} + 2\hat{j} - 3\hat{k})F$ (4) $(3\hat{i} - 2\hat{j} + 3\hat{k})F$

24. The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5 m, the angle between the force and the position vector is (in radians)

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

25. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be (Take $g = 10$ m/s^2)



(1) 8 kg-m^2/s (2) 6 kg-m^2/s

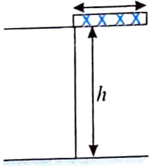
(3) 3 kg-m^2/s (4) 2 kg-m^2/s

26. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$

(3) $\frac{3}{5}$ (4) $\frac{8}{5}$

27. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slips off the table in a very short time $\tau = 0.01$ s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to



(1) 0.02 (2) 0.28

(3) 0.5 (4) 0.3

28. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is

(1) $\frac{k}{2I}\theta$ (2) $\frac{k}{I}\theta$

(3) $\frac{k}{4I}\theta$ (4) $\frac{2k}{I}\theta$

29. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of

(1) 2 s (2) 5 s

(3) 2.5 s (4) 3 s

30. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be

(1) $\frac{M\omega_0}{M+3m}$ (2) $\frac{M\omega_0}{M+m}$

(3) $\frac{M\omega_0}{M+2m}$ (4) $\frac{M\omega_0}{M+6m}$

31. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is

(1) $\frac{MR^2}{6}$ (2) $\frac{MR^2}{3}$

(3) $\frac{2MR^2}{3}$ (4) $\frac{MR^2}{2}$

32. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$ about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is

(1) $\frac{I_1\omega_1^2}{12}$ (2) $\frac{3}{8}I_1\omega_1^2$

(3) $\frac{I_1\omega_1^2}{6}$ (4) $\frac{I_1\omega_1^2}{24}$

33. A particle of mass m is moving along a trajectory given by

$x = x_0 + a\cos\omega_1 t$

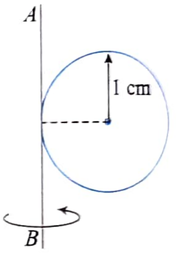
$y = y_0 + b\sin\omega_2 t$

The torque, acting on the particle about the origin, at $t = 0$ is

(1) $m(-x_0 b + y_0 a)\omega_1^2\hat{k}$ (2) $+my_0 a\,\omega_1^2\hat{k}$

(3) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$ (4) Zero

34. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5 s, is close to



(1) 4.0×10^{-6} Nm (2) 2.0×10^{-5} Nm

(3) 1.6×10^{-5} Nm (4) 7.9×10^{-6} Nm

35. The time dependence of the position of a particle of mass $m = 2$ is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time $t = 2$ is

(1) $36\hat{k}$ (2) $-34(\hat{k} - \hat{i})$

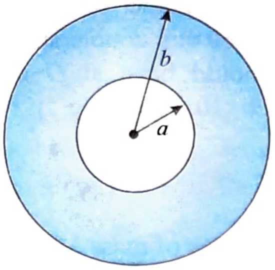
(3) $48(\hat{i} + \hat{j})$ (4) $-48\hat{k}$

36. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $7M/8$ and is converted into a uniform disc of radius $2R$. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by

(1) 185 (2) 65

(3) 285 (4) 140

37. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc vanes as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is



(1) $\frac{a+b}{2}$ (2) $\frac{a+b}{3}$

(3) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (4) $\sqrt{\frac{a^2+b^2+ab}{3}}$

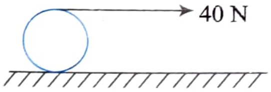
8. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance $l (l << L)$, is close to

(1) Mgl (2) $Mgl\,(1+\theta_0^2)$

(3) $Mgl\,(1-\theta_0^2)$ (4) $Mgl\left(1+\frac{\theta_0^2}{2}\right)$

Chapter 4
Rigid Body Dynamics: Part 2

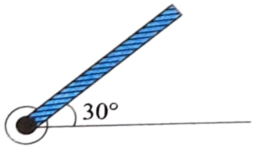
9. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string)



(1) 12 rad/s^2 (2) 16 rad/s^2

(3) 10 rad/s^2 (4) 20 rad/s^2

0. A rod of length 50 cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be ($g = 10$ ms^{-2})



(1) $\sqrt{30}$ (2) $\sqrt{\frac{30}{2}}$

(3) $\frac{\sqrt{30}}{2}$ (4) $\frac{\sqrt{20}}{3}$

1. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is

(1) $\frac{3F}{2mR}$ (2) $\frac{F}{3mR}$

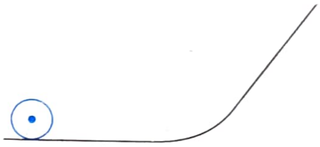
(3) $\frac{2F}{3mR}$ (4) $\frac{F}{2mR}$

42. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is

(1) $\frac{2}{3}\mu FR$ (2) $\frac{\mu FR}{3}$

(3) $\frac{\mu FR}{2}$ (4) $\frac{\mu FR}{6}$

43. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{\text{sph}}}{h_{\text{cyl}}}$ is given by



(1) $\frac{14}{15}$ (2) $\frac{4}{5}$ (3) 1 (4) $\frac{2}{\sqrt{5}}$

44. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane: (i) a ring of radius R, (ii) a solid cylinder of radius $R/2$ and (iii) a solid sphere of radius $R/4$. If in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is

(1) 4 : 3 : 2 (2) 14 : 15 : 20

(3) 10 : 15 : 7 (4) 2 : 3 : 4

Chapter 5
Gravitation

45. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is

(1) $\frac{3}{2}mv^2$ (2) mv^2

(3) $2mv^2$ (4) $\frac{1}{2}mv^2$

46. A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force exerts on a point mass 'm' at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by

(1) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)-BL\right]$

(2) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

(3) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)+BL\right]$

(4) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$

47. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be

(1) in a circular orbit of a different radius

(2) in the same circular orbit of radius R

(3) in an elliptical orbit

(4) such that it escapes to infinity

48. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is

(1) $\frac{4L}{m}$ (2) $\frac{L}{m}$

(3) $\frac{L}{2m}$ (4) $\frac{2L}{m}$

49. A satellite is revolving in a circular orbit at a height h from the earth surface, such that $h << R$ where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is

(1) $\sqrt{gR}\,(\sqrt{2}-1)$ (2) $\sqrt{2gR}$

(3) $\sqrt{gR}$ (4) $\sqrt{\frac{gR}{2}}$

50. Two satellites, A and B, have masses m and $2m$ respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius $2R$ around the earth. The ratio of their kinetic energies, T_A/T_B, is

(1) 2 (2) $\sqrt{\frac{1}{2}}$

(3) 1 (4) $\frac{1}{2}$

51. The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth = 6.4×10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is

(1) 1.28×10^4 km (2) 6.4×10^3 km

(3) 3.2×10^3 km (4) 1.6×10^3 km

52. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2 s. The period of oscillation of the same pendulum on the planet would be

(1) $\frac{2}{\sqrt{3}}$ s (2) $2\sqrt{3}$ s

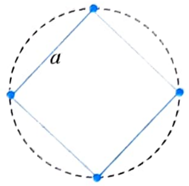
(3) $\frac{\sqrt{3}}{2}$ s (4) $\frac{3}{2}$ s

53. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is (Take gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2})

(1) 1.4×10^5 m/s (2) 24×10^4 m/s

(3) 3.8×10^4 m/s (4) 2.8×10^5 m/s

54. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of other's gravitational field in a circular orbit circumscribing the square?



(1) $1.21\sqrt{\frac{GM}{a}}$ (2) $1.41\sqrt{\frac{GM}{a}}$

(3) $1.16\sqrt{\frac{GM}{a}}$ (4) $1.35\sqrt{\frac{GM}{a}}$

55. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon

(1) $\frac{E}{4}$ (2) $\frac{E}{16}$

(3) $\frac{E}{32}$ (4) $\frac{E}{64}$

56. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness $2a$ and mass $2M$. The gravitational field at distance '$3a$' from the centre will be

(1) $\frac{2GM}{9a^2}$ (2) $\frac{GM}{3a^2}$

(3) $\frac{GM}{9a^2}$ (4) $\frac{2GM}{3a^2}$

57. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r) = \frac{K}{r^2}$.

Identify the correct relation between the radius R of the particle's orbit and its period T

(1) T/R^2 is a constant (2) TR is a constant

(3) T^2/R^3 is a constant (4) T/R is a constant

58. The value of acceleration due to gravity at Earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to (Radius of earth = 6.4×10^6 m)

(1) 1.6×10^6 m (2) 6.4×10^6 m

(3) 9.0×10^6 m (4) 2.6×10^6 m

9. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet?

[Given: Mass of planet = 8×10^{22} kg; Radius of planet = 2×10^6 m, Gravitational constant $G = 6.67\times10^{-11}\ \text{Nm}^2/\text{kg}^2$]

(1) 9 (2) 11

(3) 13 (4) 17

60. The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet? (Take the planets to have the same mass density)

(1) $\frac{R}{3}$ (2) $\frac{R}{2}$

(3) $\frac{R}{4}$ (4) $\frac{R}{9}$

Chapter 6
Fluid Mechanics

61. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be

(1) pv^2 (2) $\frac{3}{4}pv^2$

(3) $\frac{1}{2}pv^2$ (4) $\frac{1}{4}pv^2$

62. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be

(1) 1.2 (2) 0.1

(3) 2.0 (4) 0.4

63. The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to

(1) 9.6 m (2) 4.8 m

(3) 2.9 m (4) 6.0 m

64. Water flows into a large tank with flat bottom at the rate of $10^{-4}\ \text{m}^3\ \text{s}^{-1}$. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is

(1) 4 cm (2) 2.9 cm

(3) 1.7 cm (4) 5.1 cm

65. A wooden block floating in a bucket of water has 4/5 of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is

(1) 0.5 (2) 0.7

(3) 0.6 (4) 0.8

66. A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then $d_2 - d_1$ is approximately (density of water = 10^3 kg/m^3 and acceleration due to gravity = 10 ms^{-2})

(1) 500 m (2) 400 m

(3) 300 m (4) 600 m

67. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? (Take density of water = 10^3 kg/m^3)

(1) 65.4 kg (2) 87.5 kg

(3) 30.1 kg (4) 46.3 kg

68. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be (Take g = 10 ms^{-2})

(1) 1×10^{-5} m^2 (2) 5×10^{-5} m^2

(3) 2×10^{-5} m^2 (4) 5×10^{-4} m^2

Chapter 7
Elasticity

69. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is

(1) $\frac{F}{2A\alpha\Delta T}$ (2) $\frac{F}{A\alpha(\Delta T - 273)}$

(3) $\frac{F}{A\alpha\Delta T}$ (4) $\frac{2F}{A\alpha\Delta T}$

70. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is

(1) 4.0 mm (2) 3.0 mm

(3) 5.0 mm (4) zero

71. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to

(1) 10^4 Nm^{-2} (2) 10^8 Nm^{-2}

(3) 10^6 Nm^{-2} (4) 10^3 Nm^{-2}

72. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that $g = 3.1\pi \, ms^{-2}$, what will be the tensile stress that would be developed in the wire?

(1) 4.8×10^6 Nm^{-2} (2) 5.2×10^6 Nm^{-2}

(3) 6.2×10^6 Nm^{-2} (4) 3.1×10^6 Nm^{-2}

73. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to

(1) 1.9 mm (2) 1.7 mm

(3) 1.5 mm (4) 1.3 mm

74. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit?

(1) 1.16 mm (2) 0.90 mm

(3) 1.36 mm (4) 1.00 mm

75. In an experiment, brass and steel wires of length 1 m each with areas of cross section 1 mm^2 are used, the wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is (Given, the Young's Modulus for steel and brass are respectively, 120×10^9 N/m^2 and 60×10^9 N/m^2)

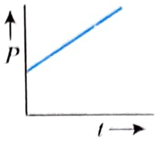
(1) 0.2×10^6 N/m^2 (2) 4.0×10^6 N/m^2

(3) 1.8×10^6 N/m^2 (4) 1.2×10^6 N/m^2

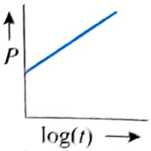
Chapter 8
Surface Tension and Viscosity

76. A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by

(1)



(2)



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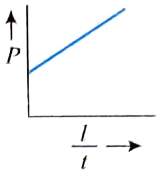

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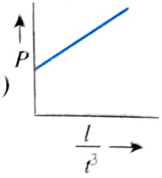

77. Water from a pipe is coming at a rate of 100 litres per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of (density of water = 1000 kg/m^3, coefficient of viscosity of water = 1 mPas)

(1) 10^6 (2) 10^3 (3) 10^4 (4) 10^2

78. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '$2r$' is

(1) $4M$ (2) M

(3) $2M$ (4) $\frac{M}{2}$

79. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to)

(1) 2/3 (2) 3/5 (3) 2/5 (4) 4/5

80. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio (v_1/v_2) equals

(1) 1/27 (2) 1/9 (3) 27 (4) 9

Answer Key

1. (1)	**2.** (1)	**3.** (4)	**4.** (3)	**5.** (3)	**6.** (1)	**7.** (3)	**8.** (1)	**9.** (1)	**10.** (3)
11. (2)	**12.** (2)	**13.** (3)	**14.** (3)	**15.** (1)	**16.** (2)	**17.** (3)	**18.** (4)	**19.** (2)	**20.** (3)
21. (1)	**22.** (3)	**23.** (4)	**24.** (2)	**25.** (2)	**26.** (4)	**27.** (3)	**28.** (4)	**29.** (1)	**30.** (4)
31. (3)	**32.** (4)	**33.** (2)	**34.** (2)	**35.** (4)	**36.** (4)	**37.** (4)	**38.** (2)	**39.** (2)	**40.** (1)
41. (3)	**42.** (1)	**43.** (1)	**44.** (*)	**45.** (2)	**46.** (2)	**47.** (3)	**48.** (3)	**49.** (1)	**50.** (3)
51. (3)	**52.** (2)	**53.** (4)	**54.** (3)	**55.** (2)	**56.** (2)	**57.** (4)	**58.** (4)	**59.** (2)	**60.** (2)
61. (2)	**62.** (3)	**63.** (2)	**64.** (4)	**65.** (3)	**66.** (3)	**67.** (2)	**68.** (2)	**69.** (3)	**70.** (2)
71. (3)	**72.** (4)	**73.** (2)	**74.** (1)	**75.** (*)	**76.** (4)	**77.** (3)	**78.** (3)	**79.** (3)	**80.** (4)

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* No option is matching.

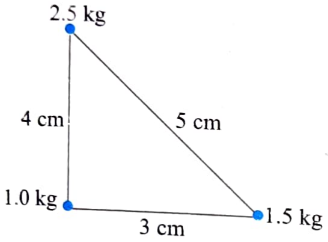
2

APPENDIX

Chapterwise Solved JEE Main January 2020 Questions (6 Sets)

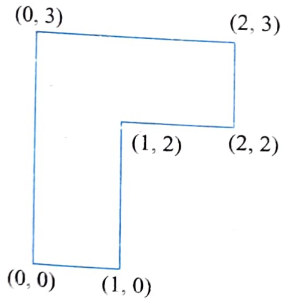
Chapter 1
(System of Particles and Centre of Mass)

1. Three-point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point



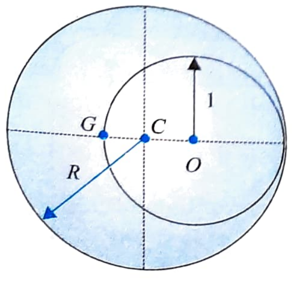
(1) 1.5 cm right and 1.2 cm above 1 kg mass
(2) 0.9 cm right and 2.0 cm above 1 kg mass
(3) 0.6 cm right and 2.0 cm above 1 kg mass
(4) 2.0 cm right and 0.9 cm above 1 kg mass

2. The coordinates of centre of mass of a uniform flag-shaped lamina (thin flat plate) of mass 4 kg (the coordinates of the same are shown in figure) are



(1) (1.25 m, 1.50 m)　(2) (1 m, 1.75 m)
(3) (0.75 m, 0.75 m)　(4) (0.75 m, 1.75 m)

3. As shown in figure, when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e., on the surface of the cavity. R can be determined by the equation



(1) $(R^2 - R + 1)(2 - R) = 1$
(2) $(R^2 + R - 1)(2 - R) = 1$
(3) $(R^2 + R + 1)(2 - R) = 1$
(4) $(R^2 - R - 1)(2 - R) = 1$

4. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at

(1) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$　(2) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$
(3) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$　(4) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$

Chapter 2
(Impulse and Collision)

5. Consider a uniform rod of mass $M = 4m$ and length l pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod–mass system just after the collision is

(1) $\frac{3}{7\sqrt{2}}\frac{v}{l}$　(2) $\frac{3\sqrt{2}}{7}\frac{v}{l}$
(3) $\frac{4}{7}\frac{v}{l}$　(4) $\frac{3}{7}\frac{v}{l}$

6. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is

(1) $\frac{1}{2}$　(2) $\sqrt{\frac{1}{2}}$
(3) $\sqrt{\frac{3}{4}}$　(4) $\sqrt{\frac{3}{2}}$

7. Two particles of equal mass m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is

(1) $\frac{3}{4}mu^2$ (2) $\frac{1}{8}mu^2$

(3) $\sqrt{\frac{2}{3}}mu^2$ (4) $\frac{1}{3}mu^2$

8. A particle of mass m is projected with a speed u from the ground at an angle $\theta = \pi/3$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity . The horizontal distance covered by the combined mass before reaching the ground is

(1) $\frac{5}{8}\frac{u^2}{g}$ (2) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$

(3) $2\sqrt{2}\frac{u^2}{g}$ (4) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$

9. A body A, of mass $m = 0.1$ kg has an initial velocity of $3\hat{i}\ \text{ms}^{-1}$. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j}\ \text{ms}^{-1}$. After collision, A moves with a velocity $\vec{v} = 4(\hat{i} + \hat{j})$. The energy of B after collision is written as $\frac{x}{10}$ J. The value of x is____________.

Chapter 3
(Rigid Body Dynamics: Part 1)

10. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is

(1) $\frac{1}{8}l$ (2) $\sqrt{\frac{7}{48}}l$

(3) $\sqrt{\frac{3}{8}}l$ (4) $\frac{1}{4}l$

11. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is

(1) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (2) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

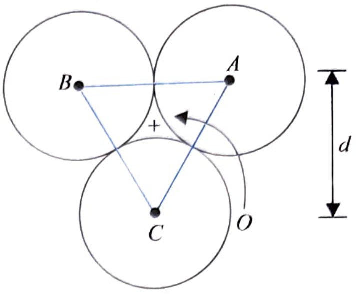
(3) $2\pi a^4\left(\frac{Aa}{4}+\frac{B}{5}\right)$ (4) $2\pi a^4\left(\frac{A}{5}+\frac{Ba}{4}\right)$

12. A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length l. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is

(1) $\frac{ml\omega^2}{k+m\omega^2}$ (2) $\frac{ml\omega^2}{k-m\omega^2}$

(3) $\frac{ml\omega^2}{k-\omega m}$ (4) $\frac{ml\omega^2}{k+\omega m}$

13. Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0/I_A of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is



(1) $\frac{13}{23}$ (2) $\frac{15}{13}$

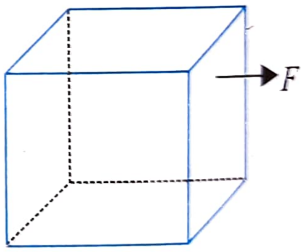
(3) $\frac{23}{13}$ (4) $\frac{13}{15}$

14. A spring mass system (mass m, spring constant k and natural length l) rests in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system rotates about its axis with an angular velocity $\omega\left(k >> m\omega^2\right)$, the relative change in the length of the spring is best given by the option

(1) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$ (2) $\frac{2m\omega^2}{k}$

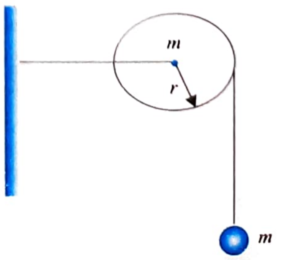
(3) $\frac{m\omega^2}{3k}$ (4) $\frac{m\omega^2}{k}$

15. Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for a box not to topple before moving is ________.



Chapter 4
(Rigid Body Dynamics: Part 2)

16. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be



(1) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$ (2) $r\sqrt{\frac{3}{4gh}}$

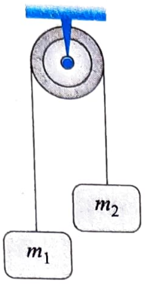
(3) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (4) $r\sqrt{\frac{3}{2gh}}$

7. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is

(1) 8.75×10^{-4} J (2) 8.75×10^{-3} J

(3) 6.25×10^{-4} J (4) 1.13×10^{-3} J

18. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see figure). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descends by a distance h is



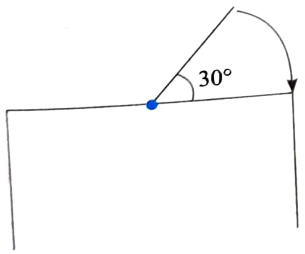
(1) $\left[\frac{(m_1 - m_2)}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}} gh$

(2) $\left[\frac{2(m_1 + m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}}$

(3) $\left[\frac{m_1 + m_2}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}} gh$

(4) $\left[\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}}$

19. One end of a straight uniform 1m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}\ \text{s}^{-1}$, where n is an integer. The value of n is ________.



Chapter 5
(Gravitation)

20. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth)

(1) $\frac{m}{20}\left(u - \sqrt{\frac{2GM}{3R}}\right)^2$ (2) $5m\left(u^2 - \frac{119}{200}\frac{GM}{R}\right)$

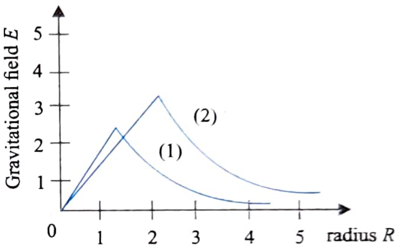
(3) $\frac{3m}{8}\left(u + \sqrt{\frac{5GM}{6R}}\right)^2$ (4) $\frac{m}{20}\left(u^2 + \frac{113}{200}\frac{GM}{R}\right)$

21. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g = 10\ \text{ms}^{-2}$ at the north pole and the radius of the earth = 6400 km)

(1) 195.66 N (2) 194.66 N

(3) 194.32 N (4) 195.32 N

22. Consider two solid spheres of radii $R_1 = 1$ m, $R_2 = 2$ m and masses M_1 and M_2, respectively. The gravitational field due to sphere (1) and (2) are shown. The value of $\frac{M_1}{M_2}$ is



(1) $\frac{1}{2}$ (2) $\frac{2}{3}$

(3) $\frac{1}{3}$ (4) $\frac{1}{6}$

23. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ of A.

The collision is completely inelastic. Then, the combined body

(1) starts moving in an elliptical orbit around the planet

(2) continues to move in a circular orbit

(3) falls vertically downwards towards the planet

(4) escapes from the planet's gravitational field

24. Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is

(1) 4 (2) 2

(3) 1 (4) 3

25. An asteroid is moving directly towards the centre of the earth. When at a distance of $10R$ (R is the radius of the earth) from the earth's centre, it has a speed of 12 km/s. Neglecting the effect of earth's atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s)? Give your answer to the nearest integer in kilometer/s ________.

Chapter 6
(Fluid Mechanics)

26. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{3}{4}$

(3) $\frac{81}{256}$ (4) $\frac{9}{16}$

27. Consider a solid sphere of radius R and mass density $\rho(r) = \rho_0\left(1-\frac{r^2}{R^2}\right)$, $0 < r \le R$. The minimum density of a liquid in which it will float is

(1) $\frac{\rho_0}{5}$ (2) $\frac{\rho_0}{3}$

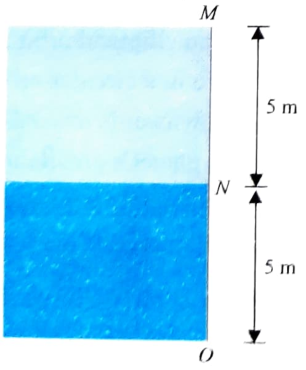
(3) $\frac{2\rho_0}{3}$ (4) $\frac{2\rho_0}{5}$

28. A leak proof cylinder of length 1 m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density of water at T = 4°C, relative to the density at T = 0°C is close to

(1) 1.01 (2) 1.04

(3) 1.03 (4) 1.26

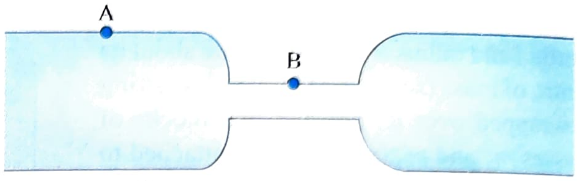
29. Two liquids of densities ρ_1 and ρ_2 ($\rho_2 = 2\rho_1$) are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing)



(1) $\frac{1}{4}$ (2) $\frac{2}{3}$

(3) $\frac{1}{3}$ (4) $\frac{1}{2}$

30. Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube.
(density of water = 1000 kgm^{-3})



(1) 1810 cm^3/s (2) 3020 cm^3/s

(3) 2720 cm^3/s (4) 2420 cm^3/s

Chapter 7
(Elasticity)

31. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is

(1) $1:\sqrt{2}$ (2) 2 : 1

(3) $\sqrt{2}:1$ (4) 1 : 2

32. A body of mass m = 10 kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in space station (Breaking stress of wire = 4.8 × 10^7 Nm^{-2} and area of cross-section of the wire = 10^{-2} cm^2) is ________.

Chapter 8
(Surface Tension and Viscosity)

33. A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet)

(1) $r = \sqrt{\frac{3T}{(2d-\rho)g}}$ (2) $r = \sqrt{\frac{T}{(d+\rho)g}}$

(3) $r = \sqrt{\frac{2T}{3(d+\rho)g}}$ (4) $r = \sqrt{\frac{T}{(d-\rho)g}}$

Answer Key

1. (2) **2.** (4) **3.** (3) **4.** (4) **5.** (2) **6.** (4) **7.** (2) **8.** (3) **9.** (1) **10.** (2)
11. (1) **12.** (2) **13.** (1) **14.** (4) **15.** (75) **16.** (3) **17.** (1) **18.** (2) **19.** (15) **20.** (2)
21. (4) **22.** (4) **23.** (1) **24.** (1) **25.** (16) **26.** (4) **27.** (4) **28.** (1) **29.** (1) **30.** (3)
31. (4) **32.** (4) **33.** (2)

3

APPENDIX

JEE Advanced 2019 Solved Questions

Single Correct Answer Type

avitation

1. Consider a spherical gaseous cloud of mass density $\rho(r)$ in a free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about their common center with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time. The particle number density $n(r) = \rho(r)/m$ is (G = universal gravitational constant)

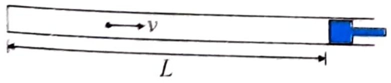
(1) $\dfrac{K}{2\pi r^2 m^2 G}$ (2) $\dfrac{K}{6\pi r^2 m^2 G}$

(3) $\dfrac{3K}{\pi r^2 m^2 G}$ (4) $\dfrac{K}{\pi r^2 m^2 G}$

Multiple Correct Answers Type

Impulse and Collision

2. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \dfrac{dL}{L} v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct?



(1) The rate at which the particle strikes the piston is v/L

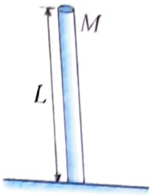
(2) After each collision with the piston, the particle speed increases by $2V$

(3) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\dfrac{1}{2}L_0$

(4) If the piston moves inward by dL, the particle speed increases by $2v\dfrac{dL}{L}$

Rigid Body Dynamics: Part 2

3. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical? [g is the acceleration due to gravity]



(1) The radial acceleration of the rod's center of mass will be $\dfrac{3g}{4}$

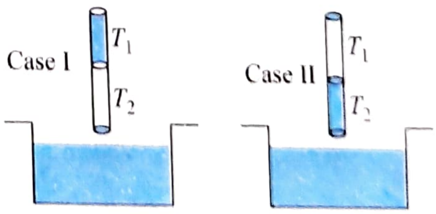
(2) The angular acceleration of the rod will be $\dfrac{2g}{L}$

(3) The angular speed of the rod will be $\sqrt{\dfrac{3g}{2L}}$

(4) The normal reaction force from the floor on the rod will be $\dfrac{Mg}{16}$

Surface Tension and Viscosity

4. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 0° and 60°, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is (are) correct ? (Surface tension of water = 0.075 N/m, density of water $= 1000\,\text{kg/m}^3$, take $g = 10\ \text{m/s}^2$)



(1) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

(2) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

(3) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

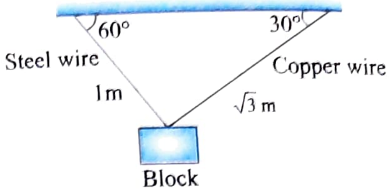
(4) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

Numerical Value Type

Elasticity

5. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm^2 and, length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is (Δl_C) and elongation in steel wire is (Δl_S), then the ratio $\frac{\Delta l_C}{\Delta l_S}$ is __________.

[Young's modulus for copper and steel are 1×10^{11} N/m^2 and 2×10^{11} N/m^2 respectively]



Answer Key

Single Correct Answer Type

1. (1)

Multiple Correct Answers Type

2. (2, 3)
3. (1, 3, 4)
4. (1, 3, 4)

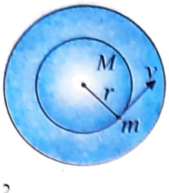
Numerical Value Type

5. (2.00)

Hints and Solutions

(1) Let total mass included in a sphere of radius r be M.

For a particle of mass m,



$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$= \frac{2}{r}\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \quad \frac{GMm}{r^2} = \frac{2K}{r}$$

$$\Rightarrow \quad M = \frac{2Kr}{Gm}$$

$$\Rightarrow \quad dM = \frac{2K}{Gm}dr$$

$$\Rightarrow \quad 4\pi r^2 dr \rho = \frac{2K}{Gm}dr$$

$$\therefore \quad \rho = \frac{K}{2\pi Gmr^2}$$

Particle number density $= \dfrac{\rho}{m} = \dfrac{K}{2\pi Gm^2r^2}$

2. (2, 3)

Let piston is at a distance L from the fixed wall. During time dt, the piston moves a distance dL.

Hence, $Vdt = -dL$

$$\text{Or} \quad dt = -\left(\frac{dL}{V}\right) \qquad \text{...(i)}$$

Negative sign as L decreases

Number of collision per second at this moment.

$$n = \frac{v}{2L} \qquad \text{...(ii)}$$

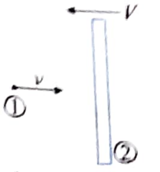
v is the speed of the particle at this time.

Total number of collision during the time dt

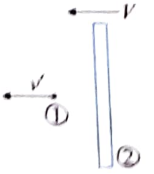
$$N = n \cdot dt = \left(\frac{v}{2L}\right)\left(-\frac{dL}{V}\right)$$

$$\text{Or} \quad N = -\left(\frac{v}{2L}\right)\frac{dL}{V} \qquad \text{...(iii)}$$

Considering collision of the particle with piston.



Just before collision



Just after collision

$$\text{As} \quad e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$1 = \frac{V - v'}{-v - V} \Rightarrow v' = v + 2V$$

After each collision the speed of the particle changes by $2V$.

Change in speed in time dt

$= 2V \times$ Number of collision in time dt

$$dv = 2V \times \left[-\left(\frac{v}{2L}\right)\cdot\frac{dL}{V}\right]$$

$$dv = -v\cdot\frac{dL}{L} \Rightarrow \frac{dv}{v} = -\frac{dL}{L} \qquad \text{...(iv)}$$

Hence option (4) is wrong.

Integrating Eq. (iv)

$$\int_0^v \frac{dv}{v} = -\int_{L_0}^{L_0/2}\frac{dL}{L}$$

$$\ln\frac{v}{v_0} = -[\ln L]_{L_0}^{L_0/2} = \ln 2$$

The kinetic energy at this position

$$(\text{KE})_{L_0/2} = \frac{1}{2}m(2v_0)^2 = 4\left(\frac{1}{2}mv_0^2\right) = 4(\text{KE})_{L_0}$$

Hence option (3) is correct.

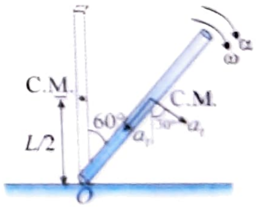
3. (1, 3, 4)

We can treat contact point as hinged. Applying work energy theorem

$$W_g = \Delta\text{KE}$$

$$mg\frac{L}{4} = \frac{1}{2}\left(\frac{mL^2}{3}\right)\omega^2$$

$$\omega = \sqrt{\frac{3g}{2L}}$$



Radial acceleration of C.M. of rod $= \left(\dfrac{L}{2}\right)\omega^2 = \dfrac{3g}{4}$

Using $\tau = I\alpha$ about contact point

Net vertical acceleration of C.M. of rod

$$\frac{mgL}{2}\sin 60^\circ = \frac{mL^2}{3}\alpha \Rightarrow \alpha = \frac{3\sqrt{3}}{4L}g$$

Net vertical acceleration of C.M. of rod

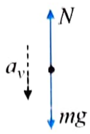
$$a_y = a_r\cos 60^\circ + a_t\cos 30^\circ$$

$$= \left(\frac{3g}{4}\right)\left(\frac{1}{2}\right) + \left(\alpha\frac{L}{2}\right)\cos 30^\circ$$

$$= \frac{3g}{8} + \frac{3\sqrt{3}g}{4L}\left(\frac{L}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{3g}{8} + \frac{9g}{16} = \frac{15}{16}g$$

Applying $F_{net} = ma$ in vertical direction on rod as system



$$mg - N = ma_y = m\left(\frac{15}{16}g\right)$$

$$\Rightarrow N = \frac{mg}{16}$$

(1, 3, 4)

Height of water column in capillary tube

$$h = \frac{2T\cos\theta}{\rho g R}$$

For tube T_1, $h_1 = \dfrac{2\times 0.075\times\cos 0^\circ}{1000\times 10\times 0.2\times 10^{-3}}$

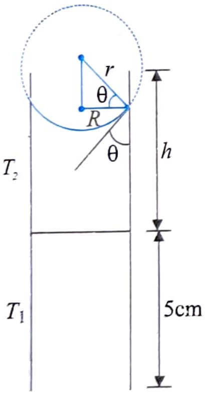
$\Rightarrow h_1 = 75$ mm (in T_1) [If we assume entire tube of T_1]

For tube T_2, $h_2 = \dfrac{2\times 0.075\times\cos 60^\circ}{1000\times 10\times 0.2\times 10^{-3}} = 37.5\,\text{mm}$

(in T_2) [If we assume entire tube of T_2]

Option (1): Since contact angles are different so correction in the height of water column raised in the tube will be different in both the cases, so option (1) is correct.

Option (2): If joint is 5 cm above water surface, then let's say water crosses the joint by height h, then



$$\Rightarrow P_0 - \frac{2T}{r} + \rho g h + \rho g \times 5\times 10^{-2} = P_0$$

$$\Rightarrow \rho g(h \times 5\times 10^{-2}) = \frac{2T}{r}$$

$$\Rightarrow \cos\theta = \frac{R}{r}, r = \frac{R}{\cos\theta}$$

$$\Rightarrow \rho g(h \times 5\times 10^{-2}) = \frac{2T\cos\theta}{R}$$

$$\Rightarrow h = \frac{2\times 0.075\times\cos 60^\circ}{0.2\times 10^{-3}\times 1000\times 10} - 5\times 10^{-2}$$

$\Rightarrow h = -$ve, not possible, so liquid will not cross the interface, but angle of contact at the interface will change, to balance the pressure,

So option (2) is wrong.

Option (3): If interface is 8 cm above water then water will not even reach the interface, and water will rise till 7.5 cm only in T_1, so option (3) is right.

Option (4): If interface is 5 cm above the water in vessel, then water in capillary will not even reach the interface. Water will reach only till 3.75 cm, so option (4) is right.

5. (2.00)

Let T_s = tension in steel wire

T_C = Tension in copper wire in x direction

$$T_C\cos 30^\circ = T_S\cos 60^\circ$$

$$T_C \times \frac{\sqrt{3}}{2} = T_S \times \frac{1}{2}$$

$$\sqrt{3}T_C = T_S \quad \text{...(i)}$$

in y direction

$$T_C\sin 30^\circ + T_S\sin 60^\circ = 100$$

$$\frac{T_C}{2} + \frac{T_S\sqrt{3}}{2} = 100 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii)

$$T_C = 50\,\text{N}$$

$$T_S = 50\sqrt{3}\,\text{N}$$

We know $\Delta L = \dfrac{FL}{AY}$

$$= \frac{\Delta L_C}{\Delta L_S} = \frac{T_C L_C}{A_C Y_C} \times \frac{A_S Y_S}{T_S L_S} = \left(\frac{T_c}{T_s}\right)\left(\frac{L_c}{L_s}\right)\left(\frac{Y_s}{Y_c}\right)$$

On solving above equation

$$\frac{\Delta L_C}{\Delta L_S} = 2$$